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XXIX.—*Description of the Lithoscope, an Instrument for distinguishing Precious Stones and other bodies.* By Sir DAVID BREWSTER, K.H., F.R.S. (Plate XIX.)

(Read 18th January 1864.)

In examining the light reflected from the surface of calcareous spar, when in contact with different fluids, I observed several phenomena, both of light and colour, which led me to make the same experiments on the natural and artificial surfaces of the precious stones and other minerals. In these experiments, the intensity of the reflected pencil varied, as might have been expected, with the refractive power of the fluid; but I was not prepared for the curious fact, that when the reflective power of the surface was reduced almost to nothing, the surface was no longer able to reflect white light, but reflected pencils of different colours, depending on the approximation of the refractive power of the fluid to that of the solid. When the crystal had much double refraction, the colour of the reflected light varied with the inclination of the plane of reflection to the plane passing through the axis of the crystal.

In making these experiments, a drop of fluid is placed upon the surface of the body to be examined, and upon this drop is laid the broad surface of a rectangular prism of glass. A parallel film of fluid will thus be formed between the crystal and the prism, and any luminous image reflected from the two combined surfaces will consist of two images superposed, one reflected from the common surface of the crystal and the fluid, and the other from the common surface of the fluid and the glass. By a slight inclination of the prism, the two images are separated, so that we can examine and compare them, the pencil from the prism and fluid surface being a standard light, with which the pencil reflected from the common surface of other crystals and the same fluid may be compared, in reference to colour and intensity of light.

By the use of fluids, therefore, of known refractive power, we can distinguish precious stones and other minerals, the standard of intensity and of colour being the invariable pencil reflected from the prism and fluid surface. A continuous scale of refractive power, embracing a great variety of minerals and other bodies, may be obtained, by mixing fluids in different proportions. The fixed oils combine readily, not only with one another, but with many of the volatile oils; and the aqueous solutions of alcohol and sugar, will to some extent, be useful auxiliaries in reducing the refractive power of the surfaces on which they are placed.

This method of distinguishing bodies is not limited to those which are solid. By standard prisms of crystals, or of different kinds of glass, we may distinguish fluids of *all* kinds, because the range of refractive powers from tabaheer to

diamond and chromate of lead, is much greater than from water to oil of cassia. In this case, a standard pencil will be obtained from a standard fluid of a known refractive power, placed on a surface of the same glass as the prism, as will be presently explained. Impurities in oils and other substances may be thus detected, and specific gravities approximately ascertained.

In order to make these experiments easily and correctly, I contrived the instrument now on the table, which was constructed for me many years ago by the late Mr DOLLOND, and which may be called a *Lithoscope*, from its application to the discrimination of precious stones.

It consists of a rectangular prism PR, mounted as shown in Plate XIX., where AB is a pillar, carrying the horizontal and vertical branches CD, CE. The prism turns vertically round a joint at E, and may be lifted round that joint by the hand, from its horizontal position, or raised slowly by the milled-head F of a screw FG, resting at G upon the branch CD. A small circular plate H, at the top of a screwed rod I, is made to rise or fall by means of the milled head KL; and has also a motion of rotation round the top of the rod. Two steel rods *de*, *cf*, carry two small aperture tubes *e*, *f*, through one of which, *f*, the incident pencil falls nearly perpendicularly upon one side of the prism, and at an angle of 45° upon its base. These rods, with their tubular apertures, slide through short tubes at *m* and *n*, in order that they may receive the incident and reflected pencils. Bottles containing oil of cassia and other standard oils are placed at *a*, *b*, *c*.

In using the Lithoscope, the mineral upon whose surface the observation is to be made is attached, if necessary, by cement or otherwise to the plate H. A drop of the proper oil is then put upon the surface of the mineral, and the prism brought into a horizontal position. If it does not touch the oil, it is brought into contact with it by the milled head KL, which raises the plate H. A plate or film of oil, with parallel surfaces, is thus formed between the mineral and the prism; and if we now view the image of the sun or of a small bright flame, transmitted through the aperture *f*, reflected from the film, and reaching the eye through the aperture *e*, we shall see one image consisting of two coincident images, the one reflected from the surface of the prism and the oil, and the other from the surface of the oil and the mineral. In order to separate these images, the prism is slightly raised round the joint at E, by turning the milled head F. The prism image will then appear at the right hand of the other image; and by a comparison of the colour and intensity of these images, we obtain the information we desire.

It was by an apparatus of this kind, furnished with a graduated circle, that I discovered the influence of the doubly refracting force in polarising common light in planes inclined to the plane of reflexion. These experiments were published in the "Philosophical Transactions" for 1819, and have led several distinguished mathematicians—Professor MACCULLAGH of Dublin, M. SEEBECK of Berlin, and

Professor NEUMANN of Konigsberg—to important extensions of the Undulatory Theory. I shall have occasion, in another paper, to submit to the Society a series of experiments on the influence of the doubly refracting force in turning the planes of polarised light out of the plane of incidence and reflexion; but at present I confine myself to the consideration of the intensity and colour of the reflected pencils.

The following observations were made chiefly with oil of cassia and oil of anise seeds, on account of their great refractive power, and will be sufficient to show the use and application of the Lithoscope:—

With Oil of Cassia.

DIAMOND.—The colour from the crystal is *yellow*, and the intensity of the pencil *four* or *five* times greater than that of the pencil from the prism. The Diamond may therefore be easily distinguished from *Quartz* and from Glass of very high refractive power, for which it is often mistaken.

ZIRCON.—The colour from the crystal is white, and the intensity of the pencil *two* or *three* times greater than that of the pencil from the prism.

RUBY, *Oriental.*—The colour from the crystal is *yellow* in artificial light, and the intensity of the pencil nearly double that from the prism.

CHRYSOBERYL, *Cymophane.*—The colour from an artificial face in candle-light is *yellowish*, and the intensity of the pencil greater than that from the prism.

BERYL.—The colour from the crystal is *slightly blue*, and fainter than the pencil from the prism.

BLUE TOPAZ.—The colour from the crystal is a *brilliant lilac*, approaching to *blue*, when the plane of reflexion passes through the axis of the prism, and the surface is *artificially polished*. On the same face, and in a plane at right angles to this, the colour of the pencil is *bluish pink*. The intensity of the pencil is two or three times less than that of the pencil from the prism.

Upon a *natural face*, the colour of the pencil is *bright blue* in every azimuth.

Above the polarising angle, the reflected pencil consists of polarised *red* light and of unpolarised *blue* light.

TOPAZ *from Brazil, colourless.*—The colour from a cleavage plane is a brilliant *blue*, in every direction.

TOPAZ, *yellow, from Brazil.*—Upon natural and artificial faces, the colour is *a faint red*, and the image scarcely perceptible. On another specimen, I found the colour to be a *bright pink*, the intensity being much less than that of the pencil from the prism.

CINNAMON STONE.—On an artificial surface of this variety of garnet, the colour of the pencil was *yellowish red*, and its intensity less than that from the prism.

GARNET, *precious.*—The colour of the pencil is *reddish yellow*, and its intensity a little greater than that from the prism.

GARNET, *from Elie, in Fife.*—The colour of the pencil is *reddish yellow*, but the intensity very much less than that from the prism.

PERIDOT.—The colour of the pencil is *brownish yellow*, and its intensity much less than that from the prism.

EUCLASE.—The colour of the pencil is *brilliant yellow*, and its intensity less than that from the prism.

PREHNITE.—The colour of the pencil is *blue*, but brighter than that from topaz.

QUARTZ.—The colour of the pencil is a *faint blue*, both on natural and artificial faces, and its intensity less than that from the prism.

AMETHYST.—The colour of the pencil is a *faint blue*, as in quartz, and its intensity less than that of the prism.

DICHROITE.—The colour of the pencil is a *blue*, fainter than in quartz, and its intensity less than that of the prism.

BLACK TOURMALINE.—The colour of the pencil in a plane perpendicular to the axis is a *brownish yellow* in candle-light, and its intensity much less than that from the prism.

AUGITE.—The colour of the pencil, in candle-light, is *yellow*, and its intensity less than that from the prism.

SULPHATE OF BARYTES.—The colour of the pencil is *brick-red*, less faint than in topaz, and the same at all incidences and in all azimuths. The intensity of the pencil is very much less than that from the prism.

SULPHATE OF LEAD.—The colour of the pencil, in candle-light, is *brilliant yellow*, and its intensity twice as great as that from the prism.

LEELITE.—The colour of the pencil is *greenish blue* when the oil is warm, the intensity of the pencil increasing with the incidence.

JET.—The colour of the pencil is a very faint *yellowish red*, and the same at all incidences.

The following experiments were made with *Oil of Anise Seeds*, a less refractive oil than *Oil of Cassia*:—

QUARTZ, on a face of the prism. When the plane of reflexion passes through the axis, the image of a candle is just perceptible, and of a *brick-red* colour.

In sun-light the colour is *blue*, of little intensity, but *pink* when the oil is first put on!

When the plane of reflexion is perpendicular to the axis, the image of a candle is distinctly visible, and of a *brick-red* colour. In sun-light the colour is *red*, but yellow when the oil is first put on.

On a face perpendicular to the axis of the crystal, the colour in sun-light is *blue*.

On a face of the pyramid, the colour is *pink* in a plane passing through the axis, but *light red* in a plane perpendicular to the axis.

AMETHYST.—The colour of the pencil is a *fine blue* at all incidences and in all azimuths.

DICHROITE.—The colour of the pencil varies from *reddish yellow*, when the oil is warm, to a *bluish* colour when the oil is cold, the intensity being the same at all incidences.

OPALS, *Common and Precious*.—The colour of the pencil a *pale yellow*, but bright.

BLOODSTONE.—Colour of the pencil *lemon yellow*.

LEELITE.—Colour of the pencil *purple*, becoming *bluish* at great incidences.

LABRADOR FELDSPAR.—Colour of the pencil *sky blue*.

ROCK SALT.—The colour of the pencil *blue*, at all incidences.

The following experiment was made with *Sulphate of Lime*:—

With *Balsam of Capivi* the pencil was colourless, and faint at all incidences.

With *Castor Oil*, the pencil was colourless at all incidences.

The following experiments were made on *Quartz* with pencils of polarised light, O and E, polarised in planes + and − 45° to the plane of reflexion:—

On the face of the *prism*, and in a plane passing through the axis of the crystal, with *Canada Balsam*, E was *orange yellow*, and not so bright as O, E being polarised at a greater incidence than O.

In a plane at right angles to this O=E, and both vanish together.

On the face of the *pyramid*, and in a plane passing through the axis of the crystal, O is *bluish* and E a *straw yellow*, O being much brighter than E, and both vanishing together.

In order to reduce the reflective power of the *quartz*, I mixed *Oil of Sassafras* and *Oil of Anise Seeds*, in such proportions that the combination had nearly the same refractive power as the mineral. The image of a candle placed a few inches from the reflecting surface was scarcely visible.

In sun-light, however, and on a face of the prism in a plane passing through the axis, the pencil E was a bright *greenish blue* at incidences below the polarising angle, and it consisted of two pencils of different colours, the one a *bright blue*, polarised in the plane of reflexion, and the other a *yellow*, polarised in a plane perpendicular to it.

In a plane perpendicular to that passing through the axis, the colour of E was a *bright purple* at various incidences.

On the face of the pyramid, the colour was *blue* in the one plane, and *purple* in the other.

The use of the Lithoscope, in distinguishing both solids and liquids, may be extended in two ways—

1*st*, By dividing the reflecting surface of the prism into two, three, or more parts, by means of grooves, so that two, three, or more oils may be used at once; and,

2*d*, By using a prism composed of two, three, or more portions of glass with different refracting powers.

By either of these means, or by the two in combination, the experimental results may be more readily and accurately compared.

In the preceding experiments our attention is called to two different phenomena—the intensity and the colour of the reflected pencil. When the pencil is colourless, which is generally the case when one of the surfaces is that of glass, its intensity depends on the index of refraction of the surfaces in contact, which is always equal to the quotient of the greater index of refraction divided by the lesser.

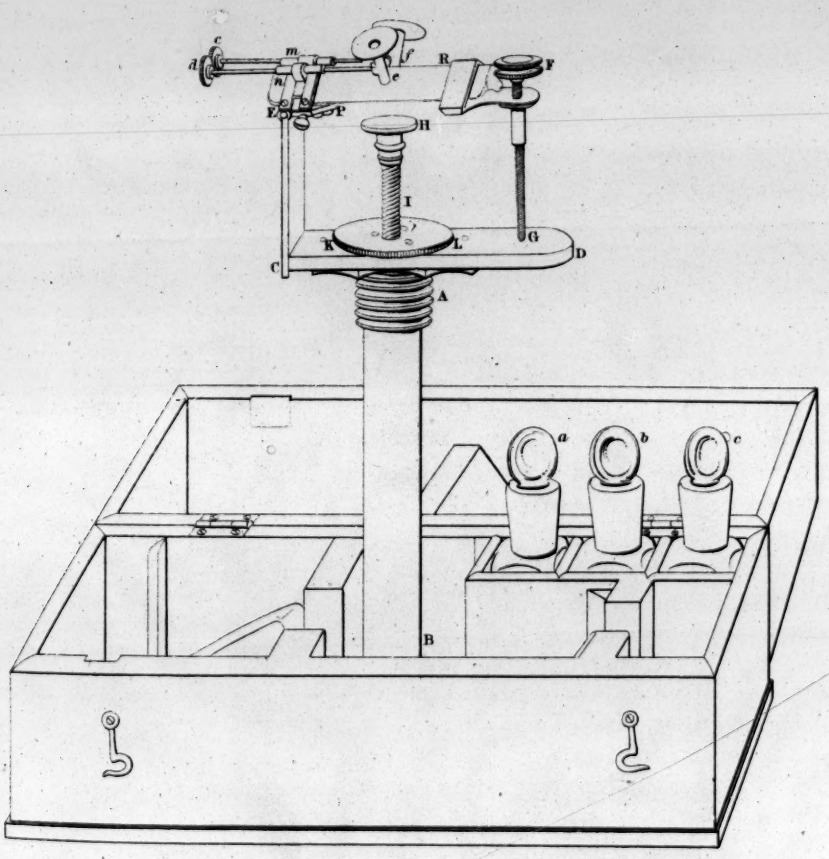
The phenomena of colour produced by feebly reflecting surfaces, and first described in my paper of 1819, already referred to, arise from two different causes—

1*st*, In doubly refracting crystals, from the influence of the doubly refracting force in turning through different angles the planes of the polarisation of the differently coloured rays; and,

2*d*, In the same class of bodies, and in those which have no double refraction, from the different dispersive powers of the surfaces in contact, and also from the irrationality of dispersion, in consequence of which the coloured spaces in different spectra have not the same proportion to each other.

To the subject of crystalline reflexion in which the phenomena of colour are exhibited, I shall have occasion to direct the attention of the Society in another paper. When the colour is produced by difference of dispersive power, or by irrationality in the spectra, it is easily explained. In a combination, for example, of flint glass and oil of cassia, the index of refraction for red light is the same in both bodies, but different for all the other colours. The *red* light, therefore, in a perfectly colourless beam, will pass through the surface of the oil and the glass, without suffering reflexion; and, consequently, the light reflected must be *bluish*, while the transmitted light will be *yellowish*. As there is no reason for believing that in any two bodies, whether gaseous, fluid, or solid, the dispersive powers are exactly the same, or the coloured spaces exactly proportional, we may assert that when pure white light is either transmitted by, or reflected from, bodies perfectly colourless, the transmitted and reflected pencil must be coloured, however inappreciable the colour may be.*

* See "Memoirs of the Life and Writings of Sir Isaac Newton," vol. i. p. 163. Second edition.



W. & A. K. Johnston, Edinburgh.

XXX.—*On the Agrarian Laws of Lycurgus, and one of Mr Grote's Canons of Historical Criticism.* By Professor BLACKIE.

(Read 4th January 1864.)

The History of Greece, by Mr GROTE, perhaps the most notable production of modern English scholarship, is characterised, amongst many great virtues, by what has always appeared to me, in a historian, a great fault—a tendency to undervalue traditional authority, and to over-rate the importance of conjectural ingenuity, in the reconstruction of the past. One of the most remarkable instances of this tendency which has fallen specially under my view, is his treatment of LYCURGUS and his legislation, as it occurs near the end of his second volume. The fallacies which seem to me to lie in this treatment, it is the object of this paper shortly to set forth.

Mr GROTE's views with respect to the Spartan laws and customs are sufficiently indicated by his general proposition, vol. ii. p. 515, that "LYCURGUS, or the individual to whom this system was owing, whoever he was, is the founder of a warlike brotherhood, rather than the lawgiver of a political community;" and by his special assertion (p. 524) that "the idea of LYCURGUS as an equal partitioner of lands, belongs to the century of AGIS IV. and CLEOMENES;" that is, to the middle, and towards the end of the third century before Christ, and is to be regarded altogether as a political fiction, not as a historical fact.

Now, by what method of inquiry does the learned gentleman arrive at this conclusion? His statement of his method, on the first blush, seems remarkably fair and reasonable. On examining the witnesses for this alleged historical fact, he has discovered that all the weighty authorities who live nearest to the event are silent on the subject, or even directly contradict the current modern belief, which can in fact be traced to only two of the most recent and least reliable authorities. The canon implied in this method of historical criticism is no doubt, taken broadly, perfectly just. But all such canons, in their application, are liable to be seriously modified by various considerations. In the first place, with respect to the fact here disputed, we must bear in mind that anything like authentic cotemporary evidence is altogether out of the question; and so far as nearness to the time in which the alleged fact took place is concerned, ARISTOTLE is a witness not a whit more reliable than POLYBIUS. According to the lowest calculation, that of THUCYDIDES (i. 18), LYCURGUS flourished 400 years before the end of the Peloponesian war, that is about 800 years B.C.; and if, as the historian asserts, the Lycurgean laws remained in force during this period, their character at the

time of their original institution, as contrasted with any changes which, in the declension of Spartan influence, they afterwards suffered, could no more be known as an existing fact by ARISTOTLE than by POLYBIUS, who lived nearly 200 years later. There was no such destruction of books and documents in the period intervening between these two great political writers, as to put the one in a more favourable position with regard to written testimony than the other; nay, so far as parchment documents were concerned, from the accumulated stores of the Alexandrian Library, POLYBIUS had, in all likelihood, much more ample means of information at hand than ARISTOTLE. POLYBIUS, therefore, who* along with PLUTARCH, in his Life of LYCURGUS, is the principal direct and positive authority for the Agrarian laws, as a part of the Spartan constitution, is, so far as the lapse of time is concerned, a witness by no means to be postponed even to the Stagyrite. As little in respect of any other virtue of a historical authority can any such inferiority be maintained. There is, indeed, no historical author among the ancients, after THUCYDIDES, whose political sagacity and penetration is more generally acknowledged; and the book in which his remarks on the Spartan constitution occur is expressly directed to the comparison of different forms of civil polity; besides, as a native of Megalopolis in Arcadia, and living in the age immediately succeeding the great Agrarian movements of AGIS and CLEOMENES, he had the most ample means of being locally well informed as to the characteristic points of the Spartan constitution. As to PLUTARCH,† though I may not deny that his account of the Spartan institutions is tinged strongly with rose colour, yet I can by no means share in the light temper of those who habitually talk of the wise old Cheronæan as an amiable, indeed, but superficial and ignorant writer. I think it plain, on the contrary, that his writings everywhere bear the stamp of good sense, of just discrimination, of honest research, and various reading. His point of view, if not always leading to a perfectly just appreciation of his object, was the point of view of the ancient world generally; and on no occasion, so far as my reading goes, can he be suspected of having lightly committed himself to a serious historical assertion. But whatever may be the particular weak points of this most agreeable and most popular, and now most undeservedly neglected writer, we must bear in mind, that, in reference to such a matter of old historical belief as the Spartan constitution, what he does give us, is not to be regarded as his opinion merely, but as the condensed and concentrated result of all the historical testimony that had come through his hands; and as he constantly quotes

* Τῆς μὲν δὴ Λακεδαιμονίων πολιτείας ἴδιον εἶναί φασι, πρῶτον μὲν τὰ περὶ τὰς ἐγγαίους κτήσεις, ὧν οὐδαμὰ μέτεστι πλεῖον, ἀλλὰ πάντας τοὺς πολίτας ἴσον ἔχειν δεῖ τῆς πολιτικῆς χώρας.—vi. 45.

† Δεύτερον δὲ τῶν Λυκούργου πολιτευμάτων καὶ νεανικώτατον ὁ τῆς γῆς ἀναδασμός ἐστι. Δεινῆς γὰρ οὔσης ἀνωμαλίας καὶ πολλῶν ἀκτημόνων καὶ ἀπόρων ἐπιφερομένων τῇ πόλει, τοῦ δὲ πλούτου παντάπασιν εἰς ὀλίγους συνερρυηκότος, ὕβριν καί φθόνον καὶ κακουργίαν καὶ τρυφὴν καὶ τὰ τούτων ἔτι πρεσβύτερα καὶ μείζω νοσήματα πολιτείας, πλοῦτον καὶ πενίαν, ἐξελαύνων, συνέπεισε τὴν χώραν ἅπασαν εἰς μέσον θέντας ἐξ ἀρχῆς ἀναδάσασθαι καὶ ζῆν μετ' ἀλλήλων ἅπαντας ὁμαλεῖς καὶ ἰσοκλήρους τοῖς βίοις γενομένους.—Vit. Lyc. viii.

ARISTOTLE, it is not to be doubted, that in composing two such important Lives as those of LYCURGUS and SOLON, he had before him that very book of πολιτεῖαι, or political constitutions, in which the Spartan polity was fully discussed, but of which now, unfortunately, only a very few fragments remain. So far, therefore, as direct positive evidence is concerned, I consider that we have in POLYBIUS and PLUTARCH, two independent witnesses of as good faith and authority as can be adduced for any historical fact of similar antiquity. And let it be remembered, that the fact here in question is not a small incidental matter, or a piece of personal gossip, which might easily have been ignored altogether, and with equal ease have been blown into existence out of nothing; it is a grand central fact with regard to the social condition of a people who played, in ancient Hellenic life, a part equally prominent and peculiar, and which, if true at all, must have been as well known to the ancient Greeks, as the aristocratic refinement of the Episcopal, and the democratic plainness of the Presbyterian Church are to modern Christendom. On the basis of these two authorities alone, therefore—one of which, by the way, the Edinburgh Reviewer of Mr GROTE's work, with a characteristic itch for sceptical novelties, most superficially ignores (Edinr. Review, vol. lxxxiv. p. 371,)—on the testimonies of POLYBIUS and PLUTARCH alone, I see no reason why we should hesitate to receive the famous Agrarian laws of LYCURGUS as one of the most reliable traditions of the ancient world. Of course, I do not mean to say, that, while accepting this general fact on the testimony of two such writers, I mean to adopt along with it all the dressings and trappings with which it has been tricked out in the course of time. Every one conversant with historical evidence knows that the fact is true in a hundred cases—witness the story of Macbeth,—where the decorations of the fact are fabulous. I build, therefore, as little as Mr GROTE on the details of these Agrarian laws as given in the Life of LYCURGUS; but while willingly with him tearing away the ornate frippery, and striking off the painted gauds that envelope the old sacred image, I deny his right to conclude that the piece of fine old carved wood which lies beneath the dress is a mere fiction and a phantom. But let us now consider those more ancient witnesses, in deference to whom only, as he would have it appear, this distinguished writer has thrown the valuable testimony of PLUTARCH and POLYBIUS aside. Of these, the most formidable, indeed the only important one, is ARISTOTLE, who, in the second book of his master-work (chap. ix., Bekker) on Political Science, has described some points of the Spartan polity with considerable detail, and among others, in alluding to this very matter of the distribution of landed property, not only does not confirm what the other two authorities say about the equal ἀναδασμός of the lands, to them so striking and essential a fact—but actually asserts the direct contrary; among other views of the Spartan polity, enumerating as one of the chief, the diminution of small proprietors, and the accumulation of immense landed properties in the hands of a few persons, especially women. This is no doubt

a very startling assertion to a reader coming fresh from the somewhat Utopian descriptions of Spartan simplicity, equality, and fraternity, in the kindly pages of old PLUTARCH. But this is not the only startling assertion about Spartan things and Spartan persons that presents itself in the same important chapter; for, whereas STRABO informs us (x. p. 449) that there was a common proverb current in Greece, "A horse from Thessaly, a woman from Sparta, and a man from the banks of the fair flowing Arethusa," whereby the best article of the several kinds was expressed, the Stagyrite, on the other hand, here allows the famous Spartan mothers no characteristic qualities, but three of the worst,—great luxury, unbounded extravagance, and, what no Athenian could tolerate for a moment, even in conception, an unwomanly mastery over their husbands. These strong statements, coming from a man generally so calm and cool and judicially impartial, naturally suggest to the thoughtful student that the great father of Encyclopædic science is betrayed for a moment into a little fit of amiable human weakness, and is speaking, not as a philosopher altogether, but partly also as an Athenian. But more than this. What ARISTOTLE here says with such decision was no doubt strictly true,—for it is not for us to pretend to find him nodding as to a matter of fact,—but it is not the whole truth. We can only explain his strong, one-sided, and, as it appears to the reader, extremely prejudiced language, by supposing, what is no doubt the fact, that he is speaking only of the Spartans of his day, not of the days of LEONIDAS or of LYCURGUS, and is no more to be considered as giving a general portraiture of Sparta and the Spartans, than a THACKERAY of the time of TACITUS would have had his sketches of Roman life under the Emperors taken for the living counterfeits of the CATOS, the SCIPIOS, and the FABII of the Punic wars. And this view of the matter will appear the only natural and just one, so soon as the reader has taken up the position and attitude of the great author of the Politics, in this second book, as distinctly enunciated in the very first sentence of this section of the work.* The intention of the work, the author declares, is to ascertain tentatively, and by approximation at least, the best polity, or, as we would phrase it, the ideal commonwealth; but as it might appear impertinent to attempt this if the best form of government had already been realised, the philosopher thinks it only reasonable that he should preface the exposition of his ideal plan by showing that the most bepraised commonwealths already existing, so far from being perfect models, are bristling with glaring defects, which a passing glance may readily discover. ARISTOTLE, therefore, by the very conception of this book, has put himself into the not very philosophical position of a systematic fault-

* Ἐπεὶ δὲ προαιρούμεθα θεωρῆσαι περὶ τῆς κοινωνίας τῆς πολιτικῆς, ἣ κρατίστη πασῶν τοῖς δυναμένοις ζῆν ὅτι μάλιστα κατ' εὐχήν, δεῖ καὶ τὰς ἄλλας ἐπισκέψασθαι πολιτείας, αἵς τε χρῶνταί τινες τῶν πόλεων τῶν εὐνομεῖσθαι λεγομένων, κἂν εἴ τινες ἕτεραι τυγχάνωσιν ὑπὸ τινῶν εἰρημέναι καὶ δοκοῦσαι καλῶς ἔχειν, ἵνα τό τ' ὀρθῶς ἔχον ὀφθῇ καὶ τὸ χρήσιμον, ἔτι δὲ τὸ ζητεῖν τι παρ' αὐτὰς ἕτερον μὴ δοκῇ πάντως εἶναι σοφίζεσθαι βουλομένων, ἀλλὰ διὰ τὸ μὴ καλῶς ἔχειν ταύτας τὰς νῦν ὑπαρχούσας διὰ τοῦτο ταύτην δοκῶμεν ἐπιβαλέσθαι τὴν μέθοδον.—Pol. ii. init.

finder; his scheme in this introductory chapter does not in any wise require that he should give a perfect account of the rise and growth, and complete development of the Spartan commonwealth, but only that he should show how, in the result, it has found itself very far removed from the model commonwealth, which XENOPHON, PLATO, and other anti-Athenian laudators upheld it to be. "By their fruits ye shall know them." If the harvest has brought forth rotten apples, the spring may have been genial,—we are not careful to inquire about that; but there must have been something wrong somewhere, either in the seed or in the summer. And not only does the philosopher content himself with indicating what time has shown to be rotten in the Spartan constitution, but he incidentally drops a remark, which receives its full significance only on the supposition of an original state of things altogether different from that which he is criticising. For, while discussing the point of the undue accumulation of landed property in his time, he blames the legislator for forbidding or discouraging the sale of lands by the hereditary holder, while at the same time, he allowed them freely to pass from family to family by testamentary conveyance—a liberty by which his prohibition was rendered in a great measure nugatory. Now, this remark plainly implies the existence, in ancient times, of a state of things in which every Spartan citizen, like the old Hebrew yeoman, had his own ancestral allotment of a certain moderate size—for there is no need of supposing mathematical equality—which, through the operation of an ill-regulated law of succession, as ARISTOTLE thought, had in the course of time produced the monstrous accumulation of property in a few hands which he considers so pernicious.

The evidence of the great Encyclopædic philosopher being thus placed, as it is hoped, in perfect harmony with the testimonies of PLUTARCH and POLYBIUS, the other witnesses, on whom the English historian founds his sceptical views, may be shortly dismissed. Their weight in the present question in fact amounts to nothing more than this, that certain authors, who might have been expected to allude to the Agrarian laws of Sparta, say nothing about them. But this sort of evidence, or rather want of evidence, in the case of writers who are not writing formal treatises on the Spartan polity, proves nothing. A modern writer, for instance, might say many true things of the social state of modern France, and yet not once allude to the very important matter of the compulsory division of landed property in that country after the demise of the holder. Not a few things in ancient writers about ancient matters are not mentioned, simply because they were so well known as not to require to be spoken about. A striking proof of this occurs, I think, in the Panathenaic oration of ISOCRATES, which contains a detailed comparison of Sparta and Athens, but where the Agrarian laws are alluded to only incidentally in a single clause, which Mr GROTE has somewhat strangely overlooked; nay, rather he quotes a passage from the oration which, next to the evidence of ARISTOTLE, seems to make most in favour of his sceptical

position. For, towards the close of the discourse (p. 287) the orator makes one of the advocates of Spartan usages say, "that whereas every one of the other states of Greece had been disturbed by frequent aristocratic and democratic revolutions, in Sparta alone no one could point out either mutinies or massacres, or unjust banishments, or confiscations of property, or violations of women, or abolition of debts, or change in the form of the constitution, or Agrarian laws (γῆς ἀναδασμούς), or any of the incurable diseases to which bodies politic are subject." True; but this means only what we have already heard from THUCYDIDES, that the constitution, as settled by LYCURGUS, had remained stable in its great supports —a remarkable phenomenon in ancient Greece—for four hundred years, undisturbed by violent changes and sudden revolutions in the ownership of property; but that the Spartan land, at the great Dorian immigration, was originally divided into equal lots, is incidentally asserted by the orator in a previous part of his discourse, p. 270 (τῆς χώρας ἧς προσῆκεν "ισον "εχειν ἑκαστον), with which passage the more vague language of PLATO in the laws (III. 684 D.) is perfectly consistent.

The silence of XENOPHON on the subject of the Spartan Agrarian laws might appear, at first sight, more difficult to account for. In a special treatise, such as the little book περὶ Λακεδαιμονίων πολιτείας unquestionably is, a characteristic feature of national life so prominently mentioned by PLUTARCH and POLYBIUS, might naturally have been expected to find a place. But when the extremely slight and flimsy character of this treatise is considered, and when the special fact is also borne in mind, that, though professing to give some idea not only of the Spartan education, but of the Spartan political constitution, the existence of the ἐκκλησία or congregation of the Commons, is not even mentioned; and when we consider farther, that the inequality of land in Lacedæmon, so glaring to the eye of ARISTOTLE, must already, in the days of XENOPHON, have become so great as to render the Lycurgean legislation on this point a matter more of historical learning than of present social significance, we must hold the silence of this writer on this particular point to be a matter of the smallest moment, certainly not of sufficient weight to countervail the positive and distinct testimony of such a grave and judicious writer as POLYBIUS.

So much for the authorities, or what theologians call the external evidence. As to internal probabilities and presumptions, however our modern British ideas may incline us to look with a certain suspicion on all accounts of social equality in the tenure of land, it is quite certain, not only that the ancients generally held a land-tenure essential to citizenship, but that a certain equality in this respect was looked on as the sign of a healthy condition of the body politic. The whole history of Agrarian laws amongst the Romans was only a violent exposition of one of the oldest principles of ancient citizenship, that the land which had been acquired by the exertions of common brothers in arms, should be possessed by the co-ordinate ownership of equal citizens. But in the case of the Spartans it

seems to me further, that their position as a small band of privileged nobles—for the proper Spartans were in fact a body of nobles, with exclusive privileges, as set against the great mass of the population—this difficult and slippery position, I say, rendered it in the highest degree perilous that any divisions should arise within the principal body; and such dissensions would naturally arise in those days, and in a country where landed property was the only valuable property, if the small section of the country originally occupied by the invaders had been absorbed by a few of the more powerful families, while the great mass was left with small allotments, which, by the action of well-known social laws, would have a tendency constantly to diminish. It is to dissensions of this kind, I conceive, that both PLUTARCH (chap. 2) and ISOCRATES (p. 270) allude, when they talk of the period of disorder and confusion which prevailed in Sparta before the final settlement of the constitution by LYCURGUS; and we may justly conceive the mission of that legislator to have consisted in adjusting this matter by an Agrarian law, as SOLON effected a similar compromise between the claims of debtor and creditor two hundred years later in Athens, or as Baron STEIN, after the battle of Jena (in 1808), under the pressure of a great national calamity, deprived the Prussian nobility of a great part of their landed property, and created a race of independent peasants from the appropriation. Of course, I do not mean to say to what extent the principle of absolute equality in the possession of the national acres might have been systematically carried out by the great Lacedæmonian lawgiver. Of that we can know nothing. All we can say is, that in accordance with the general spirit of ancient citizenship, and as a measure absolutely necessary for the security of a small body of nobles so situated, he would insist that every citizen, *quâ* citizen, should have his sufficient allotment; and as in those early times, before the conquest of Messene, the amount of really valuable land in Lacedæmon was not large, it seems impossible that any very large properties could have been allotted; and thus the very necessity of the case would dictate a practical equality, which historians, writing in times of monstrously accumulated wealth, might easily work up into a sort of mathematical marvel, to a British critical intellect in the nineteenth century after Christ altogether incredible.

These are the views to which, after much consideration, I have arrived on this interesting subject; and I feel a strong conviction that they are the views, which with the sound practical intellect of the British people, constitutionally averse to all conjectural novelties, however brilliant, will ultimately prevail. In the meantime, I shall not be surprised if the authority of Mr GROTE's name in matters where he is a safe guide, shall lead the majority astray for a season in matters where he is most unreliable. Generally, with regard to his whole method of estimating the contents of early historical tradition, I consider him to be unsound. As concerns the special matter of the Agrarian laws, I shall conclude here by protesting against the new canon of historical criticism, which he enunciates formally while

discussing the matter (vol. ii. p. 532). His words are as follows:—"It appears to me that the difficulties connected with this matter are best obviated by adopting a different canon of historical interpretation. We cannot accept as real the Lycurgean land-division described in the life of the lawgiver; but treating the account as a fiction, two modes of proceeding are open to us. We may either consider the fiction, as it now stands, to be the exaggeration and distortion of some small fact, and then try to guess without any assistance what that small fact was; or we may regard it as a fiction from first to last—the expression of some large idea and sentiment, so powerful in its action on men's minds at a given time, as to induce them to make a place for it among the realities of the past. Now the latter supposition, as applied to the times of AGIS IV., best meets the case before us." In regard to this new canon, I object, in the first place, to the language which states that as altogether a guess, which is merely the acceptance of a well-attested fact, the details of which only are a matter of exaggeration and conjecture. The only guess in the present case which touches the nucleus of a fact, is Mr GROTE'S, who, following the evil example of the Germans, has here smuggled into the sober pages of history an ingenious but altogether baseless conjecture, for the solid foundation of old authority. In the next place, I would lay it down as one of the most important principles to be borne in mind in the interpretation of all tradition, written or unwritten, to use the emphatic language of Professor DUNCKER, that "historic fantasies are not wont to arise without historic realities;" and the bare fact that AGIS and CLEOMENES in the third century before Christ based their famous but unfortunate schemes of reform in Sparta on alleged Agrarian laws of LYCURGUS made five hundred years before, is to me a strong proof that such laws did once actually exist. A reforming king, who, as a last resource, rouses the social conscience of a nation against the usurpation of an aristocracy, or a bureaucracy, must appeal, not to a pious dream or a brilliant imagination, but to a living record imprinted from generation to generation in the fleshly tables of the popular heart. Only by the touch of what is living can a living virtue be imparted to the dead.

XXXI.—*On the Limits of our Knowledge respecting the Theory of Parallels.*

By Professor KELLAND.

(Read December 21, 1863.)

The subject of this paper is a very old one, and may to many appear to be sufficiently worn; but I venture to hope, that there are some to whom a glimpse of the successive approaches of the human mind towards the right understanding of a question of pure logic, may have an interest,—even although the problem solved be an abstract one, and the conclusion a negative conclusion, having little practical application. Like the kindred problem of the quadrature of the circle, or the metaphysical problem of "Knowing and Being," the theory of parallels has been attacked in various directions, and although it is true that no one ever reached the goal he aimed at, yet it is not the less certain that great and positive results have followed in the history of human attainment. If no other lesson has been learnt, this at least may have been: that in reasoning it is necessary to look warily around and abroad at every step, seeing that admissions, the most obviously inadmissible, or evasions the most palpable, have foiled generations of thinkers, whilst those who have detected their errors have fallen into others of an equally destructive character.

It is not my intention to give an account of the successive failures of different geometers in their attempts on this problem. That has already been done by Colonel THOMPSON, in his "Geometry without Axioms." My object will be rather to show what has been successfully accomplished, and by going over in a *positive* form the ground which is forbidden to those who attack the problem directly, to indicate as clearly as I can the limits within which future research may be confined.

I say I am going over the negative limits of the discussion of the problem in a positive form. What I mean by this statement is, that I am about to start with the assumption, as though it were an axiom, of that very problem the incorrectness of which it is the object of future geometers to demonstrate, and by a purely logical process, to ascertain whither those false premises (if they be false) shall lead me. I am much mistaken if those who give themselves the trouble to examine the argument do not find it both interesting and instructive. So far as I know, it has never yet been developed in this country, although the circumstance that for the last seventeen years I have made it the repeated subject of Lectures and Essays in my Class may possibly take from it some of the appearance of

novelty which it would otherwise present. In truth, many of the following propositions are due to my students.

For the sake of clearness, I shall divide the different steps of the argument into propositions. LEGENDRE, in the 12th volume of the *Memoires del'Institut*, proved,—

PROP. I. *The sum of the angles of a triangle can never exceed two right angles.*

PROP. II. *If the angles of any one triangle can be proved to be equal to two right angles, then the angles of every triangle can.*

These two propositions reduce the difficulty to the very narrow requirement of proving that some one triangle has the sum of its angles equal to two right angles. As a *limit*, it is evidently true. For if CD be perpendicular to CA; and if CD be very great and CA indefinitely small, the angles of the triangle CAD approach two right angles as their limit. But this, as we shall see presently, proves nothing. For although they approach two right angles when CA is indefinitely diminished, they may, for anything that appears, approach a right angle and a half, or any similar magnitude, when CA is indefinitely increased.

Further, Mr MEIKLE has proved in the 36th volume of the Edinburgh Philosophical Journal—

PROP. III. *Triangles which have their areas equal have the sum of their angles the same.*

We shall consider these three propositions as established beyond question, and refer to them in the order here given.

Some years ago, there appeared in CRELLE's Journal, a notice of a work entitled "Imaginary or Impossible Geometry," viz., a discussion of the conclusions which would follow from the assumption as an axiom, of the hypothesis that "the three angles of a triangle are together less than two right angles." I have never met with any statement of the propositions which the author deduced from this hypothesis; but I have been accustomed, from time to time, to draw conclusions from the same hypothesis, and to induce my students to follow my example, so that, from one source and another, I believe I am in possession of most of those conclusions which are likely to bear on the theory of parallels. And as these conclusions are both curious in themselves, from their connection with the properties of the circle, and appear to point to the limits of our knowledge of the doctrine of parallels, I have thrown them together in the form of a sequence to the three propositions above enunciated.

It must be premised, that all the definitions and axioms of EUCLID are retained,

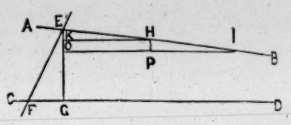
so far as they are logically correct, and make no use of the hypothesis of parallelism. As a matter of *convenience*, it is best to define a straight line, with PLAYFAIR, in the following way:—"If two lines are such that they cannot coincide in any two points without coinciding altogether, each of them is called a straight line;" for EUCLID's definition has no significance in a logical system. It is never referred to by EUCLID himself, nor indeed, could it be, seeing that it expresses nothing. The real characteristics of straightness are contained in two postulates, or, as SIMSON designates them (and it is as well to refer to SIMSON), axioms. They constitute SIMSON's 10th and 11th axioms, thus breaking into three parts—a definition and two axioms—what PLAYFAIR has reduced to one. EUCLID's definition of a square must of course be rejected. It is essentially vicious, involving both a superfluity and a want of necessary consistency. It is equivalent to the assumption, that the angles of a triangle are together equal to two right angles, or the alternative, which is demonstrably false, that a triangle may have its angles together greater than two right angles.

It is further premised, that all the Propositions of EUCLID, up to the 28th inclusive, are correctly demonstrated, and are clear of any assumption relative to the angles of a triangle, or to the doctrine of parallels. In the 29th Proposition, EUCLID has to convert the 27th and 28th, or, in other words, to show the consequences of starting with the hypothesis that two straight lines do *not* meet. The difficulty consists in deducing *positive* consequences out of *negative* premises. And the difficulty is further increased by the fact, that straightness is only known from the necessity that two lines *do* meet, whilst parallelism is only known from the necessity that they *do not* meet. This difficulty renders it imperative on EUCLID to make some additional assumption. His assumption or postulate is what SIMSON calls the 12th axiom: "If a straight line meets two straight lines, so as to make the two interior angles on the same side of them, taken together, less than two right angles, these straight lines, being continually produced, shall at length meet upon that side on which are the angles which are less than two right angles." The object of all that has been written on the subject of parallels has been to get rid of this assumption. I shall not even allude to the history of this subject. I am concerned only with that form of the argument which depends on the sum of the angles of a triangle. It has been stated above, that LEGENDRE has narrowed the requirements to the discovery of some one triangle, for which the angles shall be together equal to two right angles, by having proved that, if this be true, the same holds good of every triangle. It may perhaps be as well to add that this, once established, leads directly to EUCLID's 12th axiom.

The following appears to be the most simple and satisfactory manner of establishing this conclusion:—

PROP. IV. *Given that the angles of every triangle are together equal to two right angles, to prove Euclid's 12th axiom.*

Let AB, CD make with EF the angles BEF, EFD less than two right angles; AB, CD, shall meet, if produced, towards B, D.



Let EFG be not greater than a right angle. Draw EG perpendicular to FD. Set off equal distances along EB, viz., EH, HI, &c., and draw HK, IO, &c., perpendicular to EG; and HP perpendicular to IO.

The angles BEG, GEF, EFG, are less than two right angles, but GEF, EFG, make up a right angle; therefore BEG is less than a right angle. Now EHK and HEK make up a right angle, and EIO, IEO make up a right angle, therefore EHK = EIO; and the triangles EHK, HIP, are equal (EUC. I. 26), therefore HP = EK; but the triangles KHO, HOP, formed by joining HO, are equal (EUC. I. 26), therefore KO = HP = EK. It follows, therefore, that as we advance by equal distances along EB, we also advance by equal distances along EG; so that by going far enough along EB, we must at last advance beyond G; hence there is some point in EB produced, from which, if a perpendicular be drawn to EG produced, it shall cut EG produced beyond G; AB, therefore, meets CD, towards B, D.

Let us now examine the consequences of assuming as true the following

AXIOM. *The three angles of every triangle are together less than two right angles.*

It may be remarked, that it would have sufficed to have assumed that the three angles of some one triangle are together less than two right angles. For, that being admitted, the axiom as enunciated follows at once; inasmuch as no triangle can have the sum of its angles greater than two right angles (Prop. I.), neither can any triangle have them equal to two right angles, for then the same would be true of every triangle (Prop. II.)

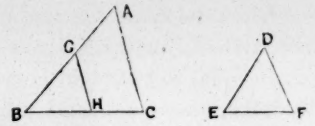
Cor. 1. The exterior angle of every triangle is greater than the sum of the two interior opposite angles.

Cor. 2. The angles of a quadrilateral are together less than four right angles.

We shall designate the deficiency of the angles of a given triangle from two right angles by the term *angular defect* of that triangle; and shall abbreviate the phrase "angular defect of the triangle ABC" by δABC: hence $\delta ABC = 180^\circ - (A + B + C.)$

PROP. V. *If two triangles have two angles of the one equal to two angles of the other, each to each, but the side adjacent to the equal angles of the one greater than the side adjacent to the equal angles of the other, then the other sides of the triangle which has the greater adjacent side are respectively greater than the other sides of the other triangle, but they contain a less angle.*

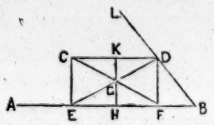
Let $\angle ABC = DEF$, and $\angle ACB = DFE$, but $BC > EF$, then is $BA > ED$, and $AC > DF$. Cut off from BC, $BH = EF$; and make the angle $BHG = EFD$ or BCA; HG shall cut BA in G; for if it do not cut BA it must cut CA, and make with HC and CA a triangle, having the exterior angle equal to the interior opposite angle, which is impossible. Now (EUC. I. 26) $BG = ED$, but $BA > BG$ $\therefore$ $BA > ED$. Similarly $AC > DF$. Also the angle BAC is less than EDF.



For $BGH + BHG + AGH + CHG$ = four right angles; but $GAC + ACH + AGH + CHG$ are less than four right angles (Axiom Cor. 2); therefore $BGH > BAC$, and $BGH = EDF$, therefore $EDF > BAC$.

PROP. VI. *If from two points without a straight line, on the same side of it, the two perpendiculars drawn to the line are equal, the straight line which joins the points is parallel to the given line.*

Let AB be the given straight line, C, D the given points without it on the same side, from which the two equal perpendiculars are drawn to the line, viz., CE, DF; CD is parallel to EF.



Join CF, ED meeting in G; and through G draw GH perpendicular to AB, and produce HG to meet CD in K.

The triangles CEF, DFE, are equal in every respect, therefore $CF = DE$, and $\angle CFE = DEF$: hence the triangles GHE, GHF are equal (EUC. I. 26), therefore $EG = FG$, and $\angle EGH = FGH$: hence, by subtraction, $CG = GD$; and $\angle CGK = DGK$, therefore (EUC. I. 4) $\angle CKG = DKG$, and each of them is a right angle; *i.e.*, HK is at right angles to both the lines AB and CD; wherefore CD is parallel to AB.

Cor. 1. The same is true if CE, DF are not perpendicular but equally inclined to AB; for then perpendiculars from C and D to AB are equal.

Cor. 2. HK bisects the sides CD, EF, and the area CEFD.

Cor. 3. A perpendicular equidistant from two equal perpendiculars is a common perpendicular to two parallels, found as above.

PROP. VII. *If a straight line falling upon two other straight lines makes the alternate angles equal to one another, these two straight lines are parallel* (EUC. I. 27.) *And from any given point in one of them there can be drawn one and only one straight line to the other, which shall make the alternate angles equal.*

Let EF make with AB and CD the alternate angles AEF, EFD, equal to one another.

Let G be any given point in AB: make FH=EG; the alternate angles which the straight line GH makes with AB and CD, are equal.

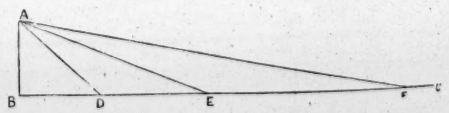
Bisect EF in K, join GK, KH: the triangles GEK, KFH, are equal in every respect (EUC. I. 4): therefore ∠GKE=HKF; to each of these add EKH, then the angles GKE and EKH together are equal to HKF and EKH; but HKF and EKH are equal to two right angles; therefore GKE and EKH are equal to two right angles, and GKH is a straight line. But the angles EGK, KHF, are equal, and they are alternate angles; therefore, through the given point G, a straight line GH has been drawn to CD, making the alternate angles equal to one another.

Also, from the same point G, no other straight line can be drawn to CD which shall make the alternate angles equal.

For, if possible, let GL be a straight line which makes the alternate angles BGL, GLC, equal to one another. Then, because BGH=GHC, by subtraction LGH is equal to the difference between GLC and GHC; that is, one of the interior opposite angles of a triangle is equal to the difference between the exterior angle and the other interior opposite angle, which is impossible (Ax. Cor. 1.)

PROP. VIII. *If from a given point without a given straight line, straight lines be drawn to the line, the angles which they make with it become less and less as the distance from the perpendicular increases; and a straight line may be drawn from the given point to the given straight line, so as to make with it an angle less than any assignable angle, by proceeding far enough.*

Let A be the given point; BC the given straight line; AB perpendicular to BC; AD any other line drawn from A to BC. Make DE=AD; join AE, and make EF=AE, and so on. The angles ADB, AEB, AFB, continually diminish (Euc. I. 16). And by proceeding far enough a line may be drawn making with the given linean angle less than any assignable angle.

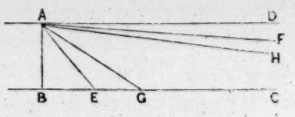


For $\angle ADB > DAE + DEA > 2AED$, because $AD = DE$: that is, $AED < \frac{1}{2} ADB$. Similarly, $AFB < \frac{1}{2} AEB < \frac{1}{4} ADB$; and it is evident that by proceeding in the same way a line may be drawn from A to BC which shall make with BC an angle less than any assignable angle.

PROP. IX. *Through a given point without a straight line there may be drawn an infinite number of straight lines all parallel to the given straight line.*

Let A be the given point, BC the given straight line; from the point A there can be drawn an infinite number of straight lines all parallel to BC.

Draw AB perpendicular to BC, and AD perpendicular to AB. Take any number of points E, G, &c., in BC, and join AE, AG, &c.

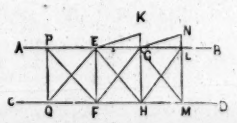


The angles BEA and BAE are less than a right angle; but the angles DAE and BAE are equal to a right angle; therefore $DAE > AEB$. Cut off $FAE = AEB$. Again, $FAE = FAG$ and GAE; but $FAE = AEB > AGE$ and GAE. Take away GAE, and $FAG > AGE$. Cut off $HAG = AGE$, &c. The straight lines AD, AF, AH, are all parallel to BC (EUC. I. 27), and it is evident that their number is unlimited.

PROP. X. *If a straight line be perpendicular to each of two parallel straight lines, it is the least distance between them; and of all other perpendiculars drawn from one of the parallels to the other, that which is nearer to the least is less than one more remote; also two equal perpendiculars can be drawn, one on each side of the shortest line.*

Let AB be parallel to CD; and let EF be perpendicular to both; EF is the shortest distance between them.

From G, any point in AB, draw GH perpendicular to CD: GH, and therefore (EUC. I. 19) any line drawn from G to CD is greater than EF.



If GH is not greater than EF, it is either equal to it or less than it.

First, Let, if possible, $GH = EF$; the triangles EFH, GHF, are equal in every respect; therefore $EH = GF$; and the triangles EFG, EHG, are equal in every respect; therefore $\angle EGH = GEF$, and each is a right angle, which is impossible (Ax. Cor. 2).

Secondly, Let $GH < EF$, and produce it to K, so that $HK = EF$. As in the former case, the triangles EFH, KHF (if KF be joined) are equal in every

respect, and EH=KF; whence also the triangles EFK, EHK are equal in every respect; and ∠HKE=FEK, and each is greater than a right angle, which is impossible (Ax. Cor. 2.) HG is therefore greater than EF.

Next if L be a point in EB beyond G, from which the perpendicular LM is drawn to CD, LM is greater than GH.

The angle LGH is greater than a right angle, and the angle GLM less; therefore LGH>GLM. Now, we can prove, exactly as in the former case, that if LM=GH, ∠GLM=LGH, which is impossible; and if LM∠GH, by producing ML to N, and joining GN, we can prove that ∠GNM=NGH, or greater than a right angle, which is impossible; therefore LM is greater than GH.

Lastly, Two equal perpendiculars can be drawn, one on each side of EF.

Take EP=EG and FQ=FH; join PQ; PQ is equal to GH, and perpendicular to CF.

For PF=FG and ∠PFE=EFG (Euc. i. 4), therefore ∠PFQ=GFH; and the triangle PFQ is equal to the triangle GFH in every respect. Hence ∠PQF =GHF, and is a right angle; and PQ=GH.

Cor. The angles MLG, HGL are less than two right angles, and the angles HGE, HGL are equal to two right angles; therefore ∠MLG is less than HGE; that is, if lines be drawn from one of the parallels perpendicular to the other, the interior angles in which they cut the first parallel continually diminish, as the distances from the common perpendicular increase.

Prop. XI. *If a straight line be perpendicular to each of two parallel straight lines, and from the points at which it meets them equal distances be set off along those lines towards the same side, the straight line which joins the points so found shall make equal angles with each of the parallels towards the same side.*

In the figure of Prop. X., let EF be perpendicular to both the parallels AB, CD; and let EG=FH, the angle EGH is equal to FHG. The triangles GEF, HFE are equal, therefore GF=EH, and consequently the triangles EGH, FHG are equal, and the angle EGH=FHG.

Cor. 1. If the angles which a straight line makes with each of two parallels towards the same side are equal, the straight line meets the parallels at points which are equidistant from the points where the common perpendicular meets them.

Cor. 2. If two straight lines be drawn, each making equal angles with two parallels towards the same side, the portions of each parallel which they cut off shall be equal.

PROP. XII. *If straight lines be drawn to two parallel straight lines on the same side of the common perpendicular, making with them equal angles respectively towards the same sides, that which is nearer to the common perpendicular is less than one more remote; and two equal lines can be drawn, making equal angles towards the same side with the given parallels, one on each side of the common perpendicular.*

Let AB be parallel to CD (fig. Prop. X.), EF perpendicular to both; GH, LM straight lines which make the angles EGH=FHG and ELM=FML; LM is greater than GH.

The angles HGL, GHM are each greater than a right angle, and GLM, HML each less than a right angle (Ax. Cor 2). Therefore $\angle$HGL=GLM.

And because GL=HM (Prop. XI. Cor. 2), therefore (EUC. I. 4) HL=GM.

Now if LM be not greater than GH, it is either equal to it or less than it. It is not equal to it, for then the triangles HGL, GLM would be equal in every respect, and the $\angle$HGL=GLM, which it is not.

Neither is LM$\angle$GH, for, if possible, produce ML to N, making MN=HG; join HN. Then, in the triangles LGH, HMN, the angle LGH is greater than HMN, therefore (EUC. i. 24) HL is greater than HN, and $\angle$HNL$>$HLN; but HLN is greater than a right angle, because HLM is less than a right angle, therefore HNL is greater than a right angle, which is absurd: LM is therefore greater than GH.

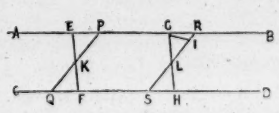
Next, if EP=EG, FQ=FH, PQ will be equal to GH, and will make equal angles with the parallels towards the same side.

For PF=FG, and $\angle$PFE=EFG, therefore $\angle$PFQ=GFH, and the triangle PFQ=GFH. Therefore $\angle$PQF=GHF and PQ=GH.

Similarly QPE=HGE.

PROP. XIII. *The straight line which makes with two parallel straight lines the alternate angles equal, is less than any other straight line which can be drawn from one of the parallels, to make with the other an angle equal to either of these alternate angles towards the same side.*

Let PQ make with the parallel straight lines AB, CD, the alternate angles equal to one another; let also any other straight line, RS, make the angle RSD=PQD; PQ is less than RS.

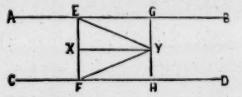


Bisect PQ in K, draw KF perpendicular to CD. make PE=QF, and join KE, then (EUC. I. 4 and 14) EKF is a straight line perpendicular to AB. Also, if SL be made =QK, and LH

be drawn at right angles to CD, and produced to meet AB in G, then LH=KF. Now (Prop. X.) GH>EF, therefore LG>KE. Also, since E, F, and H are right angles, LGE is less than a right angle, and therefore LGR greater than a right angle. Cut off the right angle LGI; then the triangles LGI, KEP have two angles of the one equal to two angles of the other, but the side LG adjacent to equal angles in the one greater than KE in the other; therefore LI is greater than KP (Prop. V.), but LR>LI ∴ LR is much greater than KP, and RS>PQ.

PROP. XIV. *If through the middle point of the common perpendicular to two parallel straight lines, a straight line be drawn perpendicular to it, this line will bisect at right angles all lines drawn so as to make equal angles towards the same side with the given parallels.*

Let EF be the common perpendicular to the two parallel straight lines AB, CD. Through X, the point of bisection of EF, let XY be drawn perpendicular to EF, meeting GH in Y. XY bisects GH perpendicularly. For the triangle EXY= FXY; therefore EY=FY, ∠XEY=XFY and ∠EYX= FYX: hence ∠YEG=YFH, and triangle YEG=YFH; therefore GY=HY, that is, GH is bisected by XY.

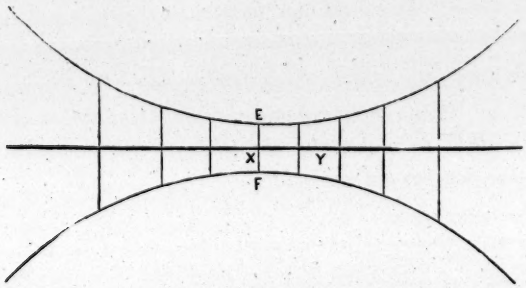


It is also bisected perpendicularly; for ∠EYX=FYX and ∠EYG=FYH; therefore ∠XYG=XYH, and each of them is a right angle.

Cor. 1. All straight lines drawn so as to make equal angles respectively with each of two parallel straight lines towards the same side are themselves parallel.

Cor. 2. And the straight line which bisects them all is parallel to the given parallels.

These propositions appear to draw the connection between parallelism and



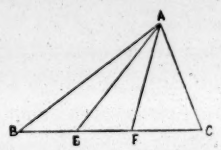
rectilinearity very close. Props. X., XI., and XII. tally very well with the (popular) view of parallels as circles ofvery large and equal radius; for instance, the equality of the angles at which any of the lines parallel to that which is perpen-

dicular to both parallels meets the parallels, the equality of lines at equal distances on either side the common perpendicular, the constant increase of these lines as they recede from that perpendicular; all these properties agree exactly with the notions attached to circles. But on the other hand, the fact proved in Prop. XIV., that a straight line parallel to each of the parallels bisects all these lines, dissipates the idea of convexity for *that* line. And when it is remembered that the properties of this bisecting line with respect to either the upper or the lower of the given parallels are precisely the same properties as those proved to exist for the given parallels themselves, we appear to have reduced parallelism very exactly to square with rectilinearity, as defined by Euclid.

We shall now approach the subject in another form.

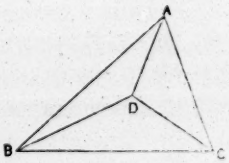
PROP. XV. *If a triangle be in any way divided into a number of triangles, the angular defect of the whole triangle is equal to the sum of the angular defects of its parts.*

Let ABC be a triangle, and,—1. Let it be divided into triangles by straight lines drawn from one of its angles A to the opposite side: $\delta ABC = \delta ABE + \delta AEF + \delta AFC$. For $A+B+C+$ all the angles at E and F = all the angles of ABE, AEF and AFC. Therefore, by subtracting from six right angles $\delta ABC = \delta ABE + \delta AEF + \delta AFC$; and in the same manner the proposition may be proved, whatever be the number of triangles into which ABC is divided.



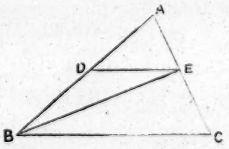
2. Let the triangle be divided into three triangles by lines drawn from any point within it: then $A+B+C+$ four right angles = the angles of the triangles ABD, BDC, CDA. Take from six right angles, and $\delta ABC = \delta ABD + \delta BCD + \delta CAD$

3. If the triangle ABC be divided into any number of triangles by lines drawn from the same point D, we shall have the same result, by combining Case 1 with Case 2.



4. The same demonstration applies, if one of the triangles into which ABC is divided be formed by joining points in AB, AC; and a similar argument would extend the proposition to other cases.

For the sake of demonstration (not of construction), we are at liberty to introduce the two following postulates.



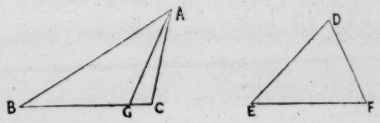
Postulate 1. From the greater of two given triangles, a triangle can be cut off equal to the less.

Postulate 2. A triangle can be divided into any number of equal triangles.

PROP. XVI. *If the angular defects of two triangles are equal, the areas of the triangles are equal.*

Let ABC, DEF, be two triangles which have the same angular defect; they have the same area.

If not, let the area of ABC be greater than that of DEF. Cut off ABG = DEF (Post. 1), then $\delta ABG = \delta DEF$ (Prop. III.),



but $\delta ABC = \delta ABG + \delta AGC$ (Prop. XV.),

$= \delta DEF + \delta AGC$;

Therefore $\delta AGC = 0$, which is contrary to the hypothesis (axiom). The areas are therefore equal.

PROP. XVII. *The areas of triangles have to one another the same ratio as their angular defects.*

Let ABC, DEF, be two triangles, area ABC : area DEF : : $\delta ABC : \delta DEF$. Let ABC be greater than DEF : Cut off from ABC the part ABG = DEF (Post 1). Suppose ABG, ABG, commensurable; and let ABC and ABG be divided into triangles of the same area (Post. 2) : let ABC contain m such triangles and ABG n of them,

δABC = sum of all the angular defects of the triangles into which it is divided

$= m$ times the angular defect of one of them

$\delta ABG = n$ times the same

$\therefore \delta ABC : \delta ABG : : m : n : :$ area ABC : area ABG.

Therefore (Prop. XVI.) $\delta ABC : \delta DEF : :$ area ABC : : area DEF.

This demonstration is only applicable when the larger triangle is *finite*.

If we desire to extend the demonstration to the case in which the larger triangle is unlimited, and the smaller triangle finite, we require to introduce a third postulate, viz., that a triangle of which the vertex is given may be increased indefinitely by increasing its base. This postulate (if admitted) would lead at once to the conclusion that $\delta ABC : \delta ABG : :$ area ABC : area ABG, whilst ABG is made as large as we please; and, consequently, if δABC be a finite quantity, δABG might be made to exceed two right angles by making ABG large enough. But as this is manifestly impossible, we should have a proof of the fact that $\delta ABC = 0$, or the three angles of a triangle taken together equal to two right angles. The postulate, however, is not legitimate as a logical canon. It evidences, however, the extreme narrowness of the limits towards the received doctrine to which the inquiry is pushed.

We may advance yet a stage further. It will suffice if the needed postulate read thus—

Postulate 3. Given a finite triangle and a finite number, it is possible to find a triangle which shall exceed the multiple of the given triangle by the given number.

If this postulate be admitted, we can prove that the angles of a triangle are together equal to two right angles as follows :—

Let ABC be a finite triangle : δABC will be a finite angle, however small; and the multiplier m, which applied to δABC shall make it greater than two right angles, will be finite. By the postulate a triangle can be found the area of which shall exceed m times the area ABC. Let DEF be this triangle; then

δDEF : δABC : : area DEF : area ABC

$$> m : 1$$

Therefore $\delta\text{DEF} > m\ \delta\text{ABC}$

$>$ two right angles; which is absurd.

Hence, the angles of a triangle cannot fall short of two right angles.

It must be remembered that this is not put forth as a demonstration: it is merely exhibited to shew what more will suffice to render the demonstration possible.

Cor. It is evident that the area of a quadrilateral is proportional to its angular defect from four right angles; and that, with this understanding, everything which has been here demonstrated of triangles is applicable to quadrilaterals.

PROP. XVIII. *If from two points in one straight line the perpendiculars drawn to another straight line are equal, so that the lines are parallel* (Prop. VI.), *and a straight line be drawn from one of the given points to a point in the given line produced on the side towards which the point lies, the exterior angle which this line makes with the line joining the points exceeds the interior opposite angle which it makes with the other line, by the angular defect of the quadrilateral formed by these three lines and the common perpendicular.*

Let CE = DF in the figure of Prop. VI., and let any point B in EF produced be joined with D and let BD be produced to L; $\angle$CDL exceeds FBD by the angular defect of KHBD. For $90° - \text{CDF} = \frac{1}{2}\delta\text{CEFD}$ (Prop. XVII., Cor.)

$$= \delta\text{KHFD (Prop. VI., Cor. 2)},$$

and $\text{CDL} + \text{CDF} + \text{FDB} = 180°$;

therefore $\text{CDL} - \text{FBD} = 90° - \text{CDF} + 90° - (\text{FBD} + \text{FDB})$

$$= \delta\text{KHFD} + \delta\text{DFB}$$

$$= \delta\text{KHBD}.$$

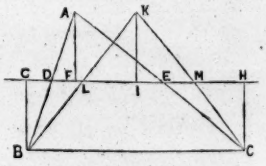
PROP. XIX. *If two triangles have the three angles of the one equal, each to each, to the three angles of the other, the triangles are equal in every respect.*

Let ABC, DEF, be two triangles, which have the angle A = D, B = E, and C = F, the triangles are equal in every respect. For their areas are equal (Prop. XVII.) If, therefore, AB be greater than DE, AC must be less than DF. From AB cut off AG = DE, and produce AC to H, making AH = DF; the triangle AHG is equal in every respect to DEF; therefore, $\angle AHG = DFE = ACB$, which is impossible. The triangles ABC, DEF, are therefore equal in every respect.

PROPS. XVI. to XIX., again recall the properties of the circle. They are in fact properties of spherical triangles on the same sphere if the phrase *angular defect* be converted into *spherical excess*. We may now apply the (popular) idea of a circle to the triangle as in the accompanying figure.

PROP. XX. *Upon one of the sides of a given triangle to construct an isosceles triangle, that shall have its area and the sum of its angles equal to those of the given triangle.* (This is Mr MEIKLE'S proposition. It is introduced for the sake of some corollaries).

Let ABC be the given triangle. Bisect AB, AC, in D and E; through DE draw GDEH, and from A, B, C, draw perpendiculars AF, BG, CH, to GDEH. Bisect GH in I, draw IK perpendicular to GH, and equal to AF, join KB, KC; KBC is the triangle required. The triangles ADF, BDG, are equal, as also the triangles AEF, CEH (EUC. i. 26). Hence the figure BGHC is equal to ABC in area, and the sum of the angles of ABC is equal to the angles GBC, BCH. Now KI = AF = BG = CH. Therefore the triangles KIL and BGL are equal, as also the triangles KIM, CHM, so that KM = MC; hence also the triangle BKC is equal in area to the figure BGHC, and the sum of its angles is the sum of the angles GBC, BCH; therefore the triangles ABC, KBC, are equal both in area and in the sum of their angles.



Also because GL = LI, IM = MH, each of them is $\frac{1}{4}$GH; therefore they are equal, and BK = KC; or BKC is an isosceles triangle.

Cor. 1. The line which joins the points of bisection of the sides of a triangle is parallel to the base.

For BG = CH; hence GH is parallel to BC (Prop. VI.)

Cor. 2. If two equal triangles be upon the same base and on the same side of

it, the straight line which bisects the sides of the one bisects also the sides of the other.

For both these lines pass through LM, the points of bisection of the *same* isosceles triangle.

Cor. 3. EUCLID's Prop. 39, "Equal triangles upon the same base, and upon the same side of it, are between the same parallels," is true.

For the perpendiculars from the vertices of the equal triangles on GH are both equal to KI, and therefore equal to one another.

Hence the line joining their vertices is parallel to GH; but GH is parallel to BC; therefore the line joining the vertices is parallel to the base.

The last part of the demonstration holds good, from the circumstance that the straight line GH, to which the two are both parallel, lies between them. EUCLID's Proposition 30 is not admissible, and has not been assumed. It will be seen by Prop. IX. that EUCLID's proposition 30 is not compatible with our axiom. Hence also the converse of this corollary is not true. Instead of it, *i. e.*, instead of EUCLID's Prop. 37, we have the following.

Cor. 4. Triangles upon the same base, and whose other sides are bisected by the same straight line, are equal to one another.

Let ABC, KBC, be any two triangles on the same base, and having their other sides bisected by the same straight line GH, area ABC=KBC.

For each triangle is equal to the figure GBCH.

Cor. 5. EUCLID's Prop. 40, "Equal triangles on the same side of bases which are equal and in the same straight line are between the same parallels," is true. The demonstration differs little from that given above for Cor. 3.

Cor. 6. EUCLID's Prop. 38 requires a similar alteration to that made on Prop. 37 in Cor. 4.

Cor. 7. If I be *any* point in the line GH, the triangle KBC is still equal both in area and in the sum of its angles to ABC.

Cor. 8. Also KM=MC as before.

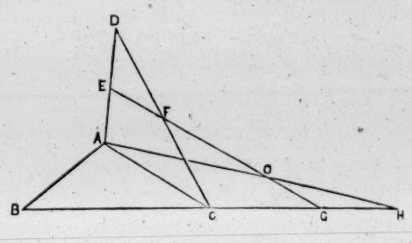
Cor. 9. The results given in Cors. 7 and 8 are also true if L be taken, any point in GH, and LK be made equal to BL, for then KI=BG as before.

Cor. 10. Hence, upon BC there can be described a triangle equal in area and in the sum of its angles to ABC, and having one side equal to a given straight line greater than one of the sides AB, BC, of the triangle. For BL may be taken equal to half that line.

PROP. XXI. *To find a point in the base of a triangle produced, such that the triangle on the part produced, of which the vertex is the same as that of the given triangle shall be equal in area to the given triangle; when it is possible.*

Let ABC be the given triangle; C the angle which is not greater than B, and

which is therefore less than a right angle. It is required to find a point H in BC produced, so that the triangle ACH may be equal in area and in the sum of its angles to ABC; whenever it is possible.



Upon AC describe the triangle DAC=BAC (Euc. i. 23.) Bisect DA, DC in E and F; join EF, and produce it till it meets BC in G, when it is possible. Make GH=CG, CAH is the triangle required, For (Prop. XX. Cor. 8) AO=OH; therefore (Cor. 9) the triangle ACH is equal in area to ADC, that is, to ABC.

Cor. By carrying on this process, we can multiply the triangle ABC to some extent, as is done in Euclid vi. 1. But here there is a limit to the multiplication. It must terminate when EF fails to meet BC produced. That this will happen after a finite number of equal triangles has been found, we now proceed to show. First, the further angle of the triangle constantly diminishes (Euc. I. 16). Let ACB (to save the trouble of drawing another figure), be *any* one of the equal triangles. The construction shows that the perpendiculars from A and C on EG are equal; therefore (Prop. XVIII.)

$\angle ACB - \angle EGB = \delta$ area included by AC, EG, CG, and the common perpendicular.

Now, the common perpendicular bisects the area included between the two perpendiculars from A and C on EG (Prop. VI. Cor. 2), and this area is equal to the area of the *given* triangle ABC; therefore $\angle ACB - EGB > \delta \frac{1}{2} ABC$. It follows, therefore, that for the addition of every new triangle equal to ACB, the angle in advance suffers a diminution of more than half the angular defect of the given triangle. It will therefore be reduced to zero, or a negative quantity, by taking a finite multiple of the triangles; after which no point can be found in BC produced which will yield another triangle equal to the original triangle.

Again, let ACB be a triangle of which C is an obtuse angle; and let AX be drawn perpendicular to BC produced; then, since both the triangles ABC, ACX are finite, a finite multiple of ABC will exceed ACX. Let ACY be the first multiple of ABC that exceeds ACX; ARY the last of the triangles, each equal to ABC, which constitute their multiple. Then the triangle ARY may be treated as the triangle ABC is treated above; and the conclusion is general: *that only a finite multiple of a given triangle can be formed by joining the vertex with successive points in the base produced.*

This very curious result is important, as maintaining the consistency of the results mentioned in Prop. XVII., and it serves as a strong caution against hasty inferences.

To retain Euclid's definition of a parallelogram, it is requisite to combine with

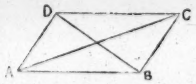
it some special definition of the particular parallels which form the parallelogram. For example, we may define a parallelogram as a four-sided figure, of which the opposite sides are parallel, *by making the alternate angles with one of the diagonals equal.* It will thus be a symmetric figure.

EUCLID'S definition of a square is at any rate vicious. To alter it consistently with EUCLID'S construction (I. 46), reading it simply as an equisided quadrilateral that has *one* angle a right angle, becomes an impossibility on our hypothesis. We may adopt the following

Definition.—A square is a four-sided figure, which has all its sides equal and all its angles equal.

PROP. XXII. *The opposite sides and angles of a parallelogram are equal to one another.*

Let ABCD be a parallelogram; AC the defining diagonal. The triangles ABC, DCA have the angles CAB, ACB respectively equal to ACD, DAC and the diagonal common; therefore they are equal in every respect; whence the truth of the proposition.



PROP. XXIII. *The alternate angles which the sides of a parallelogram make with both diagonals are equal.*

The triangles ABD, CDB are equal in every respect (EUC. I. 8); whence the truth of the proposition.

Cor. Either diagonal bisects the parallelogram.

PROP. XXIV. *Parallelograms upon the same base cannot be between the same parallels,*

For if they could, the diagonals of the two parallelograms drawn from the same point in one of the parallels would make the alternate angles equal; which is impossible by Prop. VII.

PROP. XXV. *The point of intersection of the two diagonals of a parallelogram is the middle point of the common perpendicular to each pair of parallels, which constitute its sides.*

For if from this point a perpendicular be drawn to each of the opposite parallels, there will be formed two triangles equal in every respect (EUC. I. 26); and consequently the perpendiculars will be in a straight line.

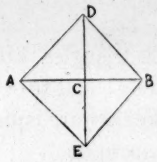
Definition. The point of intersection of the two diagonals is the *centre* of the parallelogram.

PROP. XXVI. *Parallelograms upon equal bases may be between the same parallels only when they have the same centre.*

For if other parallelograms were upon equal bases, the perpendiculars through their centres on one of the parallels would also be perpendicular on the other (Prop., XXV.) which is impossible (Ax. Cor. 2).

PROP. XXVII. *Upon a given diagonal to describe a square.*

Let AB be the given diagonal; it is required to describe a square on AB as diagonal.



Bisect AB in C; draw CD perpendicular to AB, and produce it, and make DC, CE, each equal to AC. ADBE is the square required.

The triangles are all equal (EUC. I. 4), whence the conclusion is obvious.

Cor. A square is a parallelogram.

XXXII.—*On the Temperature of certain Hot-Springs in the Pyrenees.* By R. E. Scoresby-Jackson, M.D., F.R.C.P., Lecturer on Materia Medica and Therapeutics at Surgeons' Hall, Edinburgh.

(Read 18th January 1864.)

Having determined to spend my autumn holiday in the Pyrenees, it occurred to me that I might add a link to a very interesting chain of observations which was begun by one of the Vice-Presidents of this Society in the year 1835. Twenty-eight years had elapsed since Principal Forbes made his careful observations of the temperatures of certain of the Pyrenean hot springs; and I thought that if I could repeat his experiments after so long an interval, the results might not be without some interest.*

A visit to the Pyrenees in the year 1835 was so far uncommon as to make a description of the places through which the traveller passed a source of entertainment to the general as well as the scientific reader; but now such description would be tedious. Then, too, some remarks were necessary touching the geological features of the localities where the springs existed; but it is unnecessary to repeat them now, for I am not aware that the explorations into the rocks in search of new springs, or of a more abundant supply of water from the older springs, have altered the relations as described by Principal Forbes. Therefore, respecting the natural aspect of the country, beautiful though it be, and its geological features, I shall have nothing to say; and as I have no new theory of the cause of thermal springs to propound, I need not repeat the speculations with which we are familiar.

The temperature of thermal springs is interesting in a variety of aspects. The geologist and the chemist are alike concerned in it, and to the physician it is a question of no slight importance. In the treatment of diseases by mineral waters, temperature is a cardinal element; indeed in many instances the thermal feature appears to be that by which alone the water is characterised as a medicine. Physicians who frequent the bathing places during the season when invalids resort to them, and who have unusual facilities for witnessing the effects of mineral waters, have great confidence in the curative power of heat. In esti-

* See Principal Forbes' paper "On the Temperatures and Geological Relations of certain Hot Springs, particularly those of the Pyrenees"—*Philosophical Transactions*, Part II. for 1836.

mating the influence of mineral waters three elements are to be considered,—the water itself, the substances dissolved in it, and the temperature. ANGLADA has remarked, that water alone, with the aid of certain temperatures, is capable of producing the diversified effects of a crowd of medicines. Thus, he says, the same liquid which, at a temperature of 35° to 37° Cent. may be used as an emollient, becomes powerfully excitant at from 39° to 41° Cent., and is transformed into an energetic irritant from 42° to 45°,—a temperature which the body can support only for a few seconds (Op. tome ii. p. 381, *et seq.*) M. FONTAN, medical inspector of the waters at Bagnères-de-Luchon, divides the action of thermal mineral waters into two parts; an *immediate* or *physiological* action, due to the temperature of the water, and a *mediate* or *therapeutic* action, due to its mineral ingredients. He remarks that the springs which enjoy the highest and most enduring reputation are those whose temperature approaches nearest to that of the human body, and is constant. He declares, that by means of different temperatures alone he can, by the use of mineral waters, either allay the nervous susceptibility of a Belle, or excite a very Hercules.* M. OURGAUD, the medical inspector of the waters at Ussat, amusingly describes his establishment as a thermal gamut, having a note, that is, a degree of temperature, for every phase of disease.†

But from whatever point of view the subject of the temperature of thermal springs may be regarded, it cannot but be interesting to know whether the temperature be constant or variable.

"It is a singular fact," Principal FORBES remarks, "that we are not only unacquainted with the progressive variations of temperature in springs during long periods of time, but even with the diurnal or monthly changes to which many thermal waters are probably subject. The usual statement of the constancy of the heat of such springs at all seasons is abundantly general, but perfectly vague." This statement, made in 1835, does not seem to have induced any regularity of observations. The same general but unsupported assertions of the constancy of the temperatures, from season to season and year to year, were made to me at

* "Je crois que l'on doit tenir compte avec beaucoup d'exactitude des températures auxquelles on administre les eaux thermales, et je suis persuadé, pour ma part, que les mots *fortes* et *faible*, que l'on prodigue, sans examen, à telle ou telle eau, sont appliqués le plus souvent d'une manière irreflechie quant à la composition de l'eau. Que si l'on entend par forte la propriété *immédiatement* excitante d'une eau thermale, elle doit être bien plutôt entendue de sa température que des proportions chimiques des substances qui y sont contenus. Je me chargerais, pour ma part, de calmer la susceptibilité nerveuse d'une petite-maitresse avec un bain d'eau de la Grotte de Bagnères-de-Luchon —appliqué à + 32 ou + 33 Cent., et d'exciter un Hercule avec la source de la Preste ou du Pré de Cauterets à la température de +44° et de +47° Cent."—*Recherches sur les Eaux Minérales*, p. 185.

† "D'où il résulte que ces bains, disposés successivement comme les touches d'un clavecin, offrent une série de températures decroissantes, entre 37 et 31 degrés centigrades, qui forment une sorte de gamme thermale modulée selon les divers besoins de la therapeutique."—*Précis sur les Eaux Thermo-Minerales d'Ussat-les-Bains.*

most of the thermal stations in the Pyrenees. The utmost that I have met with in the form of a continuous record is contained in work (Eaux Minérales des Pyrénées) by Professor FILHOL of Toulouse. At page 97 of his book, M. FILHOL has given a table of observations made at Bagnères-de-Luchon, at irregular intervals between 1st August 1849 and 15th April 1853; but during that period there are only eighty-six days of observation. It is to be regretted that, from one so highly respected and so eminently fitted to undertake the task, we have not a longer and more connected series of observations. But, far from charging M. FILHOL with lack of zeal, we owe him a large debt of gratitude for the labour which he has bestowed on the subject, and for his very valuable record. The value of the table is enhanced by the facts that the observations are all made by the same person, exactly at the same points of each spring, with the same instruments, they being verified on several occasions. Therefore, as the observer justly remarks, no doubt can attach to his results.

From his laborious personal observations, M. FILHOL is convinced that the temperature of springs—even those best regulated and most protected from water of infiltration—*is not absolutelg invariable.* With respect to the variability of their temperature, he divides the springs of Bagnères-de-Luchon into two groups. In the first group he places those springs which do not vary more than a few tenths of a degree at the season of snow-melting, when the level of surface water in the galleries is unusually high; and in the second group those springs whose temperature is more variable, the changes being evidently in relation with the height of the surface water. In the latter group, with a change of temperature, there is generally a corresponding change in the volume of water, a lower temperature being observed with an increase in the abundance of water. But this is not uniformly so. The increase in the volume of water of a spring, at the season of snow-melting, is not always due to water of infiltration. Occasionally the volume of water is increased without any diminution of temperature; more than that, there is sometimes a diminution of temperature with a *diminution* in the volume of water.*

M. FILHOL believes that springs with a high temperature are less susceptible of variations than those of a low temperature.†

* A diminution of temperature does not always follow the increase of surface water: Dr GARIGOU, of Ax, says:—"Plusieurs sources sont, en effet, penetrées par les eaux pluviales ou par l'eau des torrents à l'époque des fortes crues, et en même temps que leur volume augmente, on peut suivre l'abaissement de leur température et une diminution dans leur richesse en sulfure. Pour deux sources, cependant, j'ai pu, pendant l'hiver, à l'époque de la fonte des neiges, constater une élévation de température bien évidente. Pour l'une de ces sources, celle du Rossignol supérieur, la température s'est élevée de 0·7; et pour la seconde, la source Hardy du Breilh, l'augmentation dans le calorique a été de 1°."—*Etude Chimique et Médicale des Eaux Sulfureuses d'Ax.*

† "Je crois avoir remarqué que les sources les plus chaudes sont celles qui éprouvent le moins de variations; c'est ainsi que la source Bayen a oscillé, dans l'espace de trois ans, entre 67° 25, et 18 10."—*Op. cit.*, p. 96.

M. Ernest Baudrimont made a series of observations at the *Grande-Grille*, Vichy, to ascertain what changes mineral waters are subject to in their composition from day to day. At the same time he recorded the temperature of the water, and on several days in September 1850, he notes it as follows; showing an extreme range in eleven days of ·80 Cent. = 1°·44 Fahr.*

Day of Month	Temperature, Centigrade.	Day of Month.	Temperature, Centigrade.
3	33·33	9	32·80
5	33·33	12	33·25
7	33	14	33·60

Whenever I had an opportunity I put the question, whether the temperature of the springs varied from day to day, from season to season, or from year to year. The answer that I received was almost without exception a negative. When I asked the question of one of my professional brethren, the reply was often a little modified, *trivial* variations being generally admitted: but when I put the question to an *ouvrier* attached to any of the establishments, he invariably betrayed his sense of the importance of uniformity of temperature by a most emphatic denial. The prevailing opinion in the Pyrenees touching the variability of temperature of the springs is precisely what has been stated by Dr Lambron, in his comprehensive work (Les Pyrénées, et Les Eaux Thermales Sulphurées de Bagnéres-de-Luchon.) He says:—"The variations of temperature to which well secured (*bien captées*) thermo-mineral springs are liable are generally so trifling (de 1 à 2 degrés), that we may consider the temperature which they bear from the interior of the earth to be constant; and we may attribute the variations to fortuitous causes, operating either in the interior of the globe (as earthquakes), or far more commonly near the surface, as by the infiltration and admixture of surface water. And these variations are so transient, that at the end of a few days, their normal temperature is restored" (p. 414).

Suppose it were commonly admitted, however, that slight variations of temperature do occur from season to season, important as that may be to the physician and his patient, still the larger question, as to a regular and permanent loss or increase of temperature remains to be answered. M. Filhol remarks, that the changes of temperature are not always in one direction; he found the same springs to be sometimes a little above, sometimes a little below their normal temperature. It is possible, therefore, that temperatures taken at long intervals may differ, to a certain extent, without there being any really permanent change. If one observer record the temperature when the spring is abnormally low, and another when it is abnormally high; or one in winter and another in summer, a difference of several degrees may be obtained, without the water having per-

* Annales de la Société d'Hydrologie Medicale de Paris, t. ii. p. 251.

manently changed its temperature, but merely having oscillated between these extremes. One advantage derived from a comparison of my observations with those of Principal FORBES is that we both observed at the same time of year—namely, in or near the month of August.

There are certain circumstances calculated, if not satisfactorily explained, to cast discredit upon the results of an investigation of this kind: they respect the character of the instruments employed; the scrupulosity of the observer; and certain local conditions.

1. *Instruments.*—One of the difficulties that we have to contend with in comparing observations made at long intervals, is that of ascertaining the nature of the instruments employed by former observers. Unless the errors of the instruments are recognised and corrected, it is quite possible that what may appear to be a change of temperature may be nothing more than the fault of a thermometer. Principal FORBES has been most minute in describing the means adopted for the verification of the instruments which he employed; and M. FILHOL has also been at great pains to check the errors of his instruments.

Knowing that accuracy of result depended greatly upon the character of the instruments employed, I applied to one of the first houses in Edinburgh, requesting them to make for me four thermometers of the best description, two going to 140° Fahr., and two to 212° Fahr. These were marked respectively, A, B, C, and D. They were tested by Mr BUCHAN, the acting Secretary of the Scottish Meteorological Society, who having examined them and compared them with the Society's standard thermometers, pronounced three of them perfectly accurate, and the fourth very slightly faulty. The latter was rejected. All the instruments are very sensitive, the mercury rising in the tubes very rapidly when heat is applied. As thermometer D was used at every spring, returned uninjured, and was again tested by Mr BUCHAN and found perfect, I need only describe it. For convenience of carriage, its length is only (from the extreme ends) thirteen inches. It is a simple glass tube without mounting, the scale being marked on the tube, and containing eighteen degrees Fahr. to an inch. The degrees are not subdivided; but I had no difficulty in estimating the parts of a degree.

2. *The Observer.*—Having obtained trustworthy instruments, the next fear of error was from carelessness in using them. Any error from this source I was particularly anxious to avoid, and fearing that—from the awkward positions in which many of the observations were made—I might, in the reading of a figure or otherwise, make a mistake, I always endeavoured to associate myself with some person of local reputation who would accompany me to the springs, and make an observation with his own thermometer at the same time. After the observation at each of the springs was made and entered in my book, the Centigrade degrees of my associate's thermometer were reduced to those of Fahrenheit; if the observations accorded, or were within one or two-tenths of a degree, we

were satisfied; if they differed more than that, we repeated the observations, sometimes three or four times; but often failed in the end to bring the two nearer than within two or three tenths of a degree. Both observations are invariably given in the descriptive remarks and in the tables. When the temperature was taken at a buvette, or other running stream, a large tumbler was placed under the stream; the water was allowed to flow into and over it for a minute or two, until the glass assumed the temperature of the water as nearly as it could be made to do; the thermometers were then placed in the tumbler, and allowed to remain for a short time, and were then read; the water all the time flowing over the tumbler. When the temperature was taken at the *source* (to arrive at which, we usually entered into a gallery or drift, in the floor of which the springs are secured), we dipped our thermometers into the spring, moved them about from place to place to find the hottest part, and read them whilst immersed. When it was practicable, we plunged the thermometers so low as to bring the summit of the mercurial column on a level with the surface of the water, but we could seldom read them in that position, being obliged to raise them a little to bring them to the level of the eye (the head stooping to the floor of the gallery). Sometimes it occurred that the water was so far below the level of the floor of the gallery that we could not get near the thermometers to read them whilst they were immersed. In such cases we got a wine bottle, placed the thermometers in it, tied a cord round its neck, and lowered it into the water. It filled with the water, and we allowed it to remain some minutes, then hauled it up and read the instruments as quickly as possible. Sometimes we were placed in very inconvenient positions, sprawling upon the floor of the gallery, with the head and limbs awkwardly twisted, encumbered with a candle and a thermometer, and exposed to an atmosphere resembling that of a Turkish bath; but we never left a spring without carefully repeating our observations several times when necessary. I most cordially express my thanks to those gentlemen who so patiently accompanied me into these uncomfortable situations, and whose names I shall have occasion to mention hereafter. We often found it very difficult to record an exact temperature when there was an escape of gas from the waters, or where the water rushed up with force. In these cases, the rippling of the water caused the mercury to oscillate in the tube so rapidly, that it was difficult to read the instrument; but we always endeavoured to catch the highest point.

3. *Place of Observation.*—It is difficult to ascertain the exact spot at which a previous observer has plunged his thermometer. So many alterations have been made of late years at the Pyrenean spas, that I scarcely think it likely that I have in many instances succeeded in observing at the precise place occupied by Principal Forbes in 1835. In most instances, my observations have probably been made nearer the actual origin of the springs, because at most of the spas the waters have been traced, by means of excavations, further into the rocks. At

Ax, I believe the position of observation is identical, for no alterations appear to have been made there since 1835; the place answers exactly to Principal Forbes' description. In my remarks respecting each of the places visited, I have endeavoured to mention these local peculiarities.

In describing the points where I have made my observations, I frequently use the word *griffon*. I must explain the meaning of the expression. It is used rather vaguely to express a near approach to the point where the spring emerges from the ground; but I have used it to signify the point where the water passes from the fissures in the rock into the artificial apparatus constructed for its reception, where it is, for the first time, securely cut off from all external influences. This securing of the water is termed *captage*. At this point the water usually issues by several thin streams from the ground, and it is possible that these may have different temperatures. Care is therefore to be taken in immersing the thermometer, to move it about in order to ascertain whether such differences exist. I frequently found that when my companion's thermometer was placed at a little, even two or three inches, distance from my own, different temperatures were recorded. Usually, it is not until the water has become well mixed, at a little distance from the place where it emerges from the rock, that it acquires a uniform temperature.

All my observations were made in the month of August 1863.

EAUX CHAUDES.

Here I had the kind and able assistance of Dr Prosper de Pietra-Santa, physician to the Emperor, who resides during the bathing season at Eaux Bonnes. He kindly accompanied me to Eaux Chaudes. The thermometer employed by M. Pietra-Santa was made, I believe, by Fastré of Paris, and was, as I understood, quite accurate. I measured it with my pocket tape, and found it to contain twenty-five degrees of Centigrade to an inch, or forty-five degrees of Fahrenheit, which on my own thermometer (D) occupy a space equal to two inches and a half.

The thermal establishment which existed in 1835 has since been pulled down, and the waters are now administered in a handsome building, constructed in the years 1848–50 by MM. François and Latapie. The new establishment is built upon the right bank of the Gave, at a distance of only a few yards from the site of the old one. This change of position, however, must necessarily affect the temperature of the water at the place where it is employed by the invalid, the distance between the *buvettes* and the *griffon* of certain of the springs being as much greater now than it was in 1835 as the distance between the old and the new establishments. The effect of this change of position, I was informed, was decidedly injurious to these springs, whose waters are now conducted a greater distance in pipes.

Of the six springs in use at Eaux Chaudes, three are conducted into the establishment, and these are they which are affected by the change of position. They are named *Le Clot*, *L'Esquirette*, and *Le Rey*. The other three are employed at the several points at which they rise; they are called *Baudot*, *L'Arressecq*, and *Mainvielle*.

Le Clot.—This spring is received, at its point of emergence, into a reservoir which is hermetically closed. From this reservoir the water is conveyed by two conduits to the establishment—the one leading to the baths, the other to the *buvette*. The water, doubtless, loses a certain quantity of heat during its passage from the actual source to the *buvette;* and I was told, moreover, that, in spite of every effort to preserve it from decomposition, the amount of sulphuret of sodium in the water is much less at the baths and the *buvette* than at the *griffon*. The loss of temperature at the *buvette* I was unable to ascertain, as I had no opportunity of testing it at the *griffon*. The actual temperature observed by me at the *buvette* was 94·00 Fahr. At the same time, M. DE PIETRA-SANTA read his thermometer at 34·50 Cent. = 94·10 Fahr.

L'Esquirette.—This spring has been divided into two parts by a recent exploration into the rock; they are termed respectively *hot* and *temperate*. The hot stream is conducted to a buvette in the establishment, and supplies also, in combination with the temperate stream, the baths and *douches*. The distance between the *griffon* and the *buvette* of this source, I was told,* is 30 *metres*, equal to 98·427 English feet. The temperature observed by me at the *griffon* was 93·80 Fahr. M. DE PIETRA-SANTA recorded 34·20 Cent. = 93·56 Fahr. At the *buvette* I found the temperature to be 92·40 Fahr.; M. DE PIETRA-SANTA, 33·50 Cent.= 92·30 Fahr.

Le Rey.—This spring rises on the site of the old establishment, and is immediately received into a reservoir, whence it is conducted to the new establishment. The distance between the *griffon* and the *buvette* I did not ascertain. The temperature recorded by me at the reservoir was 91·40 Fahr., M. DE PIETRA-SANTA at the same time recording 33·00 Cent. = 91·40 Fahr. At the *buvette* I recorded 90·00 Fahr, and M. DE PIETRA-SANTA 32·00 cent. = 89·60 Fahr.

Baudot.—The temperature of this spring was taken at the *buvette*, close to its point of emergence from the granite. My thermometer marked 77·00 Fahr.; M. DE PIETRA-SANTA, 25·00 Cent. = 77·00 Fahr.

L'Arressecq.—The temperature of this spring was also taken at the point where it leaves the granite. My thermometer marked 74·90 Fahr.; M. DE PIETRA-SANTA, 24·00 Cent. = 75·20 Fahr.

Mainvielle.—The temperature of this spring was taken at the *buvette*, which is

* The distances mentioned throughout are such as were told me by the gentlemen who accompanied me to the several springs. I had no opportunity of measuring them.

placed immediately in contact with the *griffon*, or point of emergence, at the foot of a block of granite. The temperature recorded by me was 51·80; M. de Pietra-Santa recorded 11·00 Cent. = 51·80 Fahr.

The following table shows the temperature of the springs at Eaux Chaudes, as recorded by different observers at various periods. I give the Centigrade as well as the Fahrenheit reading of the temperature.

Name of Observer, and Date of Observation.	Le Clot. (Buvette.)		L'Esquirette. (Griffon.)		L'Esquirette. (Buvette.)		Le Rey. (Griffon.)		Le Rey. (Buvette.)		Baudot. (Griffon.)		L'Arresseeq. (Griffon.)		Mainvielle. (Griffon.)	
	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.
Prof. Forbes (July 1835), .	94·60	34·78	...	...	91·40	33·00	...	...	92·00	33·35	80·20	26·78	76·30	24·62	...	...
M. Fontan (Sept. 1835), .	97·07	36·15	...	...	89·60	32·00	...	...	92·57	33·65	81·05	27·25	77·18	25·10	...	...
M. Fontan (Sept. 1837), .	96·80	36·00	...	...	90·68	32·60	...	...	93·20	34·00	80·78	27·10	77·18	25·10	52·25	11·25
M. Gintrac (1841), . . .	96·80	36·00	...	...	...	...	...	...	93·20	34·00	...	...	...	...	...	...
Professor Filhol (1850), .	93·38	34·10	...	...	...	...	...	...	94·64	34·80	...	...	...	...	...	..
M. Lemonnier (1860), .	97·16	36·20	95·00	35·00	...	...	92·30	33·50	...	...	77·00	25·00	76·64	24·80	52·70	11·50
M. de Pietra-Santa (1863),	94·10	34·50	93·56	34·20	92·30	33·50	91·40	33·00	89·60	32·00	77·00	25·00	75·20	24·00	51·80	11·00
Dr Scoresby-Jackson (1863),	94·00	34·45	93·80	34·33	92·40	33·56	91·40	33·00	90·00	32·22	77·00	25·00	74·90	23·85	51·80	11·00

EAUX BONNES.

At Eaux Bonnes, also, I had the able assistance of M. de Pietra-Santa. There are several springs, but three only were available for my purpose, the rest being mixed in such a manner as to prevent the temperature of each being taken separately. The springs whose temperature I tested were *La Vieille, Source du Bois, ou Source Froide*, and *Source d'Orteich*.

Both the *griffon* and the *buvette* of *La Vieille Source* are within the thermal establishment, and are within four feet of each other, being separated only by a wall, through which the water is conveyed by a short pipe. At the *griffon*, the temperature was not easily taken. The chamber in which it is situated was gloomy, so that we could not read the thermometers without the aid of a candle; and even with that aid we could not read them whilst immersed in the water, the opening into the spring being very narrow, and the column of mercury, when the bulb was immersed, being below the level of the opening; that is, below the floor of the chamber. Even with my face touching the floor and close to the opening, I could not read the thermometer without raising it several inches, and when I did see the column it was subsiding so rapidly that I could not be certain of having seen it at its highest point. To obviate this difficulty, we placed the thermometers in a wine bottle, which was then lowered into the water, filled, and allowed to remain until the bottle assumed the temperature of the water. It was then withdrawn, and the thermometers were read whilst yet immersed in the water. The temperature recorded by me was 91·20 Fahr.; that by M. de Pietra

SANTA, 32·75 cent. = 90·95 Fahr. The temperature was taken also at the *burette*, and was found to be, as nearly as possible, the same as at the *griffon*.

Source du Bois, or Source Froide.—The temperature of this spring was taken at a little reservoir (about 6 feet by 3) at the point where the water issues from the rock. My thermometer recorded a temperature of 54·80 Fahr., M. DE PIETRA SANTA, 12·80 Cent. = 55·04 Fahr.

Source d'Orteich.—This spring arises in the *Vallée du Valentin*, where an establishment is now building. The temperature was taken at the *burette*, which is placed immediately over the *griffon*, and was recorded by me at 72·30 Fahr., and by M. DE PIETRA-SANTA at 22·50 Cent. = 72·50 Fahr.

The following table shows the temperature of the springs at Eaux Bonnes, as recorded by different observers at various periods:—

Name of Observer and Date of Observation.	La Vieille.		Froide.		Orteich.	
	Fahr.	Cent.	Fahr.	Cent.	Fahr.	Cent.
Professor Forbes (July 1835) . . .	91·40	33·00	54·40	12·45	...	...
M. Fontan (Oct. 1835)	92·03	33·35	55·04	12·80	...	...
M. Fontan (Sept. 1837)	91·96	33·32	55·40	13·00	...	...
M. Gintrac (1841)	91·76	33·20	55·40	13·00	...	...
Professor Filhol (1850)	89·96	32·20	53·96	12·20	...	...
Professor Filhol (1861)	90·95	32·75	55·94	13·30	71·96	22·20
MM. François and Pietra-Santa (1862)	90·95	32·75	55·04	12·80	71·96	22·20
M. de Pietra-Santa (Aug. 1863) . .	90·95	32·75	55·04	12·80	72·50	22·50
Dr Scoresby-Jackson (Aug. 1863) . .	91·20	32·89	54·80	12·67	72·30	22·39

CAUTERETS.

At Cauterets I was kindly received by Dr BURON, and had some conversation with him; but he had been for some time, and was then, suffering from severe illness, and was consequently unable to assist me in my researches. He was good enough, however, to introduce me to his friend Monsieur DE LALANDE, a gentleman who has travelled extensively, and has had great experience in chemistry and engineering; and also to Monsieur BROCA, an experienced pharmacien, who has made himself thoroughly acquainted with the mineral waters of the district. With these gentlemen I spent many hours in observing the temperature of the springs. I employed, as on all occasions, the thermometer marked D. The thermometer employed by MM. DE LALANDE and BROCA was made by SALLERON of Paris, and contained ten degrees of Centigrade to the inch of my measuring tape.

The mineral springs in the vicinity of Cauterets are numerous, and are divided, according to their position relative to the town, into two groups, named respectively *Southern* and *Eastern*. The eastern is the *northern* group of some authors, and occasionally the springs of *La Raillère* are separated into a group by them-

selves, called *Central*. Commonly, however, and sufficiently for our present purpose, they are divided into southern and eastern groups.

Southern Group.

The springs of this group emerge from the granite. We visited them in the following order :—

La Raillère.—The water of this spring is conducted into an establishment constructed at its source. We took the temperature first at the *buvette*, where my thermometer marked 101·30 Fahr., at the same time the thermometer of MM. DE LALANDE and BROCA marked 38·70 Cent.=101·66 Fahr. We then passed to the back of the building and observed the temperature at the *griffon*, which is five and a half metres (=18·044 English feet) from the buvette. Here I found the temperature to be 101·80, whilst MM. DE LALANDE and BROCA gave it as 39·00 Cent.=102·20 Fahr.

Mauhourat.—There are two drinking places to this spring,—one at a distance of two metres (= 6·562 English feet) from the griffon, and the second at a distance of 260 metres (=853·034 English feet). The buvette at the latter point is known as the *Petit Mauhourat*, and was constructed for the convenience of invalids who may find it too fatiguing to go to the principal buvette near the *griffon*, the *Petit Mauhourat* being 260 metres nearer Cauterets. The water is conducted from the source to the more distant *buvette* by means of pipes. The temperature recorded by me at the distant *buvette* (*Petit Mauhourat*) was 117·150 Fahr., that by MM. DE LALANDE and BROCA 47·50 Cent.=117·50 Fahr. At the principal *buvette*, near the source, my thermometer marked 121·00 Fahr., that of MM. DE LALANDE and BROCA 49 50=121·10 Fahr.

Les Œufs.—We observed the temperature of this water at three points,—namely, at the *griffon;* at the *buvette* 300 metres (=984·270 English feet) distant from the griffon (nearer Cauterets, and in the same building as *Petit Mauhourat*) ; and again 1400 metres (=4593·259 English feet) below the latter point (or 1700 metres from the griffon) at the *Pont de la Raillère*. It is intended to carry the water of *Les Œufs* into Cauterets by means of pipes, the operations for which had been conducted only as far as the *Pont de la Raillère*, hence I had an opportunity of recording the temperature of the water there. The distance still to be accomplished, from the *Pont de la Raillère* to Cauterets, is 1200 metres (=3937·079 English feet). The temperature of the water at the several points of observation was as follows :—

	Thermometer D.	MM. de Lalande and Broca.
Griffon, ...	131·00 Fahr.	55·00 Cent. =131·00 Fahr.
Buvette 300 metres from griffon,	128·00 „	53·50 „ =128·30 „
Escape 1700 metres from griffon,	119·00 „	48·50 „ =119·30 „

Le Pré.—The temperature of this water was taken at the *buvette*, which is

constructed at a distance of 15 metres (=49·213 English feet) from the *griffon*. The temperature recorded by me was 117·40 Fahr., that by MM. DE LALANDE and BROCA 47·50 Cent.=117·50 Fahr.

Petit St Sauveur.—The temperature of this spring was taken at the *griffon*. I found it to be 93·00 Fahr. MM. DE LALANDE and BROCA recorded 34·00 Cent.=93·20 Fahr.

Eastern Group.

These springs issue from metamorphic schist. We visited them in the following order:—

César.—We observed the temperature of this water at three points,—namely, at the *griffon*; at the *buvette* in the building near the source (distance of this *buvette* from the *griffon* 60 metres =196·854 English feet); and at the *buvette* of the establishment in the town (distance of this *buvette* from the *griffon* 350 metres =1148·350 English feet). The temperature of the water at the several points of observation was as follows:—

	Thermometer D.	MM. de Lalande and Broca.
Griffon, ...	117·40 Fahr.	47·50 Cent. = 117·50 Fahr.
Buvette near the griffon (60 metres),	116·30 „	47·00 „ =116·60 „
Buvette at the establishment (350 metres),	114·00 „	45·60 „ =114·08 „

Espagnols.—The temperature of this spring was observed only at the *griffon*, where I found it to be 115·50 Fahr., and at the same time MM. DE LALANDE and BROCA recorded a temperature of 46·60 Cent.=115·88 Fahr.

Pauze.—The temperature of this water was observed at two points,—at the *griffon*, which is not more than 80 centimetres from the *griffon* of Espagnols; and again at the *buvette*, which is distant 10 metres (=32·809 English feet) from the griffon. The temperature recorded was as follows:—

	Thermometer D.	MM. de Lalande and Broca.
Griffon, ...	109·75 Fahr.	43·25 Cent. =109·85 Fahr.
Buvette, ...	102·30 „	39·15 „ =102·47 „

The following table shows the temperature of the principal springs at Cauterets, as recorded by different observers at various periods:—

Name of Observer, and Date of Observation.	Raillère. Griffon.		Mauhourat. Buvette of Griffon.		Œufs. Griffon.		Petit St Sauveur. Griffon.		Pauze-Vieux. Griffon.		César. Griffon.	
	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.
M. Arago (1826), . . .	101·12	38·40	121·28	49·60	...	...	...	...	113·00	45·00	117·68	47·60
Professor Forbes (Aug. 1835), .	101·90	38·80	121·70	49·80	130·10	54·50	90·60	32·50	110·30	43·50	118·10	47·80
M. Fontan (Sept. 1835), . .	102·65	39·25	121·37	49·65	...	...	91·40	33·00	113·00	45·00	118·49	48·05
M. Fontan (Sept. 1837), . .	...	...	122·00	50·00	...	...	...	...	...	...	...	...
M. Gintrac (1841), . . .	101·30	38·50	122·00	50·00	131·00	55·00	...	...	...	...	118·40	48·00
Professor Filhol (1850), . .	102·20	39·00	120·20	49·00	131·00	55·00	...	...	...	...	119·30	48·50
MM. de Lalande and Broca, (Aug. 1863),	102·20	39·00	121·10	49·50	131·00	55·00	93·20	34·00	109·85	43·25	117·50	47·50
Dr Scoresby-Jackson (Aug. 1863),	101·80	38·78	121·00	49·45	131·00	55·00	93·00	33·90	109·75	43·20	117·40	47·45

ST SAUVEUR.

There are several mineral springs at St Sauveur, but only two of importance, namely, *La Source des Bains*, and *La Hontalade*. As the water of the latter is nearly cold, I did not think it of importance to test the temperature accurately.

Unfortunately, I had not the advantage of a good second thermometer at St Sauveur. M. CHARMASSON DE PUYLAVAL, to whom I was indebted for some attention, showed me a broken thermometer with which he had carefully taken the temperature two years previously. I believe the instrument was made by either SALLERON or FASTRÉ of Paris, and appeared to have been such as that used by MM. DE LALANDE and BROCA at Cauterets.

In company with M. DE PUYLAVAL, I visited the establishment which is supplied with water by the *Source des Bains*. This spring is now isolated from the rest, and has been traced for several yards into the rock, whence it is led by means of a pipe into the establishment.

I tried the temperature at the bath and douche nearest the source (according to M. DE PUYLAVAL, 10 or 12 metres from the actual source). I found the temperature to be 93·50 Fahr. M. DE PUYLAVAL tried the temperature with a common bath thermometer, and found it to be 34·50 Cent. (=94·10 Fahr.) and stated that it corresponded exactly with his previous observation made with the now broken thermometer. I then tested the temperature in the bath-room furthest from the source (about 35 metres distant from the actual source, I was informed), and found it to be 93·00 Fahr. With the bath thermometer, M. PUYLAVAL stated it to be 33·80 Cent. (=92·84 Fahr.), and that it corresponded with his previous observations.

Lastly, I obtained permission to enter into the excavated rock, and tried the temperature at a *robinet*, which I understood was within two metres of the actual source, where I found the temperature to be 94·00 Fahr. M. DE PUYLAVAL had never taken the temperature at that point, and did not accompany me into the gallery.

The observations made by Professor FORBES upon the temperature of the spring which supplies the establishment were not satisfactory. He says,—"We have stated that there are reservoirs belonging to the thermal establishment of St Sauveur. The consequence is, that the temperature perpetually varies. I have repeatedly tried it at the 'Buvette.' Thus on the 20th of July 1835, I found it to be 90·2; and on the 24th, only 88·8."

The temperature observed at the *Robinet de la Douche* by various observers at different periods is given in the following table:—

	Centigrade.		Fahrenheit.
M. Fontan (September 1835), ...	34·50	=	94·10
M. Fontan (September 1837), ...	34·50	=	94·10
M. Gintrac (1841), ...	34·50	=	94·10
Professor Filhol (1850), ...	34·20	=	93·56
Dr. Scoresby-Jackson (August 1863), . .	34·17	=	93·50

BARÈGES.

I visited Barèges after leaving St Sauveur, and on the same day observed the temperature of the springs at both places. At Barèges I had the able and cordial assistance of Dr LE BRET, the medical inspector of the establishment, and well known as one of the editors of the *Dictionnaire Général des Eaux Minérales*. The establishment at Barèges is so constructed that the *griffons* are not approachable. There are no galleries, the water being conducted immediately into the baths, which are built close to the actual sources. The only experiment which I made at Barèges was on the temperature of the *Tambour* (called also *Grande-Douche*). I observed the temperature of this spring at the *Buvette* of the new establishment, the distance, as I understood from Dr LE BRET, between the point of observation and the actual source not exceeding one *metre*. I observed the temperature with thermometer D, and at the same time Dr LE BRET observed it with his own thermometer, which was a much shorter instrument than mine, and the Centigrade degrees were closely marked. The result of our observations was as follows:—

	Fahrenheit.		Centigrade.	Difference (Fahr.)
Thermometer D, ...	109·10	=	42·83	1·20
Dr Le Bret, ...	110·30	=	43·50	

The difference between our observations could not be reduced by the most careful repetition, and therefore I concluded that the thermometers were at variance. On returning to Dr LE BRET's house, I placed both thermometers in a tumbler of cold spring water, and having allowed them to remain a sufficient length of time, we both examined the instruments with the following result:—

	Fahrenheit.		Centigrade.	Difference (Fahr.)
Thermometer D, ...	64·40	=	17·83	1·20
Dr Le Bret, ...	65·30	=	18·50	

By this experiment I conclude that our observations at the spring were both correct, and that between the temperatures 18·50 and 43·50 Centigrade, Dr LE BRET's thermometer marks 1·20 Fahr. higher than mine.

Dr LE BRET kindly furnished me with the results of observations made by Professor FILHOL in 1862; they will be found at length in the *Annales de la Société d'Hydrologie Medicale de Paris*, tome neuvième, p. 369. I give them along with others in the following tabular form:—

Name of Observer, and Date of Observation.	Tambour or Grande Douche.				L'Entree.				La Chapelle.				Polard.			
	Buvette.		Robinet of Grande-Douche.		Robinet of Baths.		R. of Cabinet 7.		Robinet of Baths.		R. of Cabinet 1.		Cistern.		Robinet of Baths.	
	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.
M. Arago (1826), . . .	...	...	111·38	44·10	99·87	37·7	...	...	...	...	...	...	...	...	98·78	37·1
Prof. Forbes (July 1835),	...	...	111·9	44·39	104·4	40·22	...	...	88·7	31·50	...	...	97·9	36·61	...	...
M. Fontan (Sept. 1835), .	...	...	112·55	44·75	104·55	40·30	...	...	89·23	31·80	...	...	...	...	99·13	37·30
M. Fontan (Sept. 1837), .	...	...	112·55	44·75	104·72	40·40	...	...	89·15	31·75	...	...	...	...	101·40	38·55
M. Gintrac (1841), . . .	...	...	113·00	45·00	...	...	...	...	87·80	31·00	...	...	...	...	...	...
Professor Filhol (1850), .	...	...	110·48	43·60	...	...	...	...	87·98	31·10	...	...	...	...	...	...
Professor Filhol (1862), .	109·40	43·00	111·38	44·10	...	...	104·00	40·00	...	...	91·40	33·00	...	...	98·60 Cabinet, No. 12.	37·00
Dr Le Bret (Aug. 1863), .	110·30	43·50	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Dr Scoresby-Jackson (1863),	109·10	42·83	...	...	...	...	...	...	...	...	...	...	...	...	...	...

BAGNÈRES-DE-BIGORRE.

At Bagnères-de-Bigorre, I had the kind and able assistance of J. MAXWELL-LYTE, Esq., an Englishman of scientific attainments, who has resided in Bagnères for eleven years. The temperature of the springs was observed very carefully by means of several instruments. I used, as on all occasions, my thermometer D. The instruments employed by Mr LYTE were A, a thermometer made by GREINER of Berlin, and sold in London by HORNE & Co. of Newgate Street. The degrees on this instrument were marked, both in Reaumur and Fahrenheit, forty-five of the latter corresponding to an inch of my pocket tape. B, a thermometer made by FASTRÉ of Paris, with a maximum index; this instrument had twelve Centigrade degrees to the inch of my tape. C, a thermometer (lent for the occasion by M. SOUBERVIE, medical inspector at Bagnères) by FASTRÉ of Paris, having eight degrees to the inch of my tape. Before leaving Mr LYTE's house, we dipped three of the thermometers (we had not then got M. SOUBERVIE's instrument) into a large beaker, containing cool spring water. The result was as follows:—

	Fahrenheit.
Thermometer D, ...	78·40
Greiner's thermometer, ...	77·50
Maximum, ... 25·70 Cent. =	78·26

In the following remarks, I shall speak only of the temperature as recorded by thermometer D and the maximum thermometer, as there was less difference between them than between the other instruments and mine; but I have kept a record of the temperature afforded by all the instruments.

We first visited the establishment known as *Le Salut*, between six and seven hundred yards from the town. There we observed the temperature of three springs, namely:—

* See explanation of difference in text.

1. *Source de la Montagne.*—The temperature of this spring was taken at its source in a gallery, excavated at a little distance from the building.

2. *Source de l'Interieur.*—Temperature taken at the *buvette* in the building, the distance from that point to the actual source, as I understood, being about one metre.

3. *Source de la Pompe.*—Temperature taken at the *buvette* in the building, being, as I understood, close to the griffon.

The temperature of the three springs was as follows :—

	Thermometer D.	Maximum.
Source de la Montagne, . . .	89·00 Fahr.	31·60 Cent. = 88·88 Fahr.
La Buvette de l'Interieur, . .	87·80 ,,	30·90 ,, = 87·62 ,,
Source de la Pompe, . . .	87·00 ,,	30·40 ,, = 86·72 ,,

Afterwards, accompanied by a workman to remove the coverings, we visited the sources of the springs which supply the principal establishment situated in the town. We visited the springs in the following order :—

1. *Source du Platane.*—At this spring we could easily read the thermometers whilst immersed in the water.

2. *Source du Foulon.*—Here the water was too far below the floor of the gallery to permit of our reading the thermometers accurately whilst in the water. We therefore placed the instruments in a quart bottle, which was lowered by means of a cord into the water, filled, and allowed to remain several minutes. It was then raised, and the thermometers were immediately read before their removal from the bottle.

3. *Source du Dauphin.*—At this spring we could readily read the instruments whilst immersed in the water.

4. *Source de la Reine.*—The approach to this spring is not so easy as to the others. We were able to read the instruments distinctly whilst immersed in the water. We also took the temperature of the water of La Reine at the *buvette* in the establishment; the distance between the latter point and the actual source where we had previously observed it, being, as I understood, about 100 metres (328 English feet).

5. *Source de Salies.*—This spring rises in the open space, about twenty paces from the right-hand corner of the establishment (the observer looking at the front of the building). At present the spring is open, half a dozen steps leading down to it; but it is proposed to extend a wing of the establishment over it.

The temperatures observed were as follows :—

	Thermometer D.	Maximum.
Source du Platane, . . .	92·30 Fahr.	33·40 Cent. = 92·12 Fahr.
Source du Foulon, . . .	95·00 ,,	35·00 ,, = 95·00 ,,
Source du Dauphin, . . .	119·40 ,,	48·40 ,, = 119·12 ,,
Source de la Reine (*griffon*), .	115·00 ,,	46·00 ,, = 114·80 ,,
,, ,, (*buvette*) .	110·40 ,,	43·50 ,, = 110·30 ,,
Source de Salies, . . .	123·00 ,,	50·45 ,, = 122·81 ,,

The following table shows the temperature of several of the springs at Bagnères-de-Bigorre, as recorded by various observers at different periods:—

Name of Observer and Date of Observation.	Foulon.				Dauphin.				La Reine.						Salies.	
	Griffon.		Bath.		Griffon.		Conduit.		Griffon.		Conduit.		Buvette.		Griffon.	
	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.
M. Arago (1826), ...	...	...	...	...	...	...	...	...	114·80	46·00	...	...	...	...	122·90	50·5
Prof. Forbes (Aug. 1835), .	...	...	93·20	34·00	...	...	119·0	48·33	...	...	114·0	45·6	...	...	...	...
M. Fontan (Oct. 1836), . .	...	...	{95·54 Robinet.}	35·30	122·00	50·00	...	...	115·88	46·60	...	...	...	...	125·24	51·80
M. Fontan (Sept. 1837) , .	...	...	94·10	34·50	118·94	48·30	...	...	115·70	46·50	...	...	...	...	123·98	51·10
Professor Filhol (1861), . .	95·90	35·50	...	...	119·75	48·75	...	...	115·70	46·50	...	...	...	...	123·44	50·80
F. Maxwell-Lyte, Esq. (1863),	95·00	35·00	...	...	119·12	48·40	...	...	114·80	46·00	...	...	110·30	43·50	122·81	50·45
Dr Sowerby-Jackson (1863),	95·00	35·00	...	...	119·40	48·55	...	...	115·00	46·11	...	...	110·40	43·55	123·00	50·55

BAGNÈRES-DE-LUCHON.

I spent several hours in the galleries at Luchon, and observed the temperature of many, but not of all, the springs. My notes of the temperatures are before me now, but I scarcely think they are calculated to fulfil the present object of determining whether the temperature of these springs be constant or not. So much engineering skill has been spent upon the excavations at Luchon that I suppose Principal Forbes, who visited the place in 1835, would scarcely recognise the places where he dipped his thermometers; and, as still further exploration is determined upon, probably in a short time the places where I made my observations will be obliterated. Another reason why I am not anxious to publish my observations made at Luchon is, that I had no second thermometer whereby to check the reading of my own. Unfortunately Dr Lambron, from whom I received much attention and kindness, had left his thermometer in Paris; and M. Fontan, the talented inspector, from whom I also received much kindness, had in like manner left his thermometer at his winter residence. I was accompanied into the galleries by an intelligent workman, who afforded me all the assistance in his power; but I am not sure that he clearly apprehended my questions, though he never failed to make a very spirited reply. For the temperatures of the springs I must refer to the works of M. Fontan, Dr Lambron, and M. Filhol.

I spent a short time at the baths of Ussat (Ariége), and noted the temperature of the springs. I was kindly aided by Dr Ourgaud, the medical inspector, in whose work will be found a notice of the temperature, and of other matters of interest connected with the mineral water.

AX ARIÉGE.

For determining the question of the variation of the temperature of mineral

springs, Ax was in some respects the most interesting of the places visited in the Pyrenees. At Bagnères-de-Luchon, art had been so well applied to render the place as attractive as possible to summer visitors, that no trace of the natural outlet of the water upon the surface of the ground remained; and even the appointed places for its escape had frequently been changed. But at Ax nature had been left almost undisturbed; the mineral waters were still oozing from their natural crevices, and the *Place du Breilh* still answered exactly to the diagrams given in Principal Forbes' paper of 1836.

At Ax I had the able assistance of Dr Garrigou, medical inspector of the waters. He mentioned that he had examined no less than seventy-eight distinct springs, and that he had since discovered many others.

I observed the temperature of several springs; but I need mention only two, namely, the *Canons* and *Rossignol*. That the temperatures of these springs were observed exactly at the same places as they were by Principal Forbes in 1835 is almost certain, for there appears to have been no change whatever made in the Place du Breilh. Dr Garrigou was decidedly of opinion that the places of observation were identical. The thermometer used by Dr Garrigou was made by Fastré of Paris, and contained eight degrees of Centigrade to the inch of my tape: I made use of my thermometer D.

Canons.—The temperature of this spring was ascertained at the taps (in the Place du Breilh), from which it issues in full streams. The water is utilised at this spot by the inhabitants of Ax, and at the time of our observations (about eight A.M.) several women were at the taps washing vegetables. We borrowed a large tin vessel from one of them, and allowed the water to fill and overflow it for a few minutes. We then immersed our thermometers, the water from the tap continuing to flow into and over the vessel. We did this at both taps, and carefully observed the temperature several times.

The temperature recorded was:—

Dr Garrigou, ...	74·80 Cent.	=	166·64 Fahr.
Thermometer D, ...	...		166·20 „
	Difference,		·44

Rossignol.—Exactly at the spot indicated by Principal Forbes (and, if I remember rightly, about 20 feet from the taps of the Canons), a large stone was lifted, thereby exposing the actual source of the Rossignol spring. Here we plunged our thermometers at once into the water as it flowed to its channel under the street.

The temperature recorded, after several observations, was,—

Dr Garrigou, ...	77·50 Cent.	=	171·50 Fahr.
Thermometer D, ...	...		171·00 „
	Difference,		·50

There is therefore a difference of half a degree of Fahrenheit between our observations in the one case, and of nearly the same amount in the other. I remember that Dr GARRIGOU explained to me that the difference was probably due to a slight alteration in his thermometer. I made a note of his remark at the time, but have mislaid it, and I can scarcely venture to repeat it from memory.

But whilst, on the one hand, Ax is one of the best places for testing the constancy of the temperature of hot springs, because of the locality not having undergone any material change; still, on the other hand, perhaps but little dependence can be placed upon the results, in consequence of the exposure of the water in its course to many external influences. Dr GARRIGOU has noticed that the temperature of most of the springs at Ax varies with the season, and he adds: *Il n'est pas étonnant qu'à Ax, où les sources sont mal captées, au milieu d'alluvions, et dans le voisinage de plusieurs rivières, on ait observé des variations fréquentes dans les indications fournies par le thermomètre.*

The following table shows the temperatures recorded by various observers at different periods:—

Name of Observer and Date of Observation.	Canons. Taps.		Rossignol. Griffon.	
	Fahr.	Cent.	Fahr.	Cent.
M. Pilhes (1786), ...	168·98	76·10	...	...
Professor Forbes (1835), ...	168·0	75·6	161·2	71·8
M. Fontan (1835), ...	168·26	75·70	166·12	74·50
M. Gintrac (1841), ...	167·00	75·00	163·40	73·00
M. Roux (1842), ...	168·26	75·50	166·12	74·50
Professor Filhol (1853), ...	167·72	75·40	171·50	77·50
Dr Garrigou (1861, summer), . . .	167·36	75·20	170·96	77·20
Dr Garrigou (1862, winter), . . .	167·72	75·40	172·04	77·80
Dr Garrigou (1863, August), . . .	166·64	74·80	171·50	77·50
Dr Scoresby-Jackson (1863, August),	166·20	74·55	171·00	77·22

From these observations it would appear that whilst there is, perhaps, in no instance a permanent change of temperature, neither is there in any an undeviating temperature. It is probable that the temperatures of these springs in the interior of the globe have undergone no change, and that the changes observable upon the earth's surface are due to superficial and evanescent causes,—such as external temperature, the infiltration of cold surface-water, and the like. To a certain extent, an allowance must be made for inaccuracies; for it is scarcely to be supposed that all the observers dipped their thermometers exactly at the same points, nor do I know that, in all cases, errors of instruments were recognised and corrected.

XXXIII.—*On Superposition.* By the Rev. PHILIP KELLAND, M.A., F.R.S., Professor of Mathematics in the University of Edinburgh. Part II. (Continued from Vol. XXI. p. 273.) (Plate XX.)

(Read 7th March 1864.)

In my former paper on the subject, I selected the following problem :—

From a given square, one quarter is cut off, to divide the remaining gnomon into four such parts that they shall be capable of forming a square.

The gnomon is, I assume, incapable of being formed into a square by being cut into three parts, and consequently the number of different ways in which it can be so formed, by cutting it into four parts, must be very limited. But, to show the fertility of the method of superposition, I exhibited the solution of the problem in twelve different manners. Many of these, no doubt, have much that is in common, whilst, on the other hand, some (such as the 12th) differ in every feature from the rest. I had thoughts of following up my plea for the study of the old geometry, by exhibiting the solutions of the 47th proposition of Euclid's first book in their beautiful variety. I have indeed temptation to do so. The modification which I gave of the demonstration of this proposition in the notes to my edition of Playfair's Geometry (edition 1846, p. 273), has had the honour of being exhibited in two different mechanical forms. The first by two rotations without sliding, whereby the two squares on the sides, when placed together, are converted into the square on the hypothenuse; the second, by two transpositions (slidings) without rotation, whereby the same change is effected The former is obvious enough, and could have escaped nobody. The latter is described by Professor DE-MORGAN in the "Quarterly Journal of Mathematics," vol. i. p. 236.

I venture, however, to think that the problem before us is still more curious, as an exemplification of the method of superposition, than the 47th of Euclid's first book. With this feeling I have overcome the hesitation I long experienced in presenting the following twelve additional solutions to the Society. The solutions are numbered in continuation of my former paper, and the values of x and a are the same as in that paper.

Constructions and Demonstrations.

XIII.—BY$=x$, EG$=x-a$. Place (3) and (4) on (1) as in the second figure, they will just fill the diagonal of (1). And since the remaining portion of the first figure is a rectangle, whose sides are a and $x-a$, it exactly completes the second figure; hence the conclusion.

XIV. BG and BY are the same as in the last method, whilst a parallel is drawn from Y instead of from G.

(1), (2), and (3) will unite as in the second figure. Also, since the sides of (3) and (4) are a and $x-a$, the square is complete (by No. XIII.); hence the conclusion.

XV. (1) and (3) are the same as in method XIII: (4) is constructed by drawing GH equal and parallel to AY, and completing the rectangle. (1), (2), and (3) will unite as in the second figure; and the conclusion is effected as in XIV.

XVI. Cut off BY, BV, each equal to x, draw VE parallel to BY, join CY and produce it to meet VE in D.

(1), (2), and (3) will unite, as in the second figure; and the length of (4) is the same as that of (3); hence the conclusion.

The division is, in this case, effected by two cuts.

Cor. A modification of this method may be produced by omitting CD, and a triangle equal to CDE out of (3), as in the dotted line.

XVII. Make $BT=x$; draw TQ parallel to AB; make $CR=DQ$, and draw FG parallel to AQ through any point F, within the limits indicated in the figure by cutting the points C and T; draw RH parallel to AB.

(1), (2), (3), and (4) will unite as in the second figure; hence the conclusion.

XVIII. Bisect PC in Q. With centre B and radius $BR=x$, describe a circle; and from Q draw QR touching the circle in R; produce QR to meet BA in S; make $BT=BS$, and draw TV parallel to BR meeting DE in V.

(1), (2), and (3), will unite as in the second figure, and the side of (4) will be in the same straight line with the sides of (1) and (2); hence the conclusion.

XIX. Bisect CD in Q. With centre A and radius $AR=x$, describe a circle; and from Q draw QR touching the circle in R; produce QR to meet BE in H; make $ET=a$, and draw TL parallel to AB.

(1) and (3) unite as in the second figure, so that the longest side of the united figure is $2a$, consequently (2) falls upon it; hence the conclusion.

The division may, in this instance, be effected by two cuts.

XX. Bisect CD in F, draw FG parallel to AB; make $AH=AY=2a-x$; join AH, and draw YZ parallel to BE.

The triangle is HGK equal to AYZ, hence (1), (2), (3), and (4) will unite, as in the second figure, and the figure is a square.

XXI. Bisect CD in G; draw GH parallel to AB; draw BJ, making an angle of 30° with BH, and AK perpendicular to BJ.

(1), (2), and (4) will unite as in the second figure, and AK and BJ are each equal to x; hence the conclusion.

XXII. Bisect CD in G, and through G draw GH parallel to AB, meeting AP in J. Draw GL, making an angle of 30° with CD, and make $LO=a$. Through

O, H, and J draw perpendiculars to GL, produce those through O and J, their own length, to X and Y; and complete the figure.

The three perpendiculars are each equal to $\frac{1}{3}x$, and the portions (4) and (iv.). (3) and (iii.), (2) and (ii.), are respectively equal; hence the conclusion.

The division may, in this instance, be effected by two cuts.

XXIII. Make $BY = x$, $CH = AY$; draw KHO parallel to BE; make $EF = KL$, and complete the rectangle JCOP; YL, KH, and LJ, JF are the cutting lines.

Because $PO = CJ = x = BY$; triangle $PKO = YLB$, also because $KO = BE$, $KH + OQ = a$; and $BJ = \frac{1}{2}BF = \frac{1}{2}(2a - EF) = \frac{1}{2}(2a - KL) = a - KH = QC$; therefore triangle $EQO = JBF$; hence the conclusion.

XXIV. Draw AK, making an angle of 30° with AB; make $AF = EK$, and from F, E and H (the fourth corner of the square) draw perpendiculars to AK AK, FG, and JL are the cutting lines.

For $\Pi L = x$, and because $AB = 2PH$, $AK = 2HJ$, therefore $BK = 2PJ = HJ$, and $FC = \frac{1}{2}AF = \frac{1}{2}EK = \frac{1}{3}(2a - BK) = a - PJ = GJ$. Again $FB = 2a - AF = 2a - FK = BK = HJ$; hence GJHO is equal to (4), and $DO = PJ$; $DE = AP$, consequently DETO is equal to (2); hence the conclusion.

The division may, in this instance, be effected by two cuts, as in No. XXII.

It will be observed that the angle of section is in all cases either 30°, or are such that the tangent of the angle is a function of $\sqrt{3}$, and of no other surd. For instance, in No. XVIII. we shall find $\tan. RAS = \frac{2\sqrt{3}-3}{2\sqrt{3}-2}$

I have only to add, that a large number of the solutions are due to various friends, including students, to whom my best thanks are tendered. It would not be easy to fix the authorship of each solution with certainty, on which account I shall not attempt it.

XV

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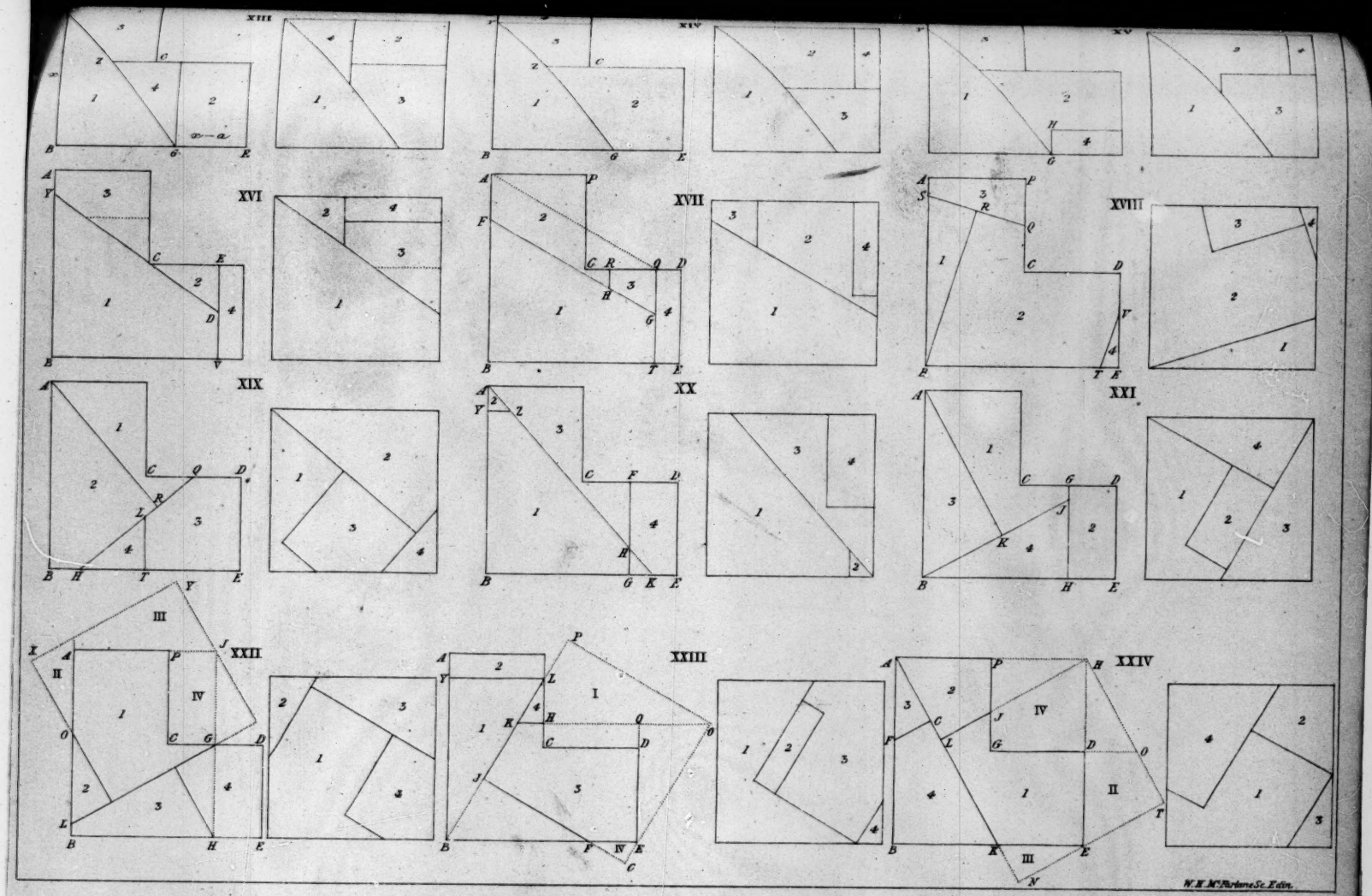
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XXXIV.—*On the Variations of the Fertility and Fecundity of Women according to Age.* By J. Matthews Duncan, M.D.

(Read 2d May 1864.)

In 1855, when the systematic registration of births in Scotland was established, the schedule used exacted from the public a variety of interesting details in connection with each return,—a circumstance which gives to the registers for that year an extraordinary value. For, in consequence, I believe, of numerous complaints regarding the irksome labour of filling up the document, it was discontinued, and a much less comprehensive schedule has been in use ever since. It is from the registers of births for 1855 that I have extracted almost all the data which have yielded the results I am now about to communicate. Similar data cannot be found in the subsequent registers. The great value of these registers has been distinctly pointed out by Dr Stark, the able assistant to the Registrar-General. I must here acknowledge, with grateful thanks, the assistance and encouragement I have received from Mr Seton and other officials of the Register-House.

The exigencies of time, labour, and expense, constrained me to restrict the number of births to be operated on within moderate limits; and I selected Edinburgh and Glasgow, with their 16,593 children legitimately born in 1855, as the field of operations. It is needless to enter fully upon the reasons of my selecting the conditions of legitimate birth in Edinburgh and Glasgow; my only object was to secure as much as possible of accuracy and completeness in the filling up of the schedules. It must be noted, that legitimate births, as registered, include only births of living children at the full term of pregnancy or near it.

The well-known difficulty of handling statistics without infringement of the rules of logic has made me be very cautious in my progress in this investigation, and I am all the more bound to be careful, because it will be necessary, in connection with my present topic, to point out great errors made by authors who have entered upon it. But although I trust no fault will be found with my mode of reasoning, I have to admit the existence of some comparatively few and unimportant errors in the details given in the registers. The chief of these will be stated in connection with the tables to be brought forward. So far as I know the errors are all in the original registers; in the elaboration of the details thence derived, I have spared nothing that could insure accuracy; and must here men-

tion my obligations to the various intelligent and assiduous gentlemen who have assisted me in the work.

The part of my investigations which is the subject of this communication is confined to the determination of the comparative fertility or productiveness and fecundity of women at different ages. It is necessary, in order to avoid confusion, here to establish a distinction, which I shall maintain as I go on, between fertility or productiveness and fecundity. By fertility or productiveness I mean the amount of births as distinguished from the capability to bear. This quality of fertility or productiveness is interesting chiefly to the statistician or the political economist. It does not involve the capability of every individual considered to bear, nor even the conditions necessary for conception. By fecundity I mean the capability to bear children; it is measured by the number born, and it implies the conditions necessary for conception in the women of whom its variations are predicated. This quality of fecundity is interesting chiefly to the physiologist and physician.

In discussing the subject of comparative fertility and fecundity at different ages, I may incidentally afford means for estimating the degree of fertility or fecundity of different ages; but I wish it to be distinctly understood, that I have not proposed to myself, in this paper, the consideration of the actual degree or amount of fertility or fecundity at any age, but only the variations of fertility or fecundity at different ages as compared with one another.

Chapter I.—*The Actual Fertility of our Female Population as a whole at Different Ages.*

The first law which I propose to establish, has reference to the ages of the mothers of legitimate children. In Edinburgh and Glasgow legitimate births form at least 90 per cent. of the whole born. The law, therefore, regards the ages of the women from whose fertility 90 per cent. of the population are recruited.

It must be observed, that this law or general statement shows nothing regarding the fecundity of women of different ages, although it has been held as doing so; it merely enunciates a truth in the doctrine of population. I place it first because it is pretty well known, because in my own investigations it was first made out, and chiefly because it is essential, before proceeding farther, to show the facts on which it is founded in their true light, avoiding the great errors of which they have been made the basis.*

The facts or data illustrating this law, with which I am best acquainted, have been derived from reports of lying-in dispensaries, as by Dr Granville, or from similar accounts of lying-in hospitals, as by Dr Collins, Drs Hardy and M'Clin-

* See Granville, Transactions of Obstetrical Society of London, vol. ii.

TOCK, and others. I here present, as an example, the table of ages of mothers of legitimate and illegitimate offspring, whether born alive or dead, from the "Practical Treatise on Midwifery" of Dr COLLINS, master of the Dublin Lying-in Hospital. The data adduced by Dr GRANVILLE in the second volume of the "Transactions of the Obstetrical Society of London," are closely similar. Judging from these data, it would appear that most children are born of women at or near the age of 30 years or the middle of the child-bearing period of life; and that the offspring of mothers of ages advancing from the commencement of child-bearing to the age of 30 or the middle of the child-bearing period gradually increases; that the climax is reached at this age, and that thereafter the offspring of mothers advancing above 30 gradually diminishes. But while the age of 30 forms the climax, there is not an equal fertility on either side of it; a much larger part of the population being born of mothers under 30 than of mothers above 30. Dividing the number of mothers at 30 years, and adding together those on each side of the division, we have on the side of the younger 12,106, and on the side of the elder women 4279, giving a majority of 7827 in favour of the younger: or, otherwise stated, we have three-fourths of the births among the younger half, and only one-fourth among the elder. The mean age of the mothers in Dr COLLINS's table is 27 years.

TABLE I.—SHOWING THE AGE OF EACH OF 16,385 WOMEN, DELIVERED IN THE DUBLIN LYING-IN HOSPITAL.

Age,	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
No. of Women,	3	19	70	237	433	926	682	1142	1023	1089	1174	1295	983	1340	517	2346	242

Age,	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	50	53
No. of Women,	467	378	384	396	379	153	217	65	326	15	21	18	17	11	5	6	5	1

TABLE II.—SHOWING THE AGES OF 16,385 WOMEN DELIVERED IN THE DUBLIN LYING-IN HOSPITAL, ARRANGED IN PERIODS OF FIVE YEARS.

Age,	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50 and over.
No. of Women,	762	4862	5309	3817	1210	397	22	6
Percentage,	4·65	29·67	32·40	23·29	7·38	2·42	·13	·03

The next table which I present shows an arranged collection of data, comprising the wives-mothers of living children born at or near the full time in Edinburgh and Glasgow in 1855. The former table has, regarding the use to be

made of it, the advantage over this table of including all mothers bearing children, whether legitimate or not, alive or dead, in the Dublin Hospital. But in every other respect, this second table presents what I judge to be more reliable data. The former table contains a class of cases selected according to complicated conditions which it is impossible to state, but which are the result of the correlated circumstances of the Hospital, and of the class from which in Dublin it draws its patients. In the second table the conditions of selection are fewer and less important, the chief being the legitimacy, life and maturity, or at least viability, of the offspring. Now the limits of the influence of these different conditions are pretty well known, and the proportional differences between the two tables are too great to be accounted for by these differences. The second table is thus shown to be the more trustworthy.

TABLE III.—Showing the Age of each of 16,301 Wives whose Children were Registered in Edinburgh and Glasgow in 1855.

Ages, .	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
Mothers,	4	28	116	228	405	543	828	888	1024	1058	1063	925	1116	875	121	545	825	645	621

Ages, .	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	57	Total
Mothers,	691	594	409	426	287	404	142	148	80	66	50	27	6	9	4	–	2	4	1	16,301

TABLE IV.—Showing the Ages of 16,301 Wives-Mothers in Edinburgh and Glasgow in 1855, arranged in Periods of Five Years.

Ages, . . .	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	Total
Mothers, . .	376	3688	5037	3850	2407	840	96	6	1	16,301
Percentage, .	2·30	22·62	30·89	23·61	14·76	5·15	·58	·03	—	

An inspection of this table shows again that the year of maternal life yielding most recruits to the general population is the thirtieth, and an easy calculation makes out that about three-fifths of the legitimately born population are derived from women of 30 years and under, while two-fifths are derived from women of 30 years and upwards. For, dividing mothers of 30 years of age, and adding together those on each side of the point of division, we have on the side of the younger 9708 mothers, and on the side of the elder 6593, giving a majority of 3115 in favour of the younger. The mean age of the wives-mothers in this table is above 29.

From these data I conclude,

1. That the actual, not the relative, fertility of our female population, as a whole, at different ages, increases from the commencement of the child-bearing period of life, until the age of 30 is reached, and then declines to its extinction with the child-bearing faculty.

2. That the actual fertility is much greater before the climax, thirty years, is reached, than after it is passed.

3. That at least three-fifths of the population are recruited from women not exceeding 30 years of age.

Before leaving these tables, it is expedient to direct attention to a striking lowness of figure at the ages of 29 and 31 respectively in Dr COLLINS's data. A similar fall on each side of the highest number occurs in Dr GRANVILLE's table, which has been referred to, and in every similar table which I know. This curiosity has given rise to very natural and ingenious speculation. Dr GRANVILLE suggests, that by the earlier decrement, nature means to rest awhile and gather strength for the enormous jump she is to make in the following year, and that by the second decrement she means to evince the exhaustion which invariably follows over-exertion. But I cannot acquiesce in this fanciful hypothesis, believing that really no such decrement, jump, and second decrement, occur. My explanation of this tabular phenomenon is suggested by the occurrence of similar falls on each side of the age of 40 years, in COLLINS's table, and in my own and in others. I am too well aware, from ample experience, of the impossibility of getting women's ages stated correctly, especially if they have passed twenty-five years, and have often observed, that when pushed, they say 30 or 40 as a round easy number; and the state of the tables appears to me merely to indicate that women of about 31 and of 41 years of age frequently say they are 30 and 40 years old respectively. In short, these decrements are evidence of the unfortunate element of error which creeps into the most carefully prepared vital statistics on a large scale.

CHAPTER II.—*The Comparative Fertility of the Female Population as a whole at Different Ages.*

Having shewn the actual fertility of women at different ages in our population, I now proceed to the question of the comparative productiveness of our whole female population at different ages. To settle this, it is only necessary to compare the number of children born of women of different ages with the number of women living at the different ages respectively. The result of the calculations involved in this comparison will be to show, not simply (as in Chapter I.,) the numbers of children born of women at different ages, but the number of mothers relatively to the number of women living at different ages—in other words, the comparative fertility or productiveness of our female population as a whole at different ages.

TABLE V.—Showing the Comparative Fertility at Different Ages of the Whole Female Population of Edinburgh and Glasgow.

Ages,	15–19	20–24	25–29	30–34	35–39	40–44	45–49
Women,	31,538	34,631	29,778	24,272	19,362	17,938	13,868
Children,	380	3731	5096	3895	2435	849	97
Proportion of latter to former is 1 in	82·9	9·2	5·8	6·2	7·9	21·1	142·9
Percentage,	1·20	10·77	17·11	16·04	12·57	4·73	·69

Table V. is constructed so as to bring out the desired results, and at the same time, be easily compared with Table VI. It must be observed that the fifth table does not exhibit results whose actual amounts are of much value, but results the value of which is very great with a view to determining the question of comparative amounts or productiveness. For, while the numbers of women at different ages include the whole women living at these ages in Edinburgh and Glasgow in 1861, the numbers of children born at different ages, as given in the table, include only children born under the conditions of legitimacy, live birth, and complete or nearly complete maturity in 1855.

In his work on Annuities* &c., Mr Milne in 1815 published a valuable table, which he describes as "shewing the fecundity of women at the different periods of life in Sweden and Finland, from 1780 to 1795, both years inclusive." It is taken from a paper by Mr Nicander, to which he gives a reference, but unfortunately I have not been able to ascertain the exact conditions (if any) under which the table was prepared.

TABLE VI.—Showing the Comparative Fertility at Different Ages of the Female Population of Sweden and Finland. [*From the Table of* Mr Nicander.]

Ages,	15–19	20–24	25–29	30–34	35–39	40–44	45–49
Females,	132,765	131,377	121,650	112,250	98,710	89,259	74,002
Deliveries,	3298	16,507	26,329	25,618	18,093	8518	1694
Proportion of latter to former, 1 in	40·256	7·959	4·620	4·382	5·456	10·479	43·686
Percentage,	2·48	12·56	21·64	22·82	18·32	9·54	2·28

* Vol. ii. p. 582.

In the last line of both tables (V. and VI.), it will be remarked that the first and last proportional numbers are very low, or that at the beginning of the scale, at the age of from 15 to 19 inclusive, fertility is comparatively very small; and that at the end of the scale, at the age of from 45 to 49 inclusive, fertility is again comparatively very small. This no doubt depends to a great degree on the circumstance, that among the women from 15 to 19 years of age, are included a large proportion of immature girls, and among the women from 45 to 49 years of age, a large proportion of women whose child-bearing powers have disappeared. Keeping in view this undoubted partial explanation of the lowness of the figure, or the lowness of fertility at these ages, the tables are seen to yield interesting results. They shew that the fertility of the populations of Edinburgh and Glasgow, and of Sweden and Finland, increases gradually, till the middle of the child-bearing period of life, or about the age of 30 years, and that then fertility gradually falls off towards its complete extinction.

My knowledge of the conditions under which my own table was framed, as already stated, being exact, as compared with my knowledge of Mr NICANDER'S, I shall, in framing conclusions, adopt the results it affords. On like grounds, I shall excuse myself from proceeding to compare the easily remarked differences of the results of the two tables.

In regard, then, to the comparative fertility of our whole female population at different ages, I conclude—

1. That it increases gradually from the commencement of the child-bearing period of life until about the age of 30 years is reached, and that then it still more gradually declines.

2. That it is greater in the decade of years following the climax of about 30 years of age, than in the decade of years preceding the climax.

CHAPTER III.—*The Comparative Fecundity of the whole Wives in our Population at Different Ages.*

I now proceed to the question of the fecundity, not fertility or productiveness, of the mass of wives of different ages in Edinburgh and Glasgow. In the two preceding chapters the fecundity or comparative power of production at different ages has not been entered on; in them have been considered merely the actual production of children by women of different ages, and the comparative amounts of production by the female population at different ages. It is known that at all ages there is a great mass of spinsters whose productiveness is not tested, and it is of course necessary, in order to determine questions of fecundity, to eliminate all women not living in married life, or not having their fecundity tested in the ordinary way, from our observations and calculations. In this chapter, therefore, the comparison is not of mothers with women living as in chapter ii., but of wives-mothers with wives.

TABLE VII.*—Showing the Comparative Fecundity at Different Ages of the Whole Wives in Edinburgh and Glasgow in 1855.

Ages,	15–19	20–24	25–29	30–34	35–39	40–44	45–49
Wives,	756	8874	14,622	14,579	11,871	10,506	7537
Wives-Mothers,	378	3709	5065	3872	2421	845	96
Proportion of latter to former is 1 in	2·0	2·4	2·9	3·7	4·9	12·4	78·5
Percentage,	50·00	41·79	34·64	26·56	20·39	8·04	1·27

The seventh table establishes a comparison between the numbers of married women of various ages, and the numbers of such women bearing living children. In Edinburgh and Glasgow, the number of wives within the ages of 15 and 49 inclusive, or who might have borne children in 1855, was 68,745, and the number of wives-mothers in the same population, in the same year, was 16,386,† or 1 in 4·2. In the table these are arranged in columns of different ages, so as to exhibit the comparative fecundity of the whole wives of different ages. It will be seen at a glance, that the table shews that the fecundity of the mass of wives is greatest in the first years of the child-bearing period of life, and I regret extremely that the data at my disposal do not permit me to condescend on the circumstances in this respect of each individual year. The table shows that, from the earliest years of child-bearing life onwards, the fecundity of the mass of married women gradually wanes to its extinction. It is also easily made out that while there were 24,252 wives under 30 years of age, and of these 9152 bore children, there were 44,493 wives of ages varying from 30 to 49 years inclusive, and of these only 7234 bore children; or, to speak in round numbers, the wives under 30 years of age were much more than twice as fecund as the wives above 30 years.

* In this table, the actual numbers are given as nearly as possible.

The numbers of wives have been arrived at in the following way. We have the population of Edinburgh and Glasgow in 1851 and 1861, and, by calculation, estimate the population in 1855. We have the actual numbers of wives of different ages in 1861, and by an easy calculation of proportions we reduce the numbers of wives of different ages to the numbers given for 1855.

The number of wives-mothers extracted from the registers of 1855, is 16,301, bearing 16,500 legitimate children. But the Registrar-General's Reports state the number of children as 16,593. Hence it appears that 93 births are omitted in the extracts. These omissions were made on account of manifest carelessness and inaccuracy in the registers. To these 93 births, corresponds the number of 92 mothers, one being deducted for a twin case. These 92 mothers have been added proportionally to the others, in order to make up the total of 16,393.

† The actual number of wives-mothers in Edinburgh and Glasgow in 1855 was 16,393. This figure is in the text reduced to 16,386, and seven wives-mothers omitted, because these seven were altogether exceptional, occurring as they did between the ages of 50 and 57, and could only damage the statement of results.

But a more interesting and valuable comparison may be made by taking the same number of 15 years before and after the middle of child-bearing life, a total period of 30 years, which includes the immense majority of child-bearing women. Doing so, we find that of 24,252 wives under 30 years of age, 9152 bore living children, and that of 36,956 wives of ages from 30 to 44 inclusive, 7138 bore living children. Had the elder women been as prolific as the younger, they would have produced 13,946 children, instead of 7138; that is, the fecundity of the younger women was almost double that of the older.*

The table given in this chapter has been prepared (see footnote) so as to give the actual amounts. I found it possible to do this with a near approach to exactness, and it is evident that in this way the results derived are not only comparative statements, with only relative value, but also statements of actual values.

From the data now given I conclude—

1. That the fecundity of the mass of wives in our population is greatest at the commencement of the child-bearing period of life, and after that epoch gradually diminishes.

2. That the fecundity of the whole wives in our population included within the child-bearing period of life, is, before 30 years of age is reached, more than twice as great as it is after that period.

3. That the fecundity of the wives in our population declines with great rapidity after the age of 40 is reached.

Some of these conclusions may be stated, with the actual numerical results, as follows:—While of all the wives living in Edinburgh and Glasgow between the ages of 15 and 45, one in 3·8 or 26·3 per cent. bore a living child; of those

* The data at my disposal enable me to give the figures for each year of age up to 20. But the numbers are so small, that little value can be placed on the results drawn from them. They seem to me to indicate that the fecundity of the mass of married women is probably highest shortly after the age of 20 is reached. For although the low fecundity of one in 2·57 at twenty years of age, is absorbed in Table VII. in the period from 20 to 24, yet this latter period shows, on the whole, the higher fecundity of 2·4.

TABLE VIII.—Showing the Comparative Fecundity of Wives at Ages of 16, 17, 18, 19, and 20, in Edinburgh and Glasgow, in 1855.

Ages,	16	17	18	19	20
Wives,	13	55	232	455	1043
Wives-Mothers,	4	28	116	228	405
Proportion of latter to former is 1 in	3·25	1·96	2·00	1·99	2·57
Percentage,	30·77	50·91	50·00	50·11	38·83

between the ages of 15 and 29 inclusive, one in 2·6, or 38·4 per cent., bore a living child; and of those between the ages of 30 and 44 inclusive, one in 5·1, or 19·6 per cent., bore a living child.

CHAPTER IV.—*The Initial Fecundity of Women at Different Ages.*

In commencing the statistical inquiry whose results I am now giving, my object was to discover the fecundity of women at different ages, and I now proceed to address myself to this point.

It is not my object to illustrate the subject of the arrival of young girls at the age of maturity, the change of the girl into the fertile woman. In the case of some peoples, facts might be collected regarding wives so young as to be in a large proportion sterile from immaturity; and their fecundity gradually appearing as age advanced, might produce a column of mothers from 10 to 20 years of age, shewing a gradually increasing fecundity of the population at these ages. Even in our tables derived from the data of wives in Edinburgh and Glasgow, some interesting results are to be found, and allowance must be made for a certain amount of immaturity in the wives of from 15 to 20 years of age. But this question of the arrival of girls at maturity is foreign to the present topic. In it all the women are supposed to be mature, and subjected to the conditions essential for procreation.

The fecundity of individual women is known to vary extremely. Some are very frequently pregnant, and repeatedly, or even constantly, have plural births, and thrive with it all. Under like conditions, other women are absolutely sterile, or a miscarriage or a dead mature child forms the climax of their fecundity, and this little may be effected at the expense of permanent constitutional exhaustion. Between these extremes of great fecundity and absolute sterility, there is an unlimited series of varying degrees of fertility. On this interesting aspect of the subject of fecundity, the present research throws little light. It is founded on the result of an aggregate of cases, and can show almost nothing as to individuals. It illustrates the fecundity at different ages of women generally, not the individual fecundity of any.

The table given in last chapter (Table VII.), affords data which cannot be applied to settle the question of the fecundity of women of different ages. For it is evident that among the mass of wives of each succeeding year, or series of years, are included the wives who were once of the former series or part of them—that is, a class of wives whose fecundity has been at least liable to be increased, diminished, or exhausted by procreation, before they have come to form part of the wives in any of the columns after the first. In order to arrive at the fecundity of women or wives of different ages, it is necessary to secure that the condi-

tions of the compared women of these different ages be as nearly the same as possible. This is not attempted in the seventh table.

TABLE IX.*—SHOWING THE INITIAL FECUNDITY OF WOMEN OF DIFFERENT AGES IN THE FIRST YEAR OF MARRIAGE.

Ages of Wives newly Married, . .	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64	65–69	Total.
No. of Wives newly Married, . . .	700	1835	1120	402	205	110	46	20	6	2	1	4447
No. of Wives Mothers within first year of Marriage, . . .	96	339	139	46	19	4	...	...	...	...	...	643
Proportion of latter to former is 1 in	7·3	5·4	8·0	8·7	10·7	27·5	...	...	...	...	...	6·9
Percentage, ...	13·71	18·48	12·41	11·44	9·27	3·63	...	...	...	...	...	14·46

Table ninth is constructed to show the relative initial fecundity of newly-married women of different ages. By the returns of the Registrar-General we calculate how many women at each succeeding year of age contracted marriage in 1855, in Edinburgh and Glasgow. My extracts from the register for 1855, shew how many of these women bore living children before they had been a year married. When the two figures are compared for each age, we have the fecundity at the outset of child-bearing at each age. The table reads as follows:—Of 700 women married between 15 and 19 years of age inclusive, 96 bore a living child before they had been wives for twelve months, or one in every 7·3; and so on.

Table tenth is in every respect the same as the former, only it shows the fecundity within 24 months of married life; or the number of women bearing living children before they were two years married is compared with the number of newly married. The observation that the fecundity within 24 months is much more than twice as much as the fecundity within 12 months of marriage, appears to me to give this table more substantial value than the former, as an indication of the actual fecundity of the outset of child-bearing at different ages.

Both these tables show the highest rate of initial fecundity to be between the

* The number of wives married at different ages in Edinburgh and Glasgow in 1855, is arrived at in the following manner. The marriages in Scotland in 1855 were 19,680. The marriages in Edinburgh and Glasgow in 1855 were 4447. The distribution of the 19,680, according to age at marriage, is given at p. 22 of the Registrar-General's Annual Report for 1861. This distribution requires a correction for the number whose ages at marriage were not known. Calculating on the ages of the whole 19,680, the proportional distribution of the 4447 married in Edinburgh and Glasgow is found to be as in the table above.

ages of 20 and 24 inclusive, and a gradual declension from that time on either side as age diminishes or increases.

TABLE X.—Showing the Initial Fecundity of Women of Different Ages within the First Two Years of Marriage.

Ages of Wives newly Married, . . .	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64	65–69	Total.
No. of Wives newly Married, . . .	700	1835	1120	402	205	110	46	20	6	2	1	4447
No. of Wives Mothers within two years of Marriage, . . .	306	1661	849	253	84	17	2	...	...	...	...	3172
Proportion of latter to former is 1 in	2·3	1·1	1·3	1·5	2·4	6·4	23·0	...	...	...	...	1·4
Percentage, ...	43·71	90·51	75·80	62·93	40·97	15·45	4·35	...	...	...	...	71·33

The two following tables (XI. and XII.) show that on the side of the women younger than 20 years, initial fecundity steadily decreases with age. In regard, however, to these young wives, it may be objected that there is a source of error from immaturity which is certainly very trifling after the age of 20 is reached. And the objection is, theoretically at least, quite just, for it is absurd to attempt to measure the fecundity of women who have not become sexually mature, and the admixture of immature with mature is a source of error, important, directly according to its amount. It is unsatisfactory merely to allege in answer, that immature girls are not likely to be found among young wives in such numbers as to form a source of great error. I have therefore taken the following means to ensure that this source of error be completely excluded.

TABLE XI.—Showing the Initial Fecundity of Women under 20 Years of Age in the First Year of Marriage.

Ages of Wives newly Married, ...	16	17	18	19
No. of Wives newly Married, ...	43	108	225	314
No. of Wives Mothers within first year of Marriage,	2	7	31	56
Proportion of latter to former is 1 in	21·5	15·4	7·2	5·6
Proportion after correction for Immaturity, is 1 in	15·5	12·8	7·0	5·4
Percentage, ...	6·45	7·77	14·70	18·30

TABLE XII.—Showing the Initial Fecundity of Women under 20 Years of Age within the First Two Years of Marriage.

Ages of Wives newly Married, ...	16	17	18	19
No. of Wives newly Married, ...	43	108	225	314
No. of Wives Mothers within two years of Marriage,	4	27	98	177
Proportion of latter to former is 1 in	10·7	4·0	2·3	1·8
Proportion after correction for Immaturity is 1 in .	7·7	3·3	2·1	1·7
Percentage, ...	12·90	30·00	46·44	57·84

The commencement of menstruation is generally considered by physicians an indication of the arrival of sexual maturity. It may be true that some are still immature in whom this phenomenon has shown itself, and it certainly is true that some are mature before its appearance. Yet it forms a generally accredited indication of maturity. The following table (XIII.), framed by Dr Whitehead, is a large collection of data, showing the age of the appearance of menstruation in 4000 individuals in this country. It is easy to calculate what fraction of the whole 4000 had begun to menstruate at 16, 17, 18, and 19 years of age respectively, or, in other words, what fraction was capable of exhibiting fecundity at these ages. This I have done, and have corrected the numbers of wives in tables eleventh and twelfth accordingly, reducing them to similar fractional parts. After making this correction for immaturity, I have calculated the proportions of wives-mothers to wives, and placed the results in the last line. They remain the same so far as to show a steady diminution of fecundity as age diminishes.

TABLE XIII.—"Showing the Age at which Puberty was accomplished in Four Thousand Individuals." (Whitehead, *on Sterility and Abortion*, p. 46.)

At Age of	10 years	9	first Menstruated.	At Age of	19 years	148	first Menstruated.
,,	11 ,,	26	,,	,,	20 ,,	71	,,
,,	12 ,,	136	,,	,,	21 ,,	9	,,
,,	13 ,,	332	,,	,,	22 ,,	6	,,
,,	14 ,,	638	,,	,,	23 ,,	2	,,
,,	15 ,,	761	,,	,,	24 ,,	1	,,
,,	16 ,,	967	,,	,,	25 ,,	1	,,
,,	17 ,,	499	,,	,,	26 ,,	1	,,
,,	18 ,,	393	,,				

From these data I conclude—

1. That the initial fecundity of women gradually waxes to a climax, and then gradually wanes.

2. That initial fecundity is very high from 20 to 34 years of age.

3. That the climax of initial fecundity is probably about the age of 25 years.

At this point my present inquiry is closed. I know of no other way of advancing our knowledge of this subject, than by the collection and analysis of statistics. The only very good quarry for such materials that I know of is the Scottish registers for 1855. The tables adduced might be improved by going over a larger field, and increasing the numbers analysed. But I do not see how the matter in the registers could be turned to more account, without encroaching on another topic which is at the same time closely connected with that under discussion,—viz., the fertility of marriage. Or, as marriage is scarcely admissible as a term in physiology, I should give this subject the title of 'sustained fecundity,' the degrees of fertility which women of different ages, beginning to live with men, continue to exhibit during the child-bearing period of life. In the meantime I shall summarily state, that, so far as I have advanced in this new topic, the evidence gained shows that, speaking generally, young women after the outset of child-bearing continue to exhibit a fecundity greater than is sustained by those married when comparatively elderly. The fourteenth table exhibits some of these results. It reads as follows:—Wives of from 20 to 24 years of age exhibit in the fifth year of marriage a comparative fecundity of 1 in 3·0; wives of from 25 to 29 years of age exhibit in the fifth year of marriage a comparative fecundity of 1 in 14·5; and so on. The table shows that at the fifth, the tenth, and the fifteenth years of marriage, the mass of women youngest married continue to show the greatest fecundity. The mass of younger women not only are more fecund at the outset of child-bearing, but after that time is past, they continue more fecund than older women who have been married the same number of years.

TABLE XIV.*—Showing the Fecundity of Wives in Edinburgh and Glasgow, after three different periods of Continuance in the Married State.

Age of Wives,	20–24	25–29	30–34	35–39	40–44
Fertility in Fifth Year of Marriage,	3·0	14·5	59·9	202·4	698·2
Fertility in Tenth Year of Marriage,	...	4·0	23·2	95·5	404·9
Fertility in Fifteenth Year of Marriage,	...	...	6·5	44·3	340·0

I have made various other inquiries with a view to throw light on the topic of this paper. They refer to variations in sex, size, and weight of newly-born children

* This Table is not prepared so as to show anything more than is described in the text. Especially the figures in one horizontal line should not be compared with those in another horizontal line. To permit this, the table requires large corrections for deaths.

according to the age of the mother, also to the frequency of twins and triplets according to the mother's age. But the results of these investigations have been so imperfect or unsatisfactory that I do not now describe them.

The views hitherto entertained regarding the influence of age on fecundity have been various. "In regard to age (says BURDACH*) fecundity is diminished in the first and last portions of the continuance of the aptitude for procreation. The elk, the bear, &c., have at first only a single young one, then they come to have most frequently two, and at last again only one. The young hamster produces only from 3 to 6 young ones, whilst that of a more advanced age produces from 8 to 16. The same is true of the pig. This rule appears to be general, since it applies also to the entomostraceæ; according to Jurine, the number of the young of the *Monoculus pulex* is at first from 4 to 5, afterwards rising gradually as high as 18. We scarcely ever encounter the births of 3 or 4 children, except in women who have passed the thirtieth year. Precocious marriages are not only less fertile, but the children also which are the result of them have an increased rate of mortality. According to Sadler, every marriage in the families of the peers of England yields 4·40 children when the woman was married below 16 years of age; 4·63 from this age to 20; 5·21 from 20 to 23; and 5·43 from 24 to 27." The notions here expressed by BURDACH are in the main correct; but it is evident that they are very indefinite. They are to be regarded, also, more in the light of happy guesses than of well-founded opinions. BURDACH evidently places chief reliance on the evidence afforded by the numbers at a birth. From many quarters I have received corroboration of BURDACH's statements regarding the increase and subsequent decrease of the number produced at a birth by pluriparous animals, and I have received similar information regarding bitches, guinea pigs, the fertility of hens, &c. When I first paid attention to this subject, the plural births of women appeared to me to form a simple key for the determination of the fecundity of women at different ages. But I soon became dissatisfied with the materials I quickly collected. Woman is not a pluriparous animal, neither does she produce so regularly, or according to season, as the animals with which she is compared. In her the occurrence of twins and triplets is an exception to the normal rule, and the number of children born by her cannot be so simple and sure a test of fecundity, as in the case of animals having multiple litters at stated periods. Indeed, it is apparent that the evidence derived from plural births alone in women may positively mislead, for a woman may be more fertile bearing one child at a time frequently than another bearing twins or triplets more seldom. In this place I shall only say, that the numerical study of twins, in reference to the age of the mother, yields interesting results, which do not confirm BURDACH's statement regarding them, yet are not hostile to the conclusions of this paper. BURDACH, in his work, describes an annual rise and fall in the fecundity of some pluriparous

* Physiologie, tom. ii. page 117.

animals. This annual variation forms a series of wavelets in the course of the great wave running from youth to old age, and culminating in middle life. This annual rise and fall of fecundity he attributes to the influence of cold.

In his "Treatise on Man," M. QUETELET has, with some care, collected the statistical materials available at the time for advancing the settlement of the question of the relation of age to fecundity. He does not allude to the opinions of BURDACH, probably because they have no sufficient foundation, but he refers to MILNE, MALTHUS, SADLER, GRANVILLE, FINLAYSON, and several foreign authors, who have more or less directly tried to throw light on the topic. QUETELET'S whole chapter on the influence of age on the fecundity of marriages is very unsatisfactory. It is at least difficult to reconcile with one another the conclusions arrived at in various parts of this chapter, and I shall not attempt to do it. It is only fair to say, that he seems conscious of the numerical deficiency of data sufficient for a basis of any conclusion, and as an example of the state of matters, the table of SADLER, which he and BURDACH both quote, may be mentioned; in it the number of marriages analysed is under 500, and they are all selected according to extraordinary conditions. The final conclusion which M. QUETELET announces is, that it is before the age of 26 years that we observe the greatest fecundity in women.

The last writer on this topic whom I know of is Dr GRANVILLE, who, in an interesting paper in the London Obstetrical Transactions, returns to the description of his former labours in the same field. In this paper, production or fertility is confounded with productive power or fecundity, and the table to which I have alluded in Chapter I. he describes not as showing the fertility at different ages of the industrial classes of the metropolis, but erroneously, as showing the alternations in the productive power of women at different ages.

In this paper, then, I have, *inter alia*, shown that the great majority of the population is recruited from women under 30 years of age; but that the mass of women in the population, of from 30 to 40 years of age, contribute to the general fertility a larger proportional share than the mass of women of from 20 to 30 years of age:—

Further, that the wives in our population, taken collectively as a mass, show a gradually decreasing fecundity as age advances; but that the average individual wife shows a degree of fecundity which increases till probably about the age of 25, and then diminishes.

The fertility of the average individual woman may be described as forming a wave which, from sterility, rises gradually to its highest, and then, more gradually, falls again to sterility.

XXXV.—*On the most Volatile Constituents of American Petroleum.* By EDMUND RONALDS, Ph.D.

(Read 15th February 1864.)

Crude American petroleum evolves, at ordinary temperatures, a quantity of combustible gas, which takes fire on contact with flame, and, when mixed in certain proportions with air, produces an explosive mixture. It is in consequence of this property that it has been thought necessary to pass a very stringent law, known as the Petroleum Bill, with a view of preventing accidents from the incautious storing and handling of the oil.

The more volatile liquid products obtained by distilling the crude oil are still more highly charged with combustible vapour, which, when these liquids are again distilled, escapes condensation even by the most powerful freezing mixtures.

The liquid constituents of petroleum have now been carefully studied by Messrs PELOUZE and CAHOURS, and some of them also by Mr SCHORLEMMER. These eminent chemists have shown that the oil consists essentially of a mixture of the homologues of marsh gas, having the general formula,

$$C_nH_{2n+1}, H.$$

It was during the collection of the more volatile of this series of compounds with a view to their analysis, in which object I have now been forestalled, that my attention was drawn to the large quantities of incondensible gas which escaped at each successive fractionation, and it appeared desirable to ascertain whether the gaseous ingredients of the oil belonged also to the same series, or were accompanied by other hydrocarbons. With this object in view, and still waiting the arrival of some specimens of oil collected and secured in hermetically sealed vessels, direct from the oil wells, I was enabled by the kind permission of Mr SHAND of Stirling, to collect the gas which floated over the surface of the crude oil in the barrels in which it is imported into this country. I also obtained from the same manufacturer some of the very first products of the stills employed in refining the petroleum on a manufacturing scale.

The gas floating over the surface of Pennsylvanian oil was collected at a temperature of — 1° C., and was observed to contain combustible ingredients. It took fire instantly on being brought into contact with flame, burning with a very faint, bluish light, but without explosion. From Canadian petroleum, which is of much thicker consistence, no combustible gas was obtained at that temperature.

The gas was collected over water by simply removing the original wooden

bung of the casks and inserting immediately a cork bung furnished with a tube, for the delivery of the gas, and a long shanked funnel tube, through which liquid petroleum was poured.

Thus obtained the gas was of course a mixture of air and hydrocarbon; it was not affected by fuming oil of vitriol, nor was bromine water discoloured by it. It was hence inferred that no perceptible quantities of the olefiant series were present, and the temperature of collection is sufficient guarantee for the absence of any known members of the benzole series.

The gas was treated over mercury, with solid potash and pyrogallate of potash successively, when it yielded—

1·27 per cent. of carbonic acid, and
6·58 „ „ oxygen.

The residue, analysed eudiometrically, gave the following results :—

Gas collected from the surface of Pennsylvanian Petroleum at a temperature of —1°C., *freed from Carbonic Acid by Potash and from Oxygen by Pyrogallic Acid.*

	Observed Volume.	Pressure m.	Temperature Cent.	Corrected vol. at 0° C.+1 m. pressure.
Gas, ...	133·1	0·3099	9°·	39·934
Do.+air, ...	392·8	0·5666	9·	215·47
Do.+do.+oxygen, ...	465·6	0·6391	8·2	288·92
After explosion, ...	421·3	0·5940	10·	245·09
After absorption of CO_2, ...	383·4	0·5515	8·	205·23
After addition of hydrogen, . . .	474·3	0·6395	3·8	299·15
After explosion, ...	346·5	0·5062	4·	172·86

Deducting the nitrogen, or 23·4 vols.=54 per cent. of the original gas, we have here a relation of hydro-carbon to condensation and carbonic acid, as—

16·534 : 43·83 : 39·86
or, 100 : 265 : 241.

The oxygen consumed amounts to 67·16 vols., or 4·06 times the volume of the hydrocarbon. The members of the olefiant and benzole series being absent, it may fairly be inferred that the hydrocarbon resembles in constitution the liquids with which it is associated; and if this be the case, the gas must be a mixture of the hydrides of ethyl and propyl, the former of which requires a relation of hydrocarbon to condensation and carbonic acid, as—

1 : 2·5 : 2

while the hydride of propyl requires a relation of 1 : 3 : 3. By calculation

from the numbers above, it can be shown that the gas analysed must have consisted of a mixture of these gases in nearly equal proportions, or of—

C_2H_6, Hydride of Ethyl 7·94
C_3H_8, Hydride of Propyl 8·01

—the correctness of which is confirmed by the amount of oxygen consumed being about the mean of the quantities required for the combustion of these hydrides separately.

Hydride of ethyl requires 3·5 times its volume of oxygen.

Hydride of propyl requires 5· times its volume of oxygen.

The gas floating over the surface of the petroleum is therefore composed of—

Carbonic acid, ...	1·27
Oxygen, ...	6·58
Nitrogen, ...	54·
Hydrocarbon, ... $\left\{\begin{matrix}C_2H_6\\C_3H_8\end{matrix}\right\}$	38·15

In this condition the gas is not explosive, and would only become so on being mixed with a large volume of air.

The most volatile liquid obtained by collecting the very first runnings from the stills employed in the process of refining petroleum has a specific gravity of 0·666. It is not sensibly affected by nitric acid, by oil of vitriol, or by bromine. When distilled, it commences to give off bubbles of gas in abundance at about 25° Cent., but after a few minutes all appearance of boiling ceases, although large quantities of gas and condensible liquid continue to pass over up to 65° or 70° Cent., and the whole liquid is evaporated below 100° Cent.

This liquid resembles very closely the kerosolene or kerosoform which an American physician of New York has introduced as an anæsthetic agent; and I am indebted to Dr Simpson for the opportunity of comparing it with a specimen of the latter. The specimen lent me by Dr Simpson was quite indifferent to the above reagents. It had a specific gravity of ·6336. It began to boil at 28° Cent., and was nearly completely volatilised at 70° Cent., so that it must have been composed almost exclusively of a mixture of the hydrides of amyl and hexyl, while the crude volatile product from the manufactory contained, in addition to these hydrides, some incondensible gaseous products, and a considerable quantity of the hydride of heptyl.

The incondensible gases dissolved in this most volatile liquid were expelled by gently warming a large quantity (about two gallons) of liquid, and passing the gases, before collecting them over water, through a long metallic worm, surrounded by a freezing mixture composed of ice and salt; the whole apparatus having been filled previously with carbonic acid to expel air.

The first two portions which were collected showed, after separating carbonic

acid and oxygen, little difference in composition from that already analysed, and which had been collected from the surface of the crude oil.

I omit the details of the analyses of these two, and submit only the results, which correspond in both cases with a mixture of the hydrides of ethyl and propyl.

	Gas.		Condensation.		Carbonic Acid.
I.	8·289	:	22·947	:	19·045
	100	:	2·77	:	242
	Oxygen consumed 32·338.				
II.	7·275	:	20·70	:	17·586
	100	:	280	:	240
	Oxygen consumed 31·07				

The gas coming over a little later from the same liquid was found to approach nearer in composition to pure hydride of propyl, as is shown by the following analysis. This portion was treated with potash before being introduced into the eudiometer, but the oxygen which it contained was not separated before combustion, but was estimated in a separate experiment, and found to amount to 2·44 per cent. of the gas burned.

	Observed Volume.	Pressure.	Temperature.	Corrected vol. at 0° + 1 m. pressure.
Gas, ...	39·723	0·2817	15·1	10·604
After addition of oxygen, ...	160·	0·3939	16·	59·548
After addition of air, ...	260·128	0·4917	14·5	121·46
After explosion, ...	236·386	0·4680	16·5	104·33
After absorption, ...	204·386	0·451	15·	87·376
After admission of hydrogen, . . .	357·161	0·602	14·	204·53
After explosion, ...	231·225	0·4643	13·6	102·29

Deducting the nitrogen and the 2·44 per cent. of oxygen contained in the gas, we have here the ratio of hydrocarbon to condensation and carbonic acid, as

5·984	:	17·13	:	16·954
100	:	286	:	283

Hydride of propyl C_3H_8=2 vols., requires a ratio of 1 : 3 : 3.

The quantity of oxygen consumed by the hydrocarbon is 4·67 times its volume, while pure hydride of propyl would require 5 times its volume.

The gas collected at a still later period from the same liquid was free from carbonic acid, oxygen, and nitrogen gases, and agreed in composition with a mixture of the hydrides of propyl and butyl.

	Observed Volume.	Pressure.	Temp. Cent.	Corrected vol. at 0° +1 m. pressure.
Gas, ...	43·034	0·2821	19·5	11·335
After addition of oxygen, ...	151·465	0·3857	19·9	54·454
After addition of air, ...	417·	0·6439	20·6	249·70
After explosion, ...	372·644	0·6038	16·7	212·05
After absorption, ...	321·	0·566	15·2	172·11
After addition of hydrogen, . . .	405·	0·649	17·	247·45
After explosion, ...	353·032	0·5846	15·2	195·52

The relation here of hydrocarbon to condensation and carbonic acid is as—

11·335	:	37·65	:	39·94
100	:	332	:	352

The oxygen consumed is 5·88 times the volume of gas burned, while hydride of butyl alone requires 6·5 times its volume of oxygen for combustion.

The gas evolved on warming the light spirit of petroleum, as it is prepared for sale, after having been kept, however, for some months in a vessel not hermetically sealed, was found to be a mixture of nitrogen and oxygen, with nearly pure hydride of butyl.

After separating by potash the carbonic acid which had been allowed to occupy the space above the liquid, the gas was analysed; the oxygen which it contained was estimated by pyrogallate of potash in a separate experiment, and amounted to 15·37 per cent.

	Observed Volume.	Pressure.	Temp. Cent.	Corrected vol. at 0° + 1m. pressure.
Gas, ...	73·2	0·2399	7·	17·25
After addition of air, ...	273·3	0·4366	5·2	117·11
After addition of oxygen, ...	334·	0·4976	5·7	162·82
After explosion, ...	288·5	0·4523	6·4	127·5
After absorption, ...	228·5	0·3995	10·2	87·998
After admission of hydrogen, . . .	330·	0·5022	13·	158·2
After explosion, ...	317·5	0·4799	12·2	145·86

Deducting the nitrogen and oxygen contained in the gas, we have here a relation of hydrocarbon to condensation and carbonic acid as,—

9·64	:	35·32	:	39·502, or as
100	:	366	:	409

Closely corresponding to the relations in hydride of butyl, which are,— 1 : 3·5 : 4.

This gas was therefore composed of—

28·74 nitrogen,
15·37 oxygen,
55·89 hydride of butyl,

and it would appear from this experiment that the light volatile liquids absorb and retain oxygen in greater proportion than that element is contained in atmospheric air.

The liquid condensed by the freezing mixture during the collection of these gases, and that obtained by subsequently heating the large body of liquid from which they were expelled to a higher temperature, not exceeding however 30° Cent., or the boiling point of hydride of amyl, was redistilled. It commenced to boil at 0° Cent.; a considerable portion passing over between 0° and 4° was collected separately; another fraction between 6° and 8° was also collected apart: the remainder nearly all distilled below 15° Cent.

The liquid distilling between 0° and 4° Cent. is nearly pure hydride of butyl, which has not yet been described. It is a perfectly clear, colourless, very mobile liquid, having an agreeable sweet smell, but eluding, by its great volatility, the sense of taste. It is insoluble in water, but dissolves in alcohol and ether, and alcohol of 98 per cent. absorbs between 11 and 12 times its volume of the vapour at a temperature of 21°·5 Cent. It burns with a yellow, not very luminous flame. Mixed in the gaseous state with twice its volume of chlorine, liquid chloride of butyl is formed, and the original 3 volumes become condensed into 2 volumes of hydrochloric acid.

The specific gravity of the liquid at 0° Cent. is 0·600. It is therefore the lightest liquid at present known.

The vapour-density determined by DUMAS' method, the vapour being absorbed by alcohol, gave the following results:—

Temperature of air,	13·8 C.	Temperature of sealing,	40° C.
Barometer,	·7615 m.	Capacity of globe,	185·6 cc.
Empty globe,	30·577 grms.	Air bubble,	7·8 cc.
Globe and substance,	30·788 grms.	Temperature of alcohol,	14° C.

Hence vapour density = 2·11.

Hydride of butyl, C_4H_{10}, requires by calculation 2·006.

The liquid, analysed eudiometrically in the gaseous state, gave the following numbers:—

Analysis of Butyl Hydride.

	Observed Volume.	Pressure.	Temperature Cent.	Corrected vol. at 0°+1 m. pressure.
Gas, ...	35·04	0·1944 M.	5° C.	6·691
After addition of oxygen, ...	326·	0·4810	4·8° C.	154·11
After explosion, ...	294·8	0·4507	4° C.	130·95
After absorption of CO_2, ...	256·9	0·4215	9° C.	104·83

Hence we have,—

	Gas.	Condensation.	Carbonic Acid.
	6·691	23·16	26·12
or,	·100	346	390

Hydride of butyl requires—

100 : 350 : 400

The liquid collected between 6°+8° Cent. is not very different from this last. It is, however, a mixture of hydride of amyl with hydride of butyl. Its sp. gr. at 0° Cent. was found to be ·6004. The vapour density was 2·178, and the composition in the gaseous state is shown by the following numbers:—

	Observed Volume.	Pressure.	Temperature Cent.	Corrected vol. at 0°+1 m. pressure.
Gas, ...	15·3	0·4392	19·3	9·39
After oxygen, ...	264·5	0·6912	19·3	185·22
After explosion, ...	223	0·6509	17·9	149·12
After absorption, ...	166·8	0·6154	19·5	106·78

Hence we have,—

	Gas.		Condensation.		Carbonic Acid.
	9·39	:	36·10	:	42·34
or,	100	:	384	:	450

Hydride of butyl requires,—

100 : 350 400

It was not to be expected, from the manner in which the gases were collected that any single portion would correspond exactly in composition with any member of the series, and some attempts which were made to separate the gases from

each other by washing with alcohol, did not yield more conclusive results than those already obtained with the mixtures.

From the foregoing experiments we may, I think, safely conclude, that all the homologues of marsh gas, excepting marsh gas itself, are present in the liquid as it comes to this country, and there appears to be little doubt that marsh gas, and perhaps even free hydrogen, will be found among the gases which are evolved with the oil at the springs.

XXXVI.—*On Sun-Spots and their Connection with Planetary Configurations.* By Balfour Stewart, Esq., M.A., F.R.S.

(Read 18th April 1864.)

In pursuance of an idea which occurred independently to Professor Tait and myself, a careful examination has been made of the solar autographs, taken at Kew and Cranford, under the superintendence of Mr Warren De la Rue. This was done with the view of detecting, if possible, some reference to planetary configurations in the behaviour of sun-spots, and in this undertaking, much aid was derived from a remark once made by Mr Beckley of Kew, when taking pictures of the sun, to the effect that, for a considerable period of time, he did not observe any spots *in the act of breaking out* on the visible disc of our luminary. A few words may not be amiss regarding the nature of the scrutiny to which the solar pictures were subjected, and also the value of this as a test of planetary action. In the first place, let us bear in mind, that by the rotation of our luminary, the different portions of his surface are successively presented to each planet in turn. Now, if the bodies of our system have any appreciable influence of this kind upon the sun, it is natural to expect that this should differ for any given portion of his surface, according as this portion is presented to the influencing body, or withdrawn by rotation, so that the sun's diameter is interposed between it and the planet. We should therefore expect to find that, for a given date, the spots should all begin to break out into visibility at or about the same ecliptical longitude. Similarly with regard to their healing up; and, generally, all spots on the sun's disc at a given date should behave in the same manner as they pass a given ecliptical longitude.

It is needless to conceal the great difficulty, if not impossibility, of a complete and final examination of sun-pictures after this method; but, on the other hand, very little consideration is required to show us its great value as a test of the *fact* of planetary action. For if it once be proved (which may easily be done by means of sun pictures), that all the spots on the sun's disc at a given date behave in the same manner, as they pass a given ecliptical longitude, *we are then compelled to resort to planetary action as the only conceivable explanation of such a phenomenon.* In the sun pictures taken by the Kew Heliograph, a vertical line denotes a north and south line through the sun's disc—that is to say, such a line denotes a section of the sun's surface, by a plane passing through the earth's axis; and therefore perpendicular to the plane of the equator. When this plane passes also through the pole of the ecliptic, which it will do twice a year, the vertical diameter of the picture will denote a line of ecliptical longitude; but in the following very

approximate investigation, this line has been used as denoting sufficiently well an ecliptical longitude at all seasons, and the behaviour of the spots has been examined with reference to it. This difference is of less consequence, when we reflect that solar spots occur in a zone extending not very far on either side of the solar equator; and therefore also not very remote from the plane of the ecliptic.

The motion of the spots, owing to rotation, is in these pictures from left to right, and the earth or point of view from which the phenomena are observed, is of course in that longitude which passes through the centre of the picture; so that if any planet were 90° to the left of the earth, it would be *opposite* that portion of the sun's disc corresponding to his left limb; and if 90° to the right, it would be next the right limb.

The motion of the planets is also from left to right, so that Mercury and Venus gain upon the earth, while on the other hand, the superior planets fall behind to the left.

In making this examination of sun-spots at Kew, it was soon seen both by Mr Beckley and myself, that if the sun's disc be filled with spots at any period, and if one of these begins to heal up before passing the central line, another does the same; if again, the disc be empty of spots, and one breaks out on the right-hand side of the disc, another spot will break out on the same side, and not on the left.

So marked is all this, that I have been enabled to construct the following table, from which it will be seen, that the same behaviour of spots often lasts for a considerable period of time.

In this table, the planets of which the action is investigated, are Mercury, Venus, and Jupiter,—the first of which, although small, is very near the sun; the second, although farther from the sun, is much larger than the first; while the third is far off, but extremely large.

The pictures (which comprise all those at Kew available for the purpose) were examined by myself, and the result obtained was confirmed by a separate examination by Messrs Beckley and Whipple of Kew Observatory.

From groups (1), (2), (4), and (5), we find that when Venus is at or near opposition, there are a good many spots, and their tendency is to increase in size up to the centre, or somewwhat past it, and then decrease.

Again, for groups (3), (6), (7), (8), Venus is at or near conjunction, and there are few spots; while for group (3), where both Venus and Jupiter are in conjunction, we observe a tendency towards the breaking out of spots on the second half of the disc. As far, therefore, as may be gleaned from this record, Venus is especially influential in promoting the formation of spots for that portion of the sun's surface, which is receding from her, and in arresting this formation for that portion which is approaching her.

TABLE, IN WHICH THE BEHAVIOUR OF SUN-SPOTS IS COMPARED WITH THE POSITIONS OF MERCURY, VENUS, AND JUPITER.

	Date.	Behaviour of Spots.	Position of Mercury.	Position of Venus.	Position of Jupiter.
(1.)	1859. Aug. 25 to Sept. 5,	Many spots, and a tendency to diminish before the central line.	Nearly in conjunction with the earth.	Much to the right.*	Much to the right.
(2.)	1859. Sept. 16 to Oct. 5,	Many spots, increase to somewhat past the central line.	A good deal to the right.	Near opposition.	Much to the right.
(3.)	1862. Feb. 7 to May 16,	Few spots, and a tendency to break out, especially towards the right side.	... ...	Has passed from the left through conjunction to the right.	At conjunction on March 13, afterward falls behind to the left.
(4.)	1862 June 3 to Dec. 31,	A good many spots, tendency to increase to the centre or past it, and then decrease.	... ...	Goes from the right to opposition, which it attains about the end of the year.	Has passed through opposition, and is at last about 100° to the right.
(5.)	1863. May 6 to June 20,	A general increase of spots to past the centre.	At first much to the left, comes up at last and passes.	Much to the left.	A little to the left.
(6.)	July, 1863,	In the beginning of July there is a great scarcity of spots, and there appears to be a tendency rather to increase in the first half, and diminish in the second.	To the right.	About 45° to the left.	80° to the left.
(7.)	1863. Aug. 26 to Nov. 9,	Very few spots.	... ...	Up and passes to the right.	Passing to opposition.
(8.)	1863. Nov. 13 to 23,	Tendency to diminish in passing over the disc.	Near opposition.	30° to the right.	Near opposition.
(9.)	Dec. 1863,	More spots, some large, which increase at first, but probably before reaching the centre diminish, and continue doing so to the border.	Much to the left.	40° or 50° to the right.	Much to the right.

* This means, that Venus having passed the Earth from left to right, the angle between the ecliptical longitude of the two planets is greater than 90°, and less than 180°.

With respect to Jupiter, these records do not enable us to come to any conclusion; perhaps the reason of this may be, that the diameter of the sun is very insignificant compared with the great distance of this planet, and that we ought rather to look for its influence in some change produced on the sun's surface, as it passes from perihelion to aphelion. In the interesting volume on Sun-Spots, recently published by Mr CARRINGTON, we have a comparison of this nature, the results of which are embraced in the following table:—

COMPARISON OF THE DATES OF GREATEST AND LEAST SUN-SPOTS, WITH THOSE OF THE GREATEST AND LEAST RADIUS VECTOR OF JUPITER. (FROM MR CARRINGTON'S CURVES.)

Greatest Elongation.		Maximum of Spots.		Difference.
1863·0	–	1860·7	=	+ 2·3
1851·1	–	1848·9	=	+ 2·2
1839·2	–	1837·4	=	+ 1·8
1827·4	–	1829·7	=	– 2·3
1815·4	–	1816·8	=	– 1·4
1791·9	–	1788·6	=	+ 3·3
1780·0	–	1779·5	=	+ 0·5
1767·9	–	1770·5	=	– 2·6
1756·2	–	1761·6	=	– 5·4
1856·8	–	1855·9	=	+ 0·9

Least Elongation.		Minimum of Spots.		Difference.
1845·1	–	1843·2	=	+ 1·9
1833·2	–	1833·5	=	– 0·3
1821·4	–	1823·0	=	– 1·6
1809·4	–	1810·0	=	– 0·6
1797·7	–	1798·0	=	– 0·3
1785·9	–	1784·4	=	+ 1·5
1773·9	–	1775·2	=	– 1·3
1762·0	–	1766·2	=	– 4·2
1750·1	–	1758·5	=	– 5·4

On this subject, also, Mr CARRINGTON makes the following remark. "It will be seen that from the year 1770 there is a very fair general agreement between maxima of frequency and maxima of Jupiter's Radius Vector, and between minima and minima, with such an amount of loose discrepancy, as to throw grave doubt on any hasty conclusion of physical connection. In the two periods which precede that date, there appears to be a total disagreement; and although the data for frequency are less certain for those years, yet the general form of the curve of Professor WOLF, is probably too well established to admit of anything like reversion, by the addition of other observations which have not yet come to hand." Now, every one must agree, that caution is very requisite in a generalization of this nature. On the other hand, I am tempted to think that the behaviour of sun-spots, to which allusion has been made in this paper, is very conclusive as to the *fact* of planetary action, and if this be allowed, it must also be admitted, that Mr CARRINGTON'S results, *on the whole*, lead us to believe, that the recession of Jupiter from the sun is favourable to the breaking out of spots, and his approach unfavourable to their production. Coupling this with what I have already endeavoured by a different process to make out regarding the action of Venus, we may perhaps conclude it probable, that the approach of a planet to any portion of the sun's surface, or that of the sun's surface to a planet, is unfavourable to spot production, while the recession from the sun of a heavenly body is favourable to the same.

Let us now see what support this conclusion derives from the phenomena of

variable stars; but before doing so, I would remark, that when the sun's disc has many spots on it, we probably derive less light from it than when it is free from spots. This is not altogether self-evident, for each spot is accompanied by faculæ, and it has been observed by Mr WARREN DE LA RUE and others, that these faculæ, when near the sun's limb, are much brighter than the neighbouring surface of the disc, but when in the centre they are not sensibly brighter than the neighbouring surface. When, therefore, a spot occurs near the limb, the loss of light which it implies may possibly be replaced by the faculæ which accompany it; but in this position, both spot and faculæ are much foreshortened, and present but a small field of view. On the other hand, the same spot, when near the centre of the disc, fills a much larger field of view, and the loss of light which this implies is not made up by any superior brightness in the accompanying faculæ beyond that of the neighbouring disc. On the whole, therefore, we have a loss of light occasioned by spots. Against this it may perhaps be argued, that when the sun's disc is full of spots the general luminosity is greater than during those periods when there are comparatively few spots; but we have no proof for such an assertion. Assuming, therefore, that a disc full of spots is deficient in luminosity, let us now turn to the phenomena of variability presented in other stellar systems. Without entering into details, it will be sufficient to mention the hypothesis which many astronomers believe in as serving well to represent these phenomena, I allude to that of rotation on an axis, where it is supposed that the body of a star is from some cause not equally luminous in every part of its surface. It is in the last clause of this sentence that the deficiency of such an hypothesis as a physical, and not a mere formal explanation, consists: for it is exceedingly difficult to conceive a body, one part of which is permanently luminous, and another part permanently dark. If, however, we conceive of a variable star as a sun round which a planet of some magnitude revolves at a small distance from its primary, and adopt the law which probably obtains in our own system, we shall have a state of things phenomenally equivalent to a body partly dark and partly bright. For as the planet moves round its primary, that portion of the disc of the latter next the planet will be brighter than that which is more remote, and this appearance will move round as the planet itself moves.

It still remains to discuss the case of a planet with a very elliptical orbit. Such a body would for a very long period of time be far removed from its primary, and for a short period of time extremely near it. We might therefore expect, according to the above law, a long period of comparative darkness in the primary, followed by a short period of brilliancy, corresponding to the perihelion of the planet. Now, this is precisely the appearance presented by temporary stars. These bodies emerge from comparative obscurity, become suddenly brilliant, and thereafter very soon begin to fade, darkness being their normal state, while their brilliancy is short-lived and exceptional.

The hypothesis herein advocated appears, therefore, to be capable of explaining, we may say, all the phenomena both of sun-spots and double stars, so far as these are at present known.

In conjunction with Professor TAIT, the author would beg to make the following suggestion before concluding this paper:—These phenomena,* appear to lead to the conclusion, that when celestial bodies approach each other, there is an evolution of light. Let us compare this with the analogous fact, that when atoms approach each other we have the same result; and we are conducted to the belief, that one great law acts in all these cases, although its *modus operandi* is no doubt circumstantially different in each. (Added 9th April.) We may also be permitted to state our impression, that in the case of atoms, as in that of systems, it is perhaps the largest body which radiates most, for when metals of which the combining equivalent is generally large, unite with oxygen for instance, which has a small equivalent, it is the vibration of the metallic particle, and not of the oxygen, which give a character to the light emitted. Possibly the following may be the explanation of this fact:—

The idea of the constitution of ether, which involves the fewest assumptions, is that which makes it a medium by means of which a body in motion parts with its motion to neighbouring bodies, and the phenomena of percussion perhaps entitle us to assume, that this property which a body has of stopping the motion of a neighbouring body depends on the size of the former, and its distance from the latter.

When, therefore, a small body is in violent motion near a large one, the preferential radiation of the motion of the former towards the large body above its radiation to the surrounding bodies will be very great. But when this motion has once been absorbed by the larger body, and taken to a great extent, the shape of heat since energy as well as momentum must be preserved, the preferential radiation of this towards the small body, as compared with its radiation to neighbouring bodies, will not be great, but it will radiate nearly equally on all sides.

The large body will thus, as it were form a reservoir into which the motion of the smaller body is emptied, and from which it is distributed nearly equally in all directions.

* And the behaviour of comets?

XXXVII.—*On the Freezing of the Egg of the Common Fowl.* By JOHN DAVY, M.D., F.R.S., Lond. and Ed., &c. (Communicated by Professor MACLAGAN.)

(Read 2d May 1864.)

In the Transactions of the Royal Society for 1778, Mr HUNTER has given an account of some experiments which he made on the freezing of the egg of the common fowl, from the results of which he inferred,—"That the fresh egg has the power of resisting heat, cold, and putrefaction, in a degree equal to many of the more imperfect animals, ... and that it is more than probable, this power arises from the same principle (a living principle) in both."

Mr PAGET, in the Transactions of the same Society, the volume for 1850, has described how he repeated and added to these experiments of HUNTER, but whilst admitting their general accuracy, the conclusion he drew from them was different, viz.—"That it is not by the power of a vital principle that eggs resist the influence of cold," but is owing to the viscidity of the albumen,—to use his own words,—"That the property which enables fresh albumen to descend below 32° Fahr. without freezing, is its peculiar tenacity or viscidity, by means of which the water combined with it is held so steadily, that the agitation favourable or even necessary to the freezing, at or near 32°, cannot take place."*

The manner in which Mr HUNTER and Mr PAGET conducted their experiments was similar. They both used strong freezing mixtures, and consequently there was a rapid cooling of the eggs, which were subjected to the cooling process; thus, in the experiments of the latter, fresh eggs placed in temperatures varying from zero to 10° Fahr. were frozen on an average in 26 minutes. In neither of their experiments does it appear that the time of the exposure of the eggs was prolonged much beyond half an hour.

Considering the importance of the subject in its physiological bearings, and reflecting on the circumstances under which their experiments were made, it appeared desirable to repeat them with some modifications. This I have done. The chief differences have been, that instead of an artificial mixture for cooling the eggs, and a rapid refrigeration, the eggs, in the trials I have made, have been exposed to the open air, and have been cooled more slowly.

1. An egg, a fortnight old, was placed on grass and left out fully exposed to

* Mr PAGET assigns for the freezing-point of the egg a temperature of 32°, or between 31° and 32°, and supposes that it cannot fall below that, unless at perfect rest; he says,—"That the egg should be unmoved, and that its albumen should be not even so much disturbed as by the introduction of the thermometer." My results, as will be seen, nowise accord with this.

the sky, on the night of the 22dof February, when a register thermometer, placed alongside of it, was 19° at its lowest. On the following morning at 10 A.M., it stood at 22°. The egg was now found fractured and frozen, the fracture of the shell proceeding from the small end towards the large end, containing the air-vesicle. With a strong knife, a section of the egg was made with some difficulty, so hard it had become. The albumen was white and crystalline, and most distinctly towards the surface. The yolk had much the same appearance it would have had had it been boiled; it showed no appearance of change of structure, none of crystallization; it was easily indented, and was softest at the centre, where its colour was orange-yellow, and was strikingly contrasted with that of its including or outer portion, which was pure yellow. During the thawing, some of the yolk became diffused in the white, seeming to indicate, at least, a partial rupture of the membrane of the former. From what is mentioned of the yolk, it is evident that in the cutting of the egg, the resistance encountered was most in the albumen.

2. On the night of the 22d of the same month, two eggs were similarly exposed, one laid that day, the other of a large size—double the ordinary size—and, as was afterwards ascertained, containing two yolks; it had been kept about six months, and during the whole time had been exposed to the air within doors. At 8·30 A.M., next day, the thermometer close to them was 25°; I am not sure that it had been lower at night. The fresh egg was fractured, the fracture proceeding from the small end, and reaching half way towards the large end. A little of the albumen had exuded, and was seen as a white frozen froth. The shell was removed with care, leaving the whole egg entire in its frozen state. It was of a pure lemon yellow, centrally darker, from the colour of the yolk transmitted through the translucent albumen. Portions of the shell removed had attached to them a thin layer of albumen, which showed distinctly a cellular or areolar structure. After thawing, the egg for some time retained its form, exhibiting the structure just mentioned; gradually, however, as if from gravitation and the contraction of the cells, the albumen separated, leaving the yolk, in its proper membrane, entire. The albumen was unusually thin, *i. e.*, less tenacious than when retained in the structure described.* The large egg was not fractured, but when opened, it was found to be frozen. There was a great deal of air in the large end, a circumstance which may account for the freezing without rupture. A thermometer placed in the white close to the shell fell to 27°; in the yolk, to 27·5°. The appearance of the two parts was much the same as that of the preceding.

3. On the 24th of the same month, two eggs laid the same day were exposed; one was smeared with a solution of gum, allowed to dry; nothing was done to

* From such observations as I have made, this filamentous cellular structure of the albumen differs but little from fibrin; it appears to possess the same contractile quality. (See the author's "Physiological Researches," p. 422.)

the other. During the night, the thermometer was as low as 24°; at 8·30 A.M. on the 25th, it was at 25°. Both eggs were found ruptured, the fracture extending from the small end towards the large; if there were any difference, the gummed egg was most ruptured. The shell removed, the albumen of each presented the same crystalline and cellular appearance as that of the day preceding. The yolk of one egg was 27·5°; the white 31°; of the other, the gummed, the yolk was 29°, the white 31°. These temperatures were ascertained when the temperature of the open air had risen to 35°.

4. On the night of the 25th, two newly laid eggs were exposed, one of them smeared with butter. During the night the thermometer was as low as 19°. At 8·30 A.M., on the 26th, it was at 20°. Both eggs were found fractured, the smeared one, at about equal distance from both ends, the other at both ends. At 11·15 A.M., when the thermometer in the open air had risen to 30°, the temperature of the smeared one at its surface within the shell was 31·5°; deeper, 31°; that of the other, in the albumen, just within the shell, was 30·5°; deeper, 30°; it was more firmly frozen than that smeared with butter, and this throughout.

These two trials were made on the idea (not supported by the results), that the coating of one of the eggs might greatly retard or prevent its freezing.

5. On the 26th, two eggs were exposed, one taken from lime-water, where it had been kept about twelve months; the other, laid the same day; both sank in water. Their temperature, ascertained before exposure by a thermometer introduced through a small hole, made about midway between the two ends, in the newly laid was 45·5°; in the other 46°, this within doors. At 10·3° A.M., they were placed on moss in the open air, the thermometer close by 26°. At 10·24 A.M., when the open air was 32°, that of each egg was 39°. At 10·40, the air 34°, that of the fresh egg was 35°; of the other 36·5. At 1·10 P.M., when the open air was 44°, that of the fresh egg was 40°, that of the other 38·5. These results do not appear congruous, I give them as obtained. They may at least tend to show the obscurity of the subject.

6. On the night of the 26th, other two eggs were exposed, one from lime-water, the other newly laid; both sank in water. Most of the time the sky was overcast. At 8·30 A.M., on the 27th, the thermometer was 31°. Both eggs were free from fracture, and it may be inferred were not frozen. All the while, from the 22d to the 25th, a calm prevailed, and the sky at night was unusually clear.

The newly laid egg which had been exposed in the last experiment to a temperature of 31°, on the 27th was put under a hen that had been sitting since the 19th of February. On the 10th of March, when most of ten eggs were hatched (six out of nine), the egg in question was taken from under her and examined. The fœtal chick was found well-developed for the period, and was evidently alive when removed warm from the nest.*

* The fœtus, well detached from the vitellus and the membrane, weighed 27·3 grs. The allantois

7. On the night of the 9th of March, two eggs were exposed to the open sky, one newly laid, the other from lime-water. The thermometer close to them fell as low as 20°. At 8·30 A.M., the following morning it had risen to 31°. There had been a fall of snow, and the eggs were found buried in the snow, as was also a glass of water, the water about equal in volume to that of an egg; the water was frozen to the depth of about an inch. Both eggs were found fractured longitudinally. The newly laid egg was easily cut in two by a knife, both the albumen and yolk being comparatively soft. The former had a crystalline appearance, and was of a light-yellow colour. The yolk was marked with concentric lines of different hues of yellow and orange, the latter most conspicuous towards the centre. The thermometer at the centre was 30·5°; next the yolk in the albumen it was 29·5°. The two parts not being firmly frozen were easily separated. The egg from lime-water was as easily divided. Its albumen exhibited much the same appearance as that of the newly laid. Its yolk was of an orange colour at centre, but not concentrically marked, like that of the preceding. Its temperature at the centre was 30°, that of the albumen, close by the yolk, was 29·75°. The newly laid egg was somewhat the largest, its long diameter being about $\frac{1}{4}$th of an inch in excess of that of the other; their shorter diameter was the same.

Besides the preceding, I have made other trials on change of temperature of the two kinds of eggs; the newly laid, and those long kept in lime-water. This was done by putting them alternately into hot and cold water, accompanied by others of the same kind which had been boiled hard. I have observed no difference in the rate of increase and diminution of temperature, except a slight one, and that, as well as I could judge, depending on the difference of the weight of the eggs used. Even between the eggs hardened by boiling and those not so hardened, the augmentation and increase of temperature hardly appreciably differed.*

or membrane next the shell, was highly vascular, as was also the vitelline membrane, their vessels conveying red blood of a florid hue. The red corpuscles were, for the most part, of the form of those of the adult fowl; some were circular and yet nucleated; these were of a larger size. The fluid between the allantois and the yolk was slightly coloured reddish, from blood corpuscles suspended in it, from the rupture of some vessels. It was limpid, very dilute, contained little or no albumen. When heated to the boiling point, it was not coagulated, nor did it become even milky, merely a minute portion of greyish matter subsided, no more than might be referred to the blood corpuscles. It had a strong alkaline reaction. 28·2 grs. of it, evaporated to dryness, left only ·3 gr. of solid matter, or 1·06 per cent., consisting chiefly of common salt and an alkaline carbonate. Contiguous to the yolk, and contained within its vascular membrane, there was some very tenacious transparent albumen, of faint alkaline reaction. By heat it was coagulated; the coagulum was milk-white, unusually dense and firm. 57 grs. of the viscid matter, evaporated to dryness, afforded a residue of 14·7, or 25·8 per cent. The yolk consisted of a thin and thicker fluid, both of which showed a faint alkaline reaction. A mixture of the two was of the sp. gr. 1022; of the thin kind, 33·9 grs., evaporated to dryness, were reduced to 8·6 grs., or 25·3 per cent. of the thicker kind; 30·4 grs. evaporated to dryness were reduced to 9·3, or 30·6 per cent. The eyes of this fœtus were fully formed; the lens of each, resting on the crystalline humour, was ·133 of an inch in diameter, a perfectly transparent sphere.

* The appearance of the yolk of the newly laid egg, and of that from lime-water, kept about twelve months after being boiled, slightly differed; that of the latter was of a paler and less bright

As the results of the foregoing experiments seem to show that there is no well-marked difference as to freezing between the newly laid egg and the egg which has been kept many months, nor any well-marked difference as to their rise or fall of temperature, they appear to support Mr PAGET'S conclusion, that the egg is not protected under these trials by a vital principle, as supposed by HUNTER.

There are other experiments which I have made, which seem to have the same tendency. These have been on the freezing of the different parts of the egg by ether, and a freezing mixture in a thin glass tube. The advantage of this method is, that what occurs is seen, and one is thus better acquainted with the particulars.

Owing to the peculiar qualities of the contents of the egg, the subject is obscure and difficult, and it is not easy to arrive at consistent and altogether satisfactory results.

My first object was to endeavour to ascertain the temperature at which the several contained parts freeze. From many trials, I am led to infer, that the freezing point of the thinner albumen is 31·75°, of the thicker 31·50°, and of the yolk 31·25°; and that comparing those of the newly laid egg with those of the egg long kept in lime-water, there is no well-marked difference. Whether such results might be expected, I hardly venture to form an opinion, not knowing to what extent the two differ in composition. That there is a difference, has already been noticed in the appearance of the yolk, and in the quantity of air contained; and another slight difference I have observed,—viz., that whilst the yolk of the newly laid egg has an acid reaction, that of the long-kept egg is neutral.

Owing to conflicting results, and many repetitions in consequence, it would be tedious to give the particulars of the many experiments which I have made. One or two examples I shall offer, and this chiefly with a view to show the variability of the freezing point.

The trials were made in a tube ·7 inch in diameter of thin glass. The thermometer, a very delicate one, was introduced with the fluid, and was often moved. As the results with a freezing mixture were least unsatisfactory, I shall confine myself to them.

In the yolk of a newly laid egg the thermometer fell to 20° without freezing occurring. It was taken out freed from adhering yolk, and again immersed; it fell to 28°, presently freezing began at the circumference, the thermometer in the centre continuing at 28°, where the yolk was still fluid; pretty rapidly it rose to

yellow. In both eggs, between the yolk and the white, there was a greyish discoloration. The quantity of air that was disengaged from the egg long kept was remarkable. I supposed that it might be carbonic acid or azote; but from one examination I made of it, it appeared to be merely common air. Owing to this circumstance, eggs thus kept, or kept long otherwise, crack, and sometimes with a little explosion, when put into boiling-water; the newly laid, which contain very little or no air, not being subject to the same effect,—the exemption may be held to be characteristic.

30° and 31°, the freezing making progress. When the greater portion was frozen, the thermometer, where the freezing had not reached, showed the temperature last mentioned.

The experiment was repeated on another portion of the same yolk. In it the thermometer fell to 22° without the occurrence of freezing; it was taken out, wiped, and returned; it fell to 28°, and continued at that degree several minutes, the yolk remaining liquid. It was again taken out and wiped, and returned, it fell to 30°, and almost immediately freezing took place at the circumference, whilst in the middle the yolk was still fluid, though the thermometer there stood at 28°, after that it rose to 31° and to 31·25°.

With the yolk from an egg kept in lime-water, the thermometer fell to 20° before freezing began at the circumference, it then rose as the freezing proceeded to 31·25°.

Another portion of the same yolk fell to 28° before freezing began; the thermometer then rose rapidly to 31·25°.

The albumen, whether thick or thin, showed the same irregularities in freezing. On what they depend I hardly venture to offer an opinion; perhaps the strength of the freezing mixture is most concerned, yet the trials made to endeavour to determine this were nowise satisfactory.

In all the experiments which I have made on the freezing of the contents of the egg by a freezing mixture and by ether, the congelation, as might be expected, always began at the circumference, gradually extending towards the centre; it was remarkable, unless the freezing mixture was powerful, how slowly it proceeded, the centre being often soft, whilst the surrounding part was hard; and from the circumstance that concentric rings were often seen in a portion of yolk entirely frozen, as noticed in the instance of the egg, it seems probable that the congelation was in a manner paroxysmal. The moving of the thermometer, even to active stirring, in the fluid, seemed to have little or no effect in hastening the freezing, even at 20° or below 20°.

Mr PAGET lays stress on the low degree of temperature which the egg bore in his experiments before its freezing took place, a result with which mine, conducted in the open air on the entire egg, hardly accord. He, as already remarked, is disposed to attribute this resistance of the egg to the peculiar viscid and tenacious state of the albumen. That this viscous state, and more especially the cellular filamentous tissue in which the albumen is retained, and to which the viscidity is very much owing, may be concerned in part, can hardly be doubted. It appears to operate by checking the motion of the fluid from change of specific gravity, depending on change of temperature, as in the instance of water. The following experiments may be mentioned in proof:—

The thick albumen of an egg was plunged into a freezing mixture; congelation took place at the bottom and sides, there the thermometer was 31·5°, whilst

towards the surface and middle, which the congelation had not reached, it was 33·5°: and more remarkable still, when the albumen was frozen hard below, above, where it was still liquid, it was 35°, a difference this strikingly illustrative of the impediment offered to the mobility of the fluid by the tissue in question. Another experiment with the thinner portion of the albumen of the same egg, one which had been kept in lime-water upwards of twelve months, may be worthy of notice in the way of further illustration. In this instance, after freezing, when thawing was going on, and the albumen was perfectly liquid below, there the thermometer was 32·5°, whilst towards the surface it was 31·75°, a difference, owing, it may be inferred, to particles of ice ascending in the act of thawing. In this trial congelation did not begin until the temperature of the fluid had fallen to 20°.

Besides the quality just mentioned, there is another, which probably has much to do in retarding the freezing of the egg,—viz., the composition of its several parts, and especially their saline elements, on the presumption of the well-ascertained property which the soluble salts have in lowering the freezing point of water, and how in aqueous solutions containing salts, such as common salt, by augmenting the cold, a gradual concentration may be effected, a strong brine may be obtained in contact with frozen water, or mingled with it. In some of my experiments the appearance of the frozen albumen, in a soft state, at a temperature below 32°, much resembled sea-water suddenly frozen.

I have mentioned in a preceding note the fluids contained in the egg in an advanced stage of development. These, with the fœtal chick, I subjected to the action of a freezing mixture. The results obtained seemed in accordance with what has been just stated.

The very thin fluid next to the allantois or outer vascular membrane, froze partially at 30·5°; it was not completely frozen at 21°, a portion was still fluid at the centre. The thermometer, as the freezing proceeded, rose to 31·5°, and it continued at that temperature whilst thawing.

The very thick viscid fluid fell to 23° before freezing began, though it was stirred; when the freezing took place, it rapidly rose to 31·25. The thin portion of the yolk fell to 23° before it began to freeze, then it pretty rapidly rose to 31·75; when the thermometer was kept in this central, more fluid part, it was 31°. The results with the thicker portion did not differ materially.

The fœtus fell to 20°, and remained at that temperature some minutes without freezing.

Whatever may be the cause or causes on which the resistance of the egg to freezing depends, its possessing this property must be acknowledged to be a happy circumstance in the animal economy, and an adequate security for the preservation of its life, or of that of the germ which the fertile egg contains, and on which its after development into the chick depends, *i.e*, under the ordi-

nary conditions of incubation, taking the season of the year into account, the habits of each kind of bird, and the kind of nest it forms. But whether it belongs to the egg of the common fowl, to the extent which Mr PAGET infers, seems to me somewhat doubtful; and also somewhat doubtful whether the freezing of the egg is compatible with *any after* development. In the following passage the first is stated, the second is inferred. Speaking of the peculiar property of the albumen, he says:—"The purpose or utility of this peculiar property of the albumen of eggs is manifest in the defence which it provides for the eggs exposed to a temperature below 32°. If an egg be frozen, the damage sustained by its structure is such that the germ cannot be fully developed; but mere cold, however intense, if freezing does not take place, does not prevent the complete development of the young bird." In proof, he adds, "I placed three eggs in a freezing mixture, varying from zero to 5° Fahr., one of them froze, and its shell was cracked from end to end; another froze, and when it thawed, its yolk was burst and mixed with the albumen. In incubation, two spots of blood were developed in the former, and an enlargement of the cicatricula ensued in the latter of these eggs—sufficient indications that the intense cold and freezing had not killed them, though it had spoiled their structure.* But in the third egg, which had been exposed for nearly an hour to a temperature below 5° Fahr., perfect development took place in incubation. Even this degree of cold had neither killed nor frozen the egg, though, according to the average rate at which eggs part with heat, its whole substance must have been for half an hour at a temperature between 5° and 10° Fahr."

These, it must be acknowledged, are very remarkable results, especially the last. But, it may be asked, is it certain that in the two eggs, in which incipient development was found to have taken place after apparent freezing, the yolk was frozen, or that the part of it containing the cicatricula was structurally damaged? Moreover, is it certain that there was no mistake as to the egg which was hatched after having been exposed to so a low a temperature? The results which I have described, show how uncertain is the temperature at which the freezing of the several parts begins. One experiment in such a matter, when the result is anomalous, is hardly to be relied on. The experience of the breeders of poultry is not in favour of eggs bearing any considerable degree of cold with impunity. It is well known to them that the tendency of eggs to

* Are, it may be asked, the above indications sufficient? In two instances I have found specks of blood on the membrane of the yolk of the newly laid egg. In that of the last, I examined the blood, and found it to contain well formed, elliptical nucleated corpuscles similar to those of the adult fowl; the inference made was, that the blood was derived from the oviduct in the descent of the yolk. As to the enlargements of the cicatricula, its evidence seems less open to objection; and yet, without a large comparison, can it be said with certainty, that the size was the result of increase from development? I have found the cicatricula of a newly laid egg that had been frozen, apparently a little enlarged.

abort, is much greater in the early spring, in cold weather, than in the advanced spring and in summer. Very lately an example of this kind occurred. On the 26th of January thirteen eggs were put under a hen, a good sitter; on the 16th of February six of these were hatched; the chickens produced were strong and healthy; of the other seven—the aborted—one only was found in a state of advanced putrefaction; in three of the remaining there were embryos from ·6 to ·7 inch in length, and in the other three, though no embryo could be seen, there was an appearance of vessels containing discoloured blood.* During the period of incubation for many days there was severe cold; but then, even in the open air, exposed to the clear sky, the thermometer never fell below 18° Fahr. The hen's nest was under cover.

Whether the germ can exist, retaining life without *vital action* of any kind, even at a temperature below the freezing point, is a physiological question, I need hardly observe, more easily asked than answered, and yet surely a question deserving of consideration. If, as some distinguished inquirers maintain, there cannot be force without action, can there be life without it? or, to quote the words of Mr PAGET, giving them in the form of a question, Can there be "a vital principle in organized bodies, such as may enable them, even when inactive and displaying no other sign of life, to resist passively the influences of physical forces?" Another question, of no less difficulty, is, Whether the germ can resume vital action after having been frozen? I will only remark, that the mere act of freezing does not necessarily imply inaction, at least the absence of chemical charge, for I have found ammonia in an unmistakeable manner evolved from the fresh egg when frozen. And further, provided the structure of the parts—that of the yolk and germ for instance—be not materially altered, and from freezing, the structure of the yolk, we have seen, has not been apparently changed, is not the condition of the germ similar to that of the ear of the rabbit and the wattles of the cock, which Hunter found could be frozen without mortifying.†

* In one of these only, were there marks of incipient putrefaction. It is noteworthy, that in three of these, the albumen was firmly coagulated, as if it had been boiled; it was quite white, and showed its usual alkaline reaction. The yolk was also coagulated, but less firmly. Mr HUNTER states (op. cit. p. 29) "that if an egg was not hatched, that egg became putrid in nearly the same time with any other dead animal matter." This does not accord with the results just mentioned, nor with others which I have obtained. In many instances I have found the aborted eggs free from putridity after incubation; whether an embryo could be detected or not, there was commonly a blending of the yolk and white, and the formation of a fluid, almost entirely destitute of viscidity. Such an admixture is certainly favourable to the putrefactive change.

† Phil. Trans., for 1778, p. 34.

LESKETH HOW, AMBLESIDE,
March 17, 1864.

XXXVIII.—*On the Morphological Relationships of the Molluscoida and Cœlenterata, and of their leading Members*, inter se. By JOHN DENIS MACDONALD, R.N., F.R.S., Surgeon of H.M.S. "Icarus."

(Read 21st December 1864.)

Few departments of zoology have recently suffered more remarkable changes, both in classification and accepted views of structure, than the Polypi or Cœlenterata, and their immediate allies in the ascending scale, the Molluscoida,—greatly depending upon the more extended study of those animals of late years. We have been thus enabled to discover natural affinities which *prima facie* evidence would scarcely ever have indicated, as well as intrinsic differences which the same kind of evidence has hitherto been incapable of revealing to the mind.

Leading from the Protozoa to the Mollusca proper, the Cœlenterata and Molluscoida constitute an unbroken series of animals forming a considerable section of invertebrata, distinguished from the Protozoa by the development of true ova, and from the Mollusca by the property of gemmation developing compound examples of the principal types. Furthermore, the motion of the blood, or its equivalent, is effected either by ciliary action or by a propulsive organ; but when the latter occurs it is unfurnished with valves, so that the course of the circulation may be reversible in the same canals.

The study of the different stages of development of a certain organ in the same animal comes within the pale of ordinary physiology: but when we pry into the progressive advance of any organ or function, taken in the abstract, we enter upon a more comprehensive branch of science, which not only embraces the common physiology of each particular animal, but its combined import in all. On comparing the relative parts of two distinct animals, one unaccustomed to a study of this kind could scarcely doubt that the mouth of one was anything more or less than the exact equivalent of the mouth of the other; but it may be clearly shown that mouths acting as such, so far as simple function is concerned, may nevertheless exhibit a remarkable difference, homologically speaking, in animals constructed on different types.

It was formerly believed that the branchial and cloacal orifices of the Ascidian were homologous with those of the siphonal tubes of Lamellibranchiata; but this very natural error has been pointed out by Professor HUXLEY, and it cannot be doubted that the orifice of ingress is in reality oral in one, while it is simply pallial in the other. In the Ascidian, moreover, the inner or commonly recognised mouth is but the œsophageal opening, though it is critically answerable to

the mouth of the polyzoon. It may be assumed that the prominent mouth of the Hydrozoon (in which, in truth, there is no stomach homologous with that of the higher types) is equivalent to the everted internal gastric opening of the Actinozoon; or, conversely, that the stomach of the Actinia is but an inversion of the oral projection of Hydra, still preserving a communication with the somatic cavity, but necessitating the formation of a new oral orifice. In the same way, the tentacula encircling the mouth, in some of the lower forms of animal life, are not in all instances homologous organs. Thus, the branchial tentacula, as they occur in the Ascidiozoa, are found in none of the other members of the series now under consideration; and in the passage from the tubularian Polyp to the Actinozoon, the oral tentacula of the former (with a single exception, so far as known to me) are suppressed in the latter; while the outer or somatic set remains, and even becomes more numerous or densely crowded as a general rule.

The true nature of the Aggregate Tunicata was first made known by SAVIGNY, and they were with great propriety removed from the zoophytes, with which they had been formerly confounded. To M. MILNE-EDWARDS is due the credit of having elevated the Polyzoa from their low estate, and ranked them with the Tunicata in his Molluscoid group. Professor HUXLEY again, from his comprehensive view of the subject, saw the propriety of associating the Brachiopoda with the Molluscoida, but more immediately with the Polyzoa, as exhibiting the "neural" intestinal flexure, in connection with many very striking points of homology which had never before been conceived.*

The delicate membrane surrounding the base of the tentacula in the *Polyzoa Hippocrepia* is considered by Professor ALLMAN as analogous to the membrane of the respiratory sac in *Tunicata;* but Mr BUSK says that this has not yet been detected in any marine Polyzoon, though I may add that it is distinctly present in the Brachiopoda; and he further considers that the membrane surrounding the base of the tentacula in Pedicellina is not homologous with it, having an entirely different import. If ever a Polyzoon resembled a Tunicary, it is the said Pedicellina, more especially when its two dorsal bends are in course of development, and a zealous observer might be very readily deceived as to the true nature of certain parts in one bearing a striking but delusive resemblance to those in the other. Yet, without going farther into the refinement of the subject, it would not be far wrong to assume, in round terms, that the pharyngeal respiratory system of the *Tunicata* is represented by the oral tentacula of the *Polyzoa*.†

The epistome of the Polyzoon, moreover, is regarded by Professor ALLMAN as homologous with the languet of the *Tunicata*.

* It will be seen, in the course of this paper, that I have availed myself of the useful terms employed by Professor HUXLEY, more particularly with reference to the *Cœlenterata*.

† See Professor ALLMAN'S remarks on this subject, in his valuable work on the fresh-water *Polyzoa*. Published by the Ray Society.

The property of gemmation in a marked manner distinguishes the *Molluscoida* from the *Mollusca* proper, on the one hand, while it associates them quite as obviously with the *Cœlenterata* on the other. We know nothing of a process of this kind as occurring in the so-called *Ctenophora*, including the families *Callianiridæ** and *Beroidæ*; but the organisation of these animals, first rightly considered by FREY and LEUKART abroad, and by Professor HUXLEY at home, referred them to the higher section of *Cœlenterata*, namely, the *Actinozoa*. Yet, the more I have studied *Cydippe*, the more it has appeared to me to hold a position between the *Actinozoa* and the *Polyzoa*, linking the two by characters which it exhibits in common with either, or both. Indeed, in any other place it would seem to be not only friendless but intrusive. A view similar to this has, I believe, been already expressed by M. VOGT, but I regret that I have not access to his observations on the subject.

It is curious to observe the progressive development of the digestive system, proceeding from the *Hydrozoon* onwards through the *Actinozoa*, *Ctenophora*, and *Polyzoa*, to the *Tunicata*.

The important part taken by the *Ctenophora* as a link in this beautiful chain, is represented in the accompanying series of diagrams. Moreover, certain points in the structure of these animals, are even made more tangible to our philosophy by the light which is thus shed upon them.

The stomach of *Cydippe* is connected with the walls of the body by two vertical septa, with two interseptal loculi. The cœcal tubes forming the lining of these loculi run forwards as far as the mouth, and are at once diverticula of an alimentary system and of a somatic cavity.

We observe here, as in *Actinia*, a well defined internal gastric opening, bounded by two crescentic folds, determined by the persistence of the before-mentioned loculi. These latter communicate below with the rudimentary intestine which is yet little more than a central tubular narrowing of the somatic cavity, from the gastric end of which also pass off the two dichotomously-branched tubes, which terminate peripherily in the fusiform sinuses,† corresponding with the ciliated bands. All this affords us a more distinct idea of the mode in which the intestinal tube is formed. Thus, instead of arising simply as an extension of the proximal end of the stomach, the whole gut is at first but a tubular process of endoderm, inclosing a portion of the somatic cavity. In *Cydippe*, the intestine is perfectly straight and axial, reaching the posterior extremity of the globose body, where it exhibits a small, but distinctly marked bifurcation, and the little nervous ganglion, with its otoconical sac, is received into the intervening recess.

* This family name is objectionable, as having been chosen from a supposititious genus founded upon a mutilated specimen of *Cydippe*, indifferently drawn.

† Would it be too far-fetched to suppose that these sinuses are, as it were, retrospective of the tentacula of Actinia?

The end of each division is well rounded, and closely applied to the ectoderm on either side of the ganglion, but I have always found it difficult to detect the so-called anal openings, though I have once or twice observed the escape of matters from within the tube at this part. The least I can say, however, is, that they are by no means so definite in nature as they are represented to be in figures and descriptions.

Professor HUXLEY first pointed out the striking original difference in the intestinal flexure in *Brachiopoda* and *Polyzoa*, as compared with that in *Tunicata*. Thus, in the two former, the intestine is simply flexed forward (*i.e.*, towards the nervous ganglion or "*neurally*"), while in the latter it is at first flexed backwards (*i.e.* towards the heart, or hæmally), and subsequently turns forward to terminate neurally in the "*atrium*." In *Cydippe*, therefore, the intestine is straight; in the *Polyzoon* and *Brachiopod*, it is once flexed (forwards), and in the *Tunicary* it is twice flexed (backwards and forwards).

The nervous ganglion appears to hold some relation to the conditions just noticed; thus, it is nearly midway between the branchial and cloacal openings in the *Tunicary*, between the oral and anal orifices in the *Polyzoon*, and at the posterior termination of the intestine in *Cydippe*.

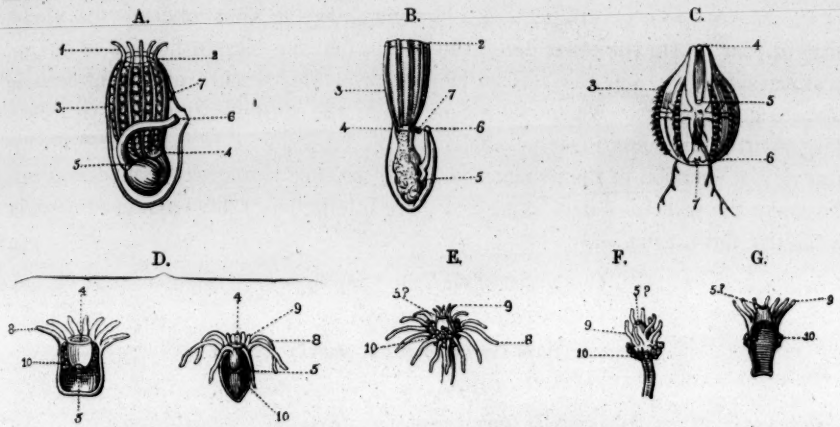
The homologies of the ciliated bands and tentacula of *Cydippe* have not yet been satisfactorily determined, though it does not appear improbable that the former organs represent the tentacula of the *Polyzoon* or of the *Brachiopod* having become retroverted, and connate as it were with the body, as next in order of suppression.

The configuration of the genus *Cestum* is strongly in favour of this view, and if the possibility of such be admitted, then the retractile racemose tentacula may be regarded as pallial. If, on the other hand, the position here supposed in relation to the ciliated bands of the *Ctenophora* be doubted without valid reason, I can only say that it is not at all more remarkable than the modifications of the ambulacra in the *Echinodermata*, passing from *Asterias* to *Spatangus* for example.

The branchial tentacula in the *Tunicata* may be regarded as ecclusory and perhaps prehensile, while respiration is effected by their pharyngeal apparatus, and the same office is probably also exercised by its homologue the oral circle of tentacula in the *Polyzoon*, and the so-called cirri of the *Brachiopod*. In the case of the latter, however, it may be subserved also by the pallial sinus system, which is ciliated within, and thus enabled to circulate the contained corpusculated fluid. The renewal of the water passing over it is of course chiefly brought about by the action of the vibratile cilia clothing the double row of tentacula ("cirri"). There is apparently, to my mind, but a short step from these latter organs to the ciliated bands of *Cydippe*, in which they are both locomotive and respiratory, being in close relation with the aquiferous system; and it must be remembered, that this, like the pallial sinus system of the *Brachiopod*, is not only lined with

cilia circulating a corpusculated fluid, but also contains the reproductive organs. Finally, the tentacula in the *Cœlenterata* are in most cases both prehensile and respiratory, while in many they are also locomotive.

The accompanying series of diagrams will show more distinctly the relationship of the various organs and parts in the five groups so imperfectly glanced at above.



A. An Ascidian.
B. A Polyzoon.
C. Cydippe.
D. Actiniæ, fixed and free.
E. Tubularia.
F. Clava.
G. Hydra.

The numerals have a corresponding signification in all the figures.

1. The outer oral, or branchial tentacula of *Tunicata*, which do not appear again in any of the inferior types.
2. The outer mouth or branchial opening of *Tunicata* only to be represented theoretically, by the conjoined summits of the tentacula of *Polyzoa*.
3. The branchial chamber or respiratory pharynx of *Tunicata*, to which the tentacula of *Polyzoa* and the ciliated bands of *Cydippe* may offer an equivalent; being thus pharyngeal in the first, oral in the second, and somatic in the last. It is, moreover, altogether absent in the other sections of *Cœlenterata*.
4. The inner or œsophageal opening of *Tunicata*, and the proper mouth of *Polyzoa*, *Ctenophora*, and *Actinozoa*.
5. Pyloric orifice of the stomach, leading into a closed intestine in *Tunicata* and *Polyzoa*; and into an intestine communicating with the somatic cavity in *Ctenophora*; and solely into the somatic cavity in the *Actinozoa*. Probably, also, in the everted state forming the oral orifice in *Hydrozoa*.
6. Anal extremity of the intestine in *Molluscoida* and *Ctenophora*.
7. Position of the nervous ganglion in the same.
8. The oral tentacula in *Actinozoa*, probably equivalent to the outer or somatic tentacula of the *Hydrozoa*, always occurring on the outer or proximal side of the reproductive organs.
9. The oral tentacula of *Hydrozoa*, always on the distal side of the young gemmules. In accordance with the views here expressed, it may be assumed that the inner circle of *Tubularia* is homologous with the single series of *Hydra*, &c.

Regarding *Cydippe* as holding a middle position in the foregoing series, the

theoretical passage of the type upon which it is constructed into higher and lower types is effected in the tentacular system by *addition*, and in the alimentary system by *modification*, and these are, of course, either progressive or retrogressive, as they tend to the higher or lower forms. Thus, the tentacula of the *Polyzoon*, and the branchial tentacula of the *Ascidian*, are progressive additions, while the oral tentacula (so-called) of the *Actinozoa*, and then the oral tentacula of the *Hydrozoa*, are retrogressive additions, however contradictory the phrase may appear.* On the other hand, the insulation and simple flexure of the intestine in *Polyzoa*, and the perfect looping or double flexure of the intestine in *Tunicata*, are progressive modifications, while the simplification and central enlargement of the somatic cavity, abolishing the intestine in *Actinozoa*, and, finally, the virtual eversion of the stomach in *Hydrozoa*, are retrogressive modifications. To make the principles here expressed more intelligible, I have arranged them in a tabular form, thus,—

Conversions of the Cydippean *Type.*

I.

MODIFICATIONS AFFECTING THE ALIMENTARY SYSTEM.

* *Progressive.*

Complete insulation of the intestine from the somatic cavity.

a. Primary type, with simple neural flexure—*Brachiopoda* and *Polyzoa*.

b. Secondary type, with primary hæmal and final neural flexure—*Ascidiozoa*.

** *Retrogressive.*

a. Primary type. Stomach more completely insulated by the abolition of the intestine—*Actinozoa*.

b. Secondary type. Eversion and consequent abolition of the true stomach—*Hydrozoa*.

II.

ADDITIONS AFFECTING THE TENTACULAR SYSTEM.

* *Progressive.*

a. Primary type. Oral tentacula of *Brachiopoda* and *Polyzoa*.

b. Secondary type. Branchial tentacula of *Ascidiozoa*.

** *Retrogressive.*

a. Primary type. Oral tentacula* of *Actinozoa*.

b. Secondary type. Oral tentacula of *Hydrozoa*.

Two suggestions present themselves in this mode of considering the subject,—viz., first, that the *Brachiopoda* and *Polyzoa* should be taken together as a group in themselves, quite as natural as the *Tunicata*, consisting of simple and compound forms; and, secondly, that the *Ctenophora* should be received within the pale of the *Molluscoida*, perhaps as a pelagic section representing *Salpa*, *Doliolum*,

* It will be observed that the homologues of the oral tentacula of *Tubularia* do not exist in *Lucernaria*, while they are the sole prehensile organs of *Coryne* and *Hydra*.

&c., amongst the *Tunicata*. In accordance with these views the following classification may be given:—

Molluscoida (including the *Ctenophora*),	Intestine insulated from the somatic cavity,	With primary hæmal and final neural flexure.	Ascidiozoa.
		With simple neural flexure,	Brachiopoda and Polyzoa.
	Intestine straight, and communicating with the somatic cavity,		Ctenophora.
Cœlenterata (excluding the *Ctenophora*),	Intestine obliterated; stomach communicating with the somatic cavity,		Actinozoa.
	True stomach obliterated, its office being answered by the somatic cavity,		Hydrozoa.

* These are probably in ordinary cases truly somatic, for, in some instances, an inner and very distinct set exists, more worthy of the name of oral, homologically speaking.

XXXIX.—*On the Great Drift Beds with Shells in the South of Arran.* By the Rev. Robert Boog Watson, B.A., F.R.S.E., Hon. Mem. Nat. Ver., Lüneburg. (Plates XXI., XXII.)

(Read 4th January 1864.)

The South of Arran in General.
Character of the Drift-Beds there.
Description of these in Detail.
1. Auchinreach Burn.
2. Glen Ashdale.
3. Torlin Burn.
4. Cloinoid Burn.
5. Slaodrigh Burn.
6. Crogcréver Burn.
7. Clachan Glen.
Conclusions :—
1. Boulder-clay derived from land glaciation.
2. Ice covered the land till submerged.
3. Boulder-clay deposited in the sea.
4. „ compressed by ice, &c.
Conclusions—(*continued.*)
5. Boulder clay contains land-formed beds.
6. „ deposited as land was subsiding.
7. Subsidence extended to 1100 or 1200 ft.
8. „ was continuous, not oscillatory.
9. „ was gradual.
10. „ was it rapid?
11. No general glaciation since re-emergence.
12. Boulder-clay beds of all ages of glacial epoch.
13. Drift and Boulder-clay contemporary.
14. Relation to sea-line, a test of age.
15. No material change on the basement-rock since glaciation.

Summary.

The whole southern part of Arran forms a field by itself, and whatever may be the deeper connections of the agencies that have fashioned the north and the south of the island, the result is a trappean area to the south, as distinct as if it lay in another hemisphere from the north, with its granitic nucleus, and encompassing rings of stratified rock.

This district is little visited, and is almost, if not quite, undescribed; it presents, however, much beautiful scenery, and for the geologist, problems of extreme difficulty and interest, which deserve more attention than they have got.

It may be divided into two belts of tilled land and moorland, above which are the hill tops. This division corresponds roughly to three regions, the lowest chiefly of sandstone, the middle where felstone prevails over the sandstone, and the highest of greenstone.

From the sea, the land rises in a precipice, of from 50 to 200 feet in height. In general, for some two or three miles up the valleys, the rock is sandstone without fossils, and of doubtful age, but probably Permian. It is soft, fine-grained, often shaley, thinly laminated, chiefly bright red or purple, and much intersected though not much disturbed by dykes of igneous rock—large isolated masses of which also occur. This sandstone region extends from the coast to a height of from 300 to 500 feet. Its rising slope towards the interior is extremely gentle both in the valley bottoms and in the hill sides above.

From the sandstone region, the rise is generally a steep one, and marked in the burns by a waterfall. This is the edge of the felstone district, where the sandstones, though present, and often considerable in depth, occupy but a small superficies. Here the land rises in faster slopes, to a height of 1200 or 1500 feet. The felstone varies very much. In the south-east it is generally a grey yellow or pink felstone, very friable, disintegrating to soft sand with great rapidity, sometimes columnar, sometimes also amygdaloidal, and frequently markedly laminated, a quality plainly due to the unequal cooling of the mass while in motion.

In the south-west again, this felstone is sparingly present, and the prevalent igneous rock is a very beautiful felstone porphyry, with a matrix of close-grained compact felstone, sometimes approaching in texture to hornstone, pinkish and yellowish, but most often grey in colour, and containing large crystals of white orthoclase and irregular rounded granules and lumps of crystalline quartz. Above the felstones rise tabular masses of greenstone, which occasionally reach a height of 2000 feet. They are of no great extent, but have been largely eroded.

The combined result of these features is a table-land with long slopes and rounded contours, rising in hummocky elevations in the distance, and much intersected by valleys.

Over the whole of this table-land lie clays and sands.

Superficial sand-beds are not common, but occur here and there, from 50 to 570 feet above the sea. Stratified gravels and clays are to be found in the basins which occur in nearly all the valleys.

Markedly distinct from these two classes of superficial beds, is the great deposit of coarse red boulder-clay, which swathes hill and valley in a dense mantle, and gives its characteristic feature of rounded outline to the scenery. (Pl. XXI. fig. 1.)

It may be found on the very edge of the beach. It lies deep over the terrace which rises steeply from the shore along the coast line, and is only absent where this terrace towers up in a great precipice of igneous rock, as at Leac-a-breac, Benan-head and Dippin.

From 50 to 300 feet above the sea, it reaches its greatest development, presenting banks in the water-courses from 80 to 140 feet high. On the hill slopes, as might be expected, it is shallower, and in the valleys too, as they rise into the uplands, the banks of boulder-clay become smaller. Even there, however, they are still considerable, and at a height of 520 feet, I found one face of the boulder-clay 110 feet high.*

The greatest measured height at which I ascertained the presence of the

* I need hardly say, that the sections formed by the burns are transverse to the bedding of the boulder-clay; and, therefore, that the above measurements do not give the true depth of the deposit, but only the height of the sections.

boulder-clay was 1100 feet, or a little more (Aneroid barometer), but I remember to have seen it 100 or 200 feet higher, and I have no doubt, patches of it may be found a good deal above this, but I had not time to seek them.

From all this it will be obvious, that the boulder-clay has not been piled up in immense irregular masses, like glacier moraines, but conforms, on the whole, to all the contours of the rock surface. Even in the valleys, where, of course, it has accumulated in masses disproportionally great compared with those on the hill faces, and where therefore the contour of the surface ceases to conform strictly to that of the rock below, there are indications, in the separate beds of the boulder-clay, of the modifying influence exercised upon them by the form of the ground on which they have been deposited,—the line of the valley forming the synclinal axis of the beds. Where wide basins exist in the valleys, which is often the case above some transverse stream of igneous rock,* the boulder-clay lies equally round its whole margin, and follows the slope of its banks.

The importance of this agreement between the lines of the boulder-clay and of the rock below is, that in glacier moraines, there is no such regular disposition of beds, and still less would such be found in the scattered droppings of floating ice.

Great irregular masses of material, resembling moraines, I only observed high up on the face of the hill, between Kildonan and Benan-head; and again, at the head of Glen Cloy, where there is a huge moraine.†

The form in which the boulder-clay beds present themselves in the burn-courses is as steep slopes thinly grass-grown,—occasionally, where water oozes over them, the face of the beds is steeper, more broken, and covered with confused heaps of stones and mud. The best view of their composition is generally to be got where the burn has cut in to the underlying rock, and the bank rises precipitously above.

In general character, the boulder-clay of the south of Arran is a coarse, red, sandy clay, full of stones, both striated and water-worn, and of all sizes, from boulders 4 or 5 feet in diameter downwards. It varies a good deal in texture, being sometimes loose and gravelly, at others dense and hard, so as to stand up in nearly perpendicular precipices, dotted over with projecting stones so firmly

* One is apt to mistake mere erosions of the boulder-clay for true basins in the basement rock, but where there has simply been an erosion, there is of course no such agreement as that spoken of between the contours of the surface of boulder-clay and the basement rock.

† I do not think this moraine has been noticed. It lies near the head of the glen, and rises to a height of 800 or 900 feet above the level of the sea. Where the glen contracts there are, on the south side, immense heaps of huge blocks of rock tumbled down in the wildest confusion,—the same appears on the north side of the valley, and all the valley bottom is obstructed by heaps of gravel and stones. The cup formed by this is about half a mile long, and above, on all sides, the hills rise perpendicularly to a height of 1100 or 1200 feet. These moraine masses extend down to the foot of Glen Dhu, more than half a mile, and in this glen, also, is some appearance of smaller moraines. In the lower part of the Glen Cloy moraine granite boulders become frequent.

fixed, that in the water-worn gullies one may often climb far up the steep bank by holding on to them. On the whole, it is perhaps less compressed and homogeneous than boulder-clay often is; it is also occasionally traversed by bands of stones or beds of sand and clay. Towards its upper surface it is often somewhat disintegrated, and less dense than below. At times it passes upwards into a dense, fine gravel. In some places it is to be found resting directly on the rock below; but at the lower levels it is often separated from the rock by a bed of sandy clay, while at a higher level it is, in at least two instances, underlain by a very hard, densely-packed, angular gravel or coarse sand, with large striated boulders, which seems to have lain directly under a glacier.

Shells occur occasionally in considerable quantities, both in the boulder-clay and in the sandy clay (not in the glacier gravel) below, at various heights, from 80 to 320 feet above the sea. No general principle explanatory either of the presence or absence of the shells is obvious, except that they seem uniformly wanting in the beds of large stones. In the boulder-clay they are very much broken. In the laminated beds, both those which rest on the rock and those which traverse the boulder-clay, the shells, when present, though of more delicate type, such as *Ledas Naticas* &c., were less destroyed. At first, from the broken state of the shells, I thought the whole deposit must have been formed on the beach; but more careful observation showed that the inference was drawn merely from bits washed out by the rains, these being by far the most abundant. If carefully dug for, the shells may often be found, crushed indeed, yet with each fragment in its own place—two-shelled species with their valves united. Some of the large specimens of Cyprina, though unbroken, are indented as by a sudden violent blow. In short, the whole condition of the shells suggests that heavy stones have been dashed down on them, or that they have yielded in the bed where they lay to the weight of sand and stones more quietly piled over them.

Of species there are sixteen determined and one doubtful, besides many fragments which do not seem to belong to any of these, but are too minute for recognition.*

As to their habitats, they belong distinctively to the coralline zone, or to even deeper water. Of the sixteen species which have been determined, seven, so far as they are known on our coasts, are never found in water shallower than 150 feet (25 fathoms); and except one, all the others, though found at a less depth, are also found in very much deeper water. The single exception was the *Litorina litorea*, but of it I found only two fragments.

In character, Mr Searles Wood, who was good enough to examine them for me, pronounces them decidedly boreal. All the species, except *Turritella communis* (which is common in the Norwegian drift-beds), extend to the arctic province.

* For list of these see end of paper.

Three—*Pecten islandicus*, *Astarte arctica*, and *Cryptodon Sarsii* (if it be really distinct from the *sinuosum*), are distinctively arctic, a character further indicated forthe whole collection by the prevalence of astartes, which, both in species and individuals, greatly outnumber all others.

Vegetable remains, apparently heather stalks, were present at two places in the boulder-clay.

In general, the material of the boulder-clay is derived from the rocks of the particular glen in which it lies. Still foreign materials are not wanting. Thus, in Cloinoid Glen, I found pieces of syenite, which must have crossed the water-shed between that glen and the Torlin Glen to the east, in which the syenite rock lies. In the same glen also were pieces of a very peculiar (carboniferous) sandstone, which must have come down across two water-sheds, from the Clachan Glen to the north. Fragments, too, have been brought down from the Silurian shales in the north-west, and from the granite in the north ; and one bit of impure coal I found, which must have come either from the little patch of coal in the extreme north-east, or from the mainland opposite. All these wanderers have probably been brought by floating-ice.

The red colour of the boulder-clay shows how largely it is indebted to the friable red shales of the district. Hardened knots of this shale, which are always striated, are frequent ; but by far the largest proportion of the stones which it contains are derived from the igneous rocks. The soft laminated felstone, indeed, which prevails in the south is not common, from its proneness to disintegration, but felstone porphyry and greenstones abound. The absence of striations on the stones from the felstone porphyry is very striking. This, however, is apparently due merely to the texture and colour of the stones, and is equally true of the whitey coarse-grained variety of greenstone.

At the lower levels the surface of the underlying rock is rarely to be seen striated, a fact partly owing to its texture, which is very soft and friable, and partly to the difficulty of getting a surface at once sufficiently exposed, clean, and unweathered. On the shore between Brodick and Lamlash, however, there are some very well-marked striated surfaces.

I have made these general remarks as full as possible, in order to avoid repetition in detail of features common to the whole boulder clay, and will therefore now only refer to individual places, in so far as they strikingly illustrate previous statements, or otherwise present something peculiar.

Below the bridge over the Auchinreach Burn, above Whiting Bay, 70 feet above the sea, the boulder-clay is 40 feet deep. It rests directly on a bed of friable shales. The stones in it are much striated ; small granite boulders are frequent. At 50 feet above the sea this bank is 50 feet high, but it is only the lower 15 feet that are boulder-clay. The underlying rock is not seen here. The

boulder-clay is very hard. Through it there runs a stratum of fine clay 3 or 4 inches thick. Above the boulder-clay is a great bed of sand, in some places 35 feet thick, but diminishing rapidly in depth as the bank slopes downwards towards the sea.

At the mouth of Glen Ashdale, the boulder-clay beds lie deep over the whole hill side to the north. They are also present, but in a very ruinous state, on the south.

Between Whiting Bay and the Torlin Burn they are everywhere to be seen, but I did not examine them minutely.

In the Torlin Burn, and especially in the Cloinoid branch of it, they are immensely developed, and present the best sections I have seen in Arran. The beds also contain shells.

At the farm of Torlin, on the edge of the sea cliff, the boulder-clay rises from the beach to 110 feet above the sea. On the road above Lag, at the schoolhouse, it is 150 feet above the sea. This is just above the edge of the great red banks, which rise steep and broken from the burn, and run on uninterruptedly for a couple of miles. The first place where I found shells in these banks was on the east side of the burn, 120 feet above the sea (Plate XXI., fig. 1), and about 15 or 20 feet below the top of the bank, which is here 50 or 60 feet high.

Just below the Church of Kilmorie, the burn is crossed by a little wooden bridge. Immediately below the bridge, on the west side of the burn, 80 feet above the sea, is an extremely interesting section. The bank is 100 feet high, but is much obscured by sludge. The boulder-clay, however, can be made out in detail nearly to the top of the bank. At the edge of the burn course, the sandstone is laid bare. It is very considerably hardened by a greenstone dyke, which lies here in the burn course. The face of sandstone slopes quietly up from the level of the burn for 4 or 5 feet. If it was ever striated, the burn has effaced the markings, the whole surface having been evidently exposed for a considerable time. Its upper edge is perpendicularly broken off on an irregular line; and at the back of this edge, the curious succession of beds shown in the accompanying sketch can be made out. (Plate XXI., fig. 2). The impression which they produced on my mind was, that the under beds to No. 6 or 7 had been formed by running water under a glacier, and had been jammed in at the back of the rock by the ice moving downwards, not directly in the line of the present burn course, but obliquely across it from the north-west, while the other beds above, from No. 8 onwards, were deposited in the sea. In any case the presence of the sea during the formation of beds Nos. 10 and 11 is certain, from the presence of shells in both of them. In No. 10 I found a few unbroken *Ledas* in pairs; and in No. 11, besides fragments of *Ledas* two broken *Turritellas*, and a small bit of a *Litorina litorea*. This last bed is harder and darker than No. 13; and the stones, which are well striated, are fewer and smaller, but both beds are distinctly boulder-clay. Bed No. 10 seems to dip

under No. 11 to the west, at an angle of 75°; but the dips in all these clay beds are very deceptive.

Above the church there is a flat open space in the bottom of the valley. In the centre of this flat a bank slopes out from the hill side above, and breaks abruptly in the middle of the field, presenting a face 20 feet high. On the top is seen coarse water-rolled gravel—below is fine clayey sand. This seems to be the wreck of the superficial deposits, which had once covered the whole flat. Just below this, in the burn course, I found, resting on the boulder-clay, a layer of fine clay, buried beneath sand and gravel, and containing dead equisetum roots, which I have no doubt are modern; but the depth to which this plant penetrates is often very deceptive. These beds seemed to form a lower member of the superficial deposits mentioned above.

In the centre of the field a boss of felstone porphyry projects above the flat. It has the form of a *roche moutonnée*, but I could not satisfy myself that it was striated.

Above this the boulder-clay banks are of great size, but I did not examine them carefully. In the upper part of the valley, at 450 feet above the sea, and near the path leading over to Lamlash, I found a well striated face of greenstone. Further on, at the same height in the burn course, is a bank of boulder-clay, rising to 40 feet above the burn, but the lower part of the bank, for 20 feet, is formed of the friable shale rock of the district. Close by, however, the boulder-clay has fully this depth, and is hard, firm, and flakey in texture. This is just below the farm of Stragael. In the valley bottom here is a flat, where the boulder-clay is covered with gravel. At 570 feet above the sea, just above the farm of Stragael, is a great bed of sand with a few stones. At 800 feet above the sea, the boulder-clay banks beside the burn are still 30 feet high. At 1100 feet it is 10 feet thick; but at 1130 it is thin and sandy, and the shales appear in broken angular fragments close to the surface. On the hill-side, north from this, at 100 or 200 feet higher, it again lies thicker, but I could not examine this locality minutely. At 1250 feet there are well striated bosses of felstone.

The Cloinoid Burn is a branch which joins the Torlin Burn from the N.N.E., about a mile, or rather less, from the sea. Between the burns the land swells up in a rounded back, which is, however, very little higher than the edge of the great boulder-clay banks which line the burns. The valley is very narrow in its lower part, and the banks high and steep; but they are much obscured by debris, so that the details of the boulder-clay can seldom be followed for any distance. The best section of them occurs about a mile above Lag, from 160 to 200 feet above the sea, the top of the bank rising from 240 to 340 feet above the sea, or from 80 to 140 feet above the burn. This section extends, with interruptions, for several hundred yards. It may best be considered in two parts. The first part (Plate XXI., fig. 3), is about 80 feet high and 200 yards long. At the

bottom is exposed a small face of soft friable sandstone, above which is a dense sandy clay apparently about a foot thick, but very little of it can be seen from the debris. It is best seen at the south end of the rock, and its laminæ correspond to the slope of the rock. About 20 feet above this the face of the boulder-clay is crossed by a band 2 feet thick of big stones, some of which are 2 feet long and broad. This bed runs downwards across the bank till it sinks to 8 or 10 feet above the burn. Ten feet above this is a less marked bed of stones, which, on the whole, keeps its relative position to the other, till, at their point of lowest depression, they seem to run into one another. As the centre of this section is obscured for 100 yards by a grassy bank, of course there is no certainty that these stone beds are the same throughout; they seem however to be so. If they are their dip is probably due to the fact that they were deposited on the slope of the hill side, and that the section which the burn has made is not in the line of their strike but transverse to it,—as indeed is obviously the case from the exposure of the rock where they are highest, and its concealment where they sink lower. The irregularity of the two beds relatively to each other is no more than might be expected. A close examination of these beds is impossible from the steepness of the bank, and the view of them from below, or from the other side, is unsatisfactory, owing to the debris, the irregularity of the surface, and the similarity of the colour throughout. Above this bed of stones is a layer of clay a few inches thick. Little bands of clay and sand traverse the boulder-clay, and about half-way up the bank is at one point (Plate XXI., fig. 4), a bed of stratified sand, overlaid by a stratum of boulder-clay, which last is separated from the mass of the boulder-clay above by a parting of sand. The sand-bed contained a few fragments of shells. The layers of sand curve sharply over upon themselves, as if they had been thrust forwards under a heavy weight from behind, and forced to over-ride one another. The boulder-clay resting on this sand seems to have shared in the thrust, but being less easily bent has merely swelled out into a club-shaped mass.

Between this and the next good section a considerable mass of felstone porphyry appears in the burn. Above this is by far the finest section of all (Plate XXI., fig. 5.) It is from 200 to 300 yards long, and the bank is 140 feet high. It presents two or three broad perpendicular faces, intersected by deep hollows channelled out by the running of water. The lower 20 feet or so is a debris talus. Just above this the rock crops out through the bank at one place. It is a soft, crumbly shale. It is covered by a bed much more sandy, and with fewer stones, than the rest of the deposits. It was in this bed that I found *Naticas* quite unbroken, but so fragile that they could not be got out uninjured. In it I also found a fragment of *Litorina*. The boulder-clay above the sand shows a tendency to bedding. A very marked line, apparently of large stones, crosses the whole face of the bank from 25 to 50 feet up, rising as it goes down the

burn to the south. It is very likely one of the beds of stones which we saw in the last section. There are also other indications of water-sorting in the beds both above and below. Shells were most abundant near the rock, and also high up the bank to the south end of the section.

Above this a great mass, 30 to 40 feet high, of laminated felstone, in a very shattered and rotten state, crosses the valley. The channel cut through this is only the breadth of the burn, and in this narrow passage are a few old rounded, but not striated surfaces; but elsewhere I could see none, so completely is the whole rock in a splintery state. The boulder-clay lies over the top of this rock, and comes down to the burn both above and below it. I unfortunately neglected to measure the height to which this rock barrier rises above the burn,—a point so far of importance that the rock, if it really cross the valley, as it seems, must have formed a dam behind which the glacier ice, coming down from the hills above, would be checked and embayed in the basin above. Of course it may have accumulated till it rose high enough to overflow the barrier, but the narrowness of the passage, obviously cut by water, indicates plainly that through it the ice found no egress.

Above this barrier is an open basin half a mile long (Plate XXI. fig. 6.), round which the boulder-clay lies deep. The curve of the bank all round faintly suggests a water-formed terrace, but I did not attempt to ascertain how far it keeps the same level. The lower end of the basin is about 300 feet above the sea, the upper about 330. It was in the boulder-clay bank, about 20 feet up from the burn at the lower end of this basin, that I found the last fragment of shell I met with.

Plate XXI. fig. 1, gives this bank. The lower part of the bank consists of a singularly hard, dense, dry, gravelly clay, derived from felstone porphyry rather than from the sandstones. It looks as if it had been jammed in dry against the felstone rock, to the east and south down the burn, by the glacier when in motion. In it are several large, well striated greenstone boulders. Above this hard gravelly clay, to the left, the red boulder-clay rises to the top of the bank. To the right this boulder-clay has been stripped off, and over the surface of the underlying bed is a stratum 2 or 3 feet thick of water-rolled boulders. This bed merely laps up on the edge of the red boulder-clay as it thins out on the flat. The relation of the two is indeed very difficult to make out from the debris and turf which conceal their junction in the corner at the rise of the bank; but it is better seen at the other end of the section of their junction-line, where it is exposed about 100 yards up the burn, and this is shown in Plate XXII. fig. 8. Here, as before, the hard yellow gravelly clay, about 6 feet thick, lies in the burn course, and, resting directly on it, is the bank of red boulder-clay. To the left, cut off by the burn, is seen the projecting edge of the valley flat. Here the yellow clay seems to have been eroded and buried under a thicker mass of rolled stones and gravel, which becomes thinner as it is followed down the edge of the bank to the

former section. In the angle of the bank and the flat, this bed of stones is overlaid by the remains of a bed of sand. Both of these beds seem to be superficial deposits of a later period, when the basin was occupied by a loch or inland sea-bay, the currents of which had deeply eroded the red and yellow clays.

Elsewhere in the flat other superficial beds besides these can be made out,—not indeed all at any one spot, yet distinctly enough, to a thickness of 8 feet: 1. At the bottom coarse water-worn gravel, what this rests on is not seen; 2. Above this fine clay, with equisetum roots apparently of the period of the clay, but this is not certain; 3. Sandy gravel; 4. Coarse gravel; 5. Sandy clay; 6. Loose coarse gravel, or small rolled stones. In this last bed nearly every stone, both greenstone and felstone, was so thoroughly disintegrated that the whole bed seemed at first to be merely sand.

At the very head of this basin a burn goes off to the west or north-west, in which a deep section of the boulder-clay is given; and between this burn and the Cloinoid Burn, just to the east of the clachan of Cloinoid, is a deep land-slip in the boulder-clay, which strongly conveys the impression of how deep the clay is.

The boulder-clay banks press closely in on the burn above this, and at 520 feet above the sea is a waterfall over the edge of a great bed of felstone porphyry, and from the foot of the fall the boulder-clay rises 110 feet perpendicular; but above this the valley rapidly opens out to a mere depression in the contour of the hill, and the boulder-clay thins out more and more.

The next great glen to the west is that of the Scoradale Burn or Slaodrigh Water, in which, and all its tributaries, the boulder-clay banks are very large. Up one of these tributaries I found a bank of the clay 130 feet high at 310 feet above the sea, and in the main burn they seem even larger. I examined them carefully, however, only in the Croghcréver Burn, which joins the Slaodrigh Burn from the north-west, about a mile above the sea. At the junction of the burns, 50 feet above the sea, one gets a very good view of the mass of the beds, which are here from 90 to 100 feet high. Just above the junction is a steep but somewhat ragged face of the boulder-clay (Plate XXII. fig. 9). Here the soft shales at the bottom rise on the left to about 15 feet, but diminish in height down the stream. Above the rock are ten or twelve feet of coarse gravel and sand plainly stratified. Above this is boulder-clay 50 feet, with traces of bedding, and perhaps somewhat looser than usual. I found a few fragments of shell about the middle of the bank. They had been washed out by the rains, and I could not trace them to any particular spot. The nature of the bank prevents a very close examination of it in detail.

Just above this is a bank which a year ago presented a remarkably fine section. Being very deeply channelled by water-courses, its details were well shown, and it was also remarkably rich in shells. Scarcely a trace, however, now remains of it, so completely has it been washed away or buried in its own

ruins. The soft sandstone rock here appeared about 40 feet above the burn, and over its face lay a dark bed of fine clay, in which shells were both abundant and remarkably fresh and unbroken, the bivalves being in pairs, and retaining both their ligament and epidermis.

Above this is a long, narrow gorge, about 60 feet deep and half a mile long, cut through the soft shales in the line of their strike, which is north-west. The boulder-clay lies down the dip of the strata on the north-east. (Plate XXII., fig. 10.) At about 130 feet above the sea the gorge turns a little more westward than before, and in the angle it expands and forms a small basin, with huge boulder-clay banks 120 feet high on the north-east side. About 60 feet up, the bank is crossed by a dense bed, a few inches thick, of dark brown clay, very hard, with small gravel in it, and very rich in shells, especially *Leda* in pairs, and *Balanus* valves.

Above the gorge is a great stretch of laminated felstone, which occupies both sides of the burn-course. It is this felstone which seems to have protected the shales lower down the burn.

The phenomena of the clay-beds for half a mile above this point, which is from 250 to 300 feet above the sea, are extremely instructive and interesting, but very difficult to explain. (Plate XXII. fig. 11.)

Cut through by the burn is a bank, 7 feet high, of intensely hard yellow clayey gravel, resembling that which I have described in Cloinoid Glen, and like it, chiefly made up of felstone porphyry. Like it, too, it has all the appearance of having been travelled over by the glacier, so compressed and yet so dry and in itself incoherent is it. The upper surface of this bed seems to be quite unconformable in its slope to any of the other contour lines, either of the rock below, or of the beds above, for it dips to the westward, while the whole land is rising in that direction. It seems, in short, just like a bank that had nestled in behind the edge of the rock, and was thus preserved from the abrasion of the glacier. On this back slope of the yellow gravel lies the much redder and somewhat softer common boulder-clay, in which I found a fragment of shell. (Plate XXII. fig. 12.) Horizontally overlying this, and abutting unconformably against the yellow gravel, is a ten-foot thick bed of coarse sand, on the top of which is a layer, 3 feet thick, of very large stones. Above this is some 6 inches of fine light-coloured sand, and over all is earth. On the east side of the burn, this bed of yellow gravel (Plate XXII. fig. 11) nestles in behind a strangely isolated mass of the felstone, close above which is another rock of rotten felstone, round which the old burn-course lies. The new channel is 3 feet deeper than the old one. Some very large and well-striated boulders of greenstone lie in the yellow gravel bank opposite this point, as shown in Plate XXII. fig. 12.

Just above this, in the burn-course, there appears on the west side, a long face of felstone, which rises 4 or 5 feet above the burn. It seems glacier-worn

from its rounded form, but its whole exposed surface is utterly shattered, while its junction with the overlying clay beds is entirely concealed by debris. (Plate XXII. fig. 13.) Resting directly on it, is a confused dark-coloured bed of gravelly clay, with large angular stones crushed rather than ground, some of them striated. In this, a year ago, I found quantities of heather-stalks, but this spring, on revisiting the place, I found it a good deal more concealed by debris, and I could only find one bit of heather-stalk about an inch long, which I fairly dug out of the clay among the hard pressed fragments of stone. The discoloration of the bed, which is very marked, may be partly due to the vegetable matter, but it seems much more owing to the disintegrated greenstone and pitchstone fragments which abound. I had great difficulty in satisfying myself what this bed was, but was greatly helped by finding it again further up the burn. It is just the hard yellow gravel formerly described, and the red boulder-clay which (Plate XXII. fig. 12), along the burn, intervenes between it and the yellow gravelly clay, must be just a tongue of the boulder-clay, lying in a depression of the surface of the yellow clay.

For some way up the burn, the felstone rock forms one side, and the boulder-clay the other of the water-course; and no doubt, the yellow gravelly clay lies hidden under the debris between, for it reappears a little higher up on the opposite (*i.e.* the east) side of the burn, and there it distinctly underlies the boulder-clay. It is intensely hard, and all the stones in it are angular. From the point where it reappears, it can be followed more or less continuously for a long way, sometimes on one sometimes on the other side of the burn, but never, I think, occupying both sides at once, so that it seems to be thin. At one place, on the east side of the burn, it actually seems to underlie a considerable mass of the red and yellow soft shale rock which unexpectedly makes its appearance amidst the felstone. At the north-west corner of the mass where alone I could get a good view of their relations, the edges of the shale strata distinctly lay upon and projected over the yellow gravel beds. On the whole, the shale rock seemed either a mass projecting amidst the felstone, and into the foundations of which the yellow gravelly clay had been violently forced, or more probably a loose mass of the strata which has either slipped or been pushed over the top of this yellow clay-bed, yet without being upset or indeed very violently disturbed. Higher up again, the yellow gravelly clay is seen directly overlaid by the red boulder-clay.

Just above this, at 280 feet above the sea, the clay banks are 100 feet high. From this point, a great shallow circular basin opens up on the hill face, and all around its margin, the boulder-clay banks go sloping down into it.*

* Just where the burn escapes from this basin, the section of the strata shown in twenty yards of the burn-course is most curious. Descending the burn, one first reaches the junction of the felstone porphyry and the underlying sandstone. The felstone porphyry crosses the burn to the N.N.W., and unconformably overlies the sandstone from which it is parted by a greenstone dyke. The sandstone is considerably hardened. A little lower down the burn the sandstone laps up on

Between Glen Scoradale and the Great Black Water Valley there are immense beds of boulder-clay, and also of water-rolled gravels but, I only marked their presence without examining them in detail.

In the Clachan Glen, the great boulder-clay beds extend from the bridge at its mouth at 190 feet above the sea upwards for a mile and a half or two miles. At 220 feet above the sea, felstone rock appears in the burn-course, but it is all so shattered on the surface and buried in debris, that no striations can be seen. At 250 feet above the sea, the shales reappear. Here, in the open and flat valley-bottom, a bank rises 20 feet above the burn which presents the section shown in Plate XXII. fig. 14. The lower part is entirely concealed by a talus of debris, above which the shales rise perpendicularly on the left. These are partly overlaid by 2 feet of dense gravel, very hard pressed, which belongs to the boulder-clay series. Above both the shales and the gravel-bed are 3 feet of large loose stones, and 2 feet of fine sand, with earth over all. The two latter beds, that, viz., of stones and that of sand, seemed to me to belong to the class of later superficial deposits thrown down when the boulder-clay was eroded by the action of a lake or bay of the sea.

A little way behind this on the north, the boulder-clay banks rise 120 feet high, while 100 yards east, or further up the burn, they reach 170 feet. The upper part of this bank consists of hard stratified gravels, less dense than usual in the boulder-clay, and the surface below the turf has three inches of fine hard clayey sand.

By far the best sections of the drift are to be seen on the other (the south) side of the burn; for there, though hardly more continuous, they are somewhat more perpendicular and less obscured by debris than on the north side. Unfortunately, however, they are so far obscured and interrupted, that the beds which they present cannot be traced continuously for any distance. At 290 or 300 feet above the sea is certainly the best and most interesting view that can be got of them, the whole bank being cleared from the burn upwards for 90 feet. (Plate XXII. fig. 15.)

At the bottom is some 15 feet of hard gravelly clay; above this a layer of 4 or 5 feet of great stones and gravel; then 15 feet of dense finely laminated clay with fine sand, becoming more sandy towards the top. In the upper part of this, almost at its junction with the overlying bed, I found some small broken twigs of exogenous wood, crushed flat. They were lying well into the bank, between the undisturbed horizontal layers of the sand, and beyond a doubt belonged to the period of the formation of the bank. Next is a bed 8 feet thick of dense hard pressed boulder-clay, more gravelly and sandy than usual. Then follow 5 feet

the back of a mass of the laminated felstone, the laminæ of which are parallel to the junction-line with the sandstone, but turn sharp round at an acute angle, where they abut against another greenstone dyke, which here occupies the opposite or left side of the burn, and which cuts through the sandstone from S.E. to N.W.

of very coarse big stones and gravel, much water-worn, and over this 30 to 40 feet of boulder-clay; the materials of which are less coherent than in any of the other glens I examined, having less clay and more stones than elsewhere, and this is a characteristic of the boulder-clay throughout the whole Clachan Glen, a peculiarity obviously due to certain specialities of the valley itself, affecting the disposition of the boulder-clay.

At 20 feet above this section, the beds are again well shown; and there the great bed of clay is either buried under the talus of debris, or is absent altogether. At this point a prodigious stretch of the banks is displayed, but too much ruined to afford much information. Most of the stones are much water-rolled, but many of them still retain traces of striation. By far the greatest number of these are greenstone and felstone porphyry, but the latter, as usual, scarcely ever *show* striations. Out of 105 stones distinctly striated, which I counted in this bank, there were,—

Greenstone,	28	= 27 per cent.
Knots from the soft red shale,	28	= 26 „ „
Laminated felstone,	4	= 4 „ „
Felstone porphyry,	3	= 3 „ „
Soft sandstone,	19	= 18 „ „
Green conglomerate, from the altered carboniferous strata,	9	= 9 „ „
Hard, purply, flinty, slatey shale,	9	= 8 „ „
Syenite,	5	= 5 „ „
	105	100

About two miles above the bridge, at 300 feet or so above the sea, is a deep narrow gorge in the valley, where the drift banks are enormous, rising 200 feet high. The gorge is formed by a great felstone dyke, and the shales which it has protected. These have made a kind of dam across the mouth of the upper valley. Both the felstone and the shales crop out in the boulder-clay banks on the south side of the burn, at various points, at 100 or 120 feet above the burn. The shales seem to be overlaid by a mass of laminated or banded felstone, which apparently has spread out from the dyke. This is best seen in a precipice on the south side of the gorge. The greatest mass of boulder-clay lies on the north side; and the top of the bank here, for 3 feet, looks like water-sorted clay and sand, beneath which are some large stones, and below this is boulder-clay. The boulder-clay banks extend up both the main valley and a side branch, which opens to the south-east, above the gorge; but they lose there the huge proportions they have below.

Before concluding this examination of these beds, I think it may be well to explain why I have so definitely spoken of them throughout, as being composed of boulder-clay. Of course, I am aware that exception may be taken to this application of the name, on the ground that these beds present traces of stratification, and contain shells. But this objection could at most only apply to the

particular portion of the clay which is stratified or shell-bearing, and cannot be held to exclude the great mass of the beds, which, in their heterogeneous stiff clay mixed with striated stones, present—if not in high development, yet beyond a doubt—all the characteristics of true boulder-clay. If parts of the beds, then, must be classed undoubtedly as boulder-clay, the whole must be so called, and the elevation of mere stratification and the presence of shells into crucial tests of what is not boulder-clay must be rejected. But, besides this, these beds belong unquestionably to the glacial period, as is proved by the striated surfaces of the rock and of the stones, and by the boreal character of the shells. Now, failing any evidence of the submergence of Arran during the glacial epoch, we may conclude that boulder-clay must have been deposited on the island; and unless it were true, as it is not, that the boulder-clay has been entirely remade, and that the existing beds are the mere wrecks of that deposit—then these must just be the boulder-clay beds of the glacial period.

Such, then, are the facts here presented to us; and with these facts before us, it is obvious that we have, to some extent, a record of the history of the land. How far can we decipher the record?

Let us start from what we know. In geologically recent times there has been a glacial period. The existence of *roches moutonnées*, of striated surfaces and of scratched boulders, proves the action of ice on the land. The presence of marine shells and of water deposits is evidence of the submergence of the land to a considerable depth. Its present state implies its re-elevation. Of these phenomena enough at least exists in the south of Arran—in the boreal shells, the striated stones, and the hard chaotic clay—to justify the belief that this district formed no exception to the general condition of the country; that any differences here are due to local circumstances; and that we may fairly bring the general information gathered elsewhere to eke out our knowledge of this district. Assuming, therefore, that in Arran, as elsewhere, the land was gradually covered by ice, let us try to conceive the result.

At first snow would accumulate on the hill tops, creeping thence as glaciers down the valleys. As it grew in depth, it would spread further and further on the slopes till the whole land was swathed deep in ice. The fact that in our country all the rock surface at all elevations, with mere local exceptions, is striated, indicates such an existence, not of glaciers merely, but of a massive ice-cake, more universal than even in Southern Greenland now. Beneath this ice-cake the soil, and all of life it supported, would be gradually harried away to the sea, any traces of it left being nests of debris nitched into corners, ground over and disturbed in every conceivable way by the ice above. Loose blocks would be carried off, corners would be rounded—the rock faces and the scrubbing-stones would be striated—quantities of stones, gravel, sand, and mud would be ground promiscuously together, and the tendency of the whole mass would be down-

wards, ever by the steepest and the fastest descent it could find, towards the valleys and the sea. At the shore the ice-cake would still sink, crushing down, and yet partly resting on, the debris which lay beneath it, until at a depth proportioned to its thickness it would at last be floated up, forming that flat terrace along the land known among Arctic travellers as the ice-foot. Beyond the ice-foot, we know what occurs in Greenland, and some of the main features are familiar even to Norwegian travellers: floating icebergs, laden with debris, driven about by winds and currents,—masses of ice floating up with a thaw, and bringing up rocks to which they had been frozen, often dropping these again at a far higher level than their parent bed,—turbid fresh water, ice cold, flowing out to sea in a shallow stream, destructive of all animal and vegetable life. At the edge of the ice-foot, a steep bank of tumultuous debris,* shelving down precipitously to a great depth; beyond this, gravel, sand, and mud irregularly distributed, according to the currents, but in the main a fine mud always present at a distance; seaweeds rare, or only locally frequent; while, from the very edge of the debris bank, animal life in abundance, but in the tumult of the debris bank itself only exceptionally present. Such are the phenomena more or less to be found along all glacial coasts; and such, no doubt, were the phenomena of our own shores in the glacial period.

This then being so, we are entitled to say—

1. That the material of the boulder-clay is the result of land glaciation. The enormous debris torn from the abraded surface of the basement rock must have gone somewhere—the huge boulder-clay heaps must have come from somewhere. Do not the two fit each other, as the broken masses of a land-slip in the valley fit the bald rock-face on the hill above?

But besides this, the shells associated with the boulder-clay are decidedly boreal in character, and indicate just such a glacial climate as the ice-covered land implies. The shells, therefore, show that the boulder-clay in which they are found belongs to the glacial period.

2. The ice covered the land till it was submerged. This certainly implies a great severity of cold, and an enormous snow-fall, and yet this seems actually to have been the case. Had it been otherwise; had the ice-cake begun to waste off the land at the lower levels before these were covered by the sea, we should probably have found traces immediately on the rock of the debris with which a glacier is charged, and which, in melting, it would have left behind it. The ice

* I have repeatedly lain on the edge of such a bank, at the mouth of glacier rivers in Norway—particularly at the Skars Fjord glacier, where nothing but warps would hold us, the face of the bank on which our anchor lay being too steep to afford any hold. Our bows were almost grating on the shingle, while we had 70 fathoms under our stern, and 83 fathoms, with a fine mud bottom, at 100 yards distance. The surface water was filthily turbid, bitterly cold, perfectly fresh, and not above 8 or 9 feet deep, if so much; below was the clear salt water, sensibly warmer, and swarming with animal life, both fish and mollusks.

seems rather to have been actually floated off by the sinking of the land; and the first deposits thrown down on the bared rock are marine and shell-bearing.

3. The boulder-clay was deposited in the sea. With the moving ice the debris entangled in it and under it must have been in constant progress seawards, and could never permanently have come to rest till thrust out into the sea under the ice-foot. From this consideration alone we might have inferred the deposition of the boulder-clay under water; but we have more direct evidence, for in Arran, as in some other places, the boulder-clay contains marine shells in considerable numbers, and in circumstances indicating that the animals lived and died in the bed where they now are. Obviously, therefore, the clay in which these are found was deposited under water. Corroborative proof of this fact is further found in those beds of stratified sands and clay which are, comparatively, so frequent in Arran, and which are seen elsewhere wherever an extensive section of the boulder-clay is visible. These plainly imply the action of water, but of themselves, and without the shell beds, would still have left it doubtful whether the water-action might not have been temporary and lacustrine. The two combined confirm each other's testimony to the deposition of the boulder-clay in the sea.

4. Though deposited in the sea, the boulder-clay has been compressed by ice. At the edge of the ice-foot there must have been a growing bank of debris, on which fresh heaps were constantly emptied. On a free and open slope, in such circumstances, the mass of course simply rises in height, till, the angle of slope become too great, the face of the bank breaks, and the upper part slips forward over the lower. But in the case of the boulder-clay, the massive shelf of ice above would prevent the free growth of the bank, which, partly under its own weight, and partly under the pressure of the ice, must constantly have been subsiding, to some extent undergoing compression, to some extent yielding internally and bulging out in front, so as to undergo a complete disarrangement and confounding of all its component parts, which is one of the characteristic features of the boulder-clay, and of which we may regard the bed shown in Plate XXI. fig. 4. as an unfinished and transition example. There the pressure has evidently been at once from behind and above, so that the beds have bulged out forwards, but the pressure had not been long enough maintained to work the various beds into one another.

The influence of the massive ice-foot, in working utter confusion in the bank which lay below it, must have been all the greater from the different conditions it must have assumed under the ever varying change of the seasons, and those slower oscillations of rain-fall and temperature which extend throughout years. At times it would crush the oozy mass below with an enormous weight, till the sludge was compressed almost to the solidity of stone. Again, it would be floated off, and leave space below it for the deposition of stratified sands and clays, in which even molluscan life might thrive, till a rush of debris, brought down by a

warmer summer, buried them, or a series of wetter and colder years swelled the mass of ice so that it rested again with its full weight on the bank, crushing down its surface, and, as it ground its way over the resisting mass, producing those "striated pavements" which are well known as one of the striking features of the boulder-clay.

5. The boulder-clay contains beds which seem to have been formed on the land. Such beds as those I have described at pages 528, 531, and 533 certainly give the impression of their having had a glacier lying directly upon them, and the nature of the position in which they lie confirms the impression of their having been thrust into an angle of the strata by the glacier. But whether it be true, as I believe, for these cases or not, it is obvious that instances of the kind must have occurred, and are to be looked for in all such corners, and other places of the basement rock, as could give shelter.

6. The boulder-clay was deposited as the land was subsiding. Of this fact, the proof which appeals to our senses is the sequence of the beds. From the sea-level to 1200 feet they can be followed uninterruptedly, with quite enough of stratification to leave no doubt that the beds higher above the sea are also higher stratigraphically, and rest on those below. The other proof, though not so direct, rests on even a broader and less fallible basis of fact. The mere presence of the boulder-clay, from the sea-line up to the mountains, implies its deposition during the subsidence of the land; for let us suppose the contrary, and imagine that at some period of the glacial epoch the land previously submerged began to rise. As each zone of the land successively came to the surface it would be subjected to the ice and glaciated. So long as the glaciation went on, every particle of soil would be stripped off the rock and accumulated at the sea-line; and when the glacial period passed away, it would have left our land like a bleached and ghastly skull projecting from the grave.

This obviously is not the case. The whole country more or less, except only the higher mountains, is covered with soil, the boulder-clay itself rising to 1100 or 1200 feet at least; and this is exactly what would occur under the other supposition, that the glaciation of the country, and the deposition of the boulder-clay, went on as the land was subsiding; for thus, as Hugh Miller has somewhere shown long ago, the sea would protect the boulder-clay from the ice, while the ice-foot would shelter it from the surf, and only when the ice was gone would the higher level beds be so far wave-beaten and eroded as to supply a coating of soil to the bare rock of the hill tops, and the higher mountain summits alone would be left in the nakedness of broken and weathered rock which characterises them. And here, if adaptation of means to an end be a proof of design, we have a marked evidence of the work of God preparing the earth for man's habitation.

7. The subsidence extended from the present sea-level, and ultimately reached

1100 or 1200 feet at least. The striated surfaces (indicative of ice) at the present sea-level prove that the land, during the boulder-clay period, stood at least no lower than it does now, and the presence of the boulder-clay—a sea deposit as we have seen—proves that to a height of 1100 or 1200 feet the sea rose over the land. There is indeed some evidence that during the glacial period the land stood higher than now, and still better proof exists, in Wales especially, that the depression extended to not less than 1400 feet; but I confine myself now to the evidence afforded by the Arran beds.

8. The subsidence was probably continuous not oscillatory. Of course there may have been oscillations here, as there have been, in recent times, at Naples and elsewhere; but, so far as I know, no evidence exists of such oscillations occurring during a protracted and long-sustained process of subsidence and upheaval: and certainly no trace of such irregularity has yet been found in connection with the movements of our country during the glacial period. At the same time it is not very obvious by what evidence such oscillations, during the deposition of the boulder-clay, could be established. Still, in the absence of any strictly analogical case, we may consider that the subsidence of the land here was continuous.

9. The subsidence was gradual. As this opinion is opposed to that which I held when I read this paper to the Society, it is right that I state somewhat fully the grounds on which it rests.

The depression of the land might take place by a succession of sudden jerks or leaps, with long intervals of rest, or by a slow, steady subsidence, as is the case at present in Greenland.*

In the former case, the ice-cake would be floated off the land as it sank, and between the top of the boulder-clay bank on which it had been resting, and the new ice-foot, there would be a zone of bare rock perpendicularly equal to the amount of the subsidence, but which, in the flatter districts, would be of considerable extent horizontally. Along the upper edge of this zone, at its junction with the ice, the boulder-clay bank would again begin to form, but all its lower expanse would lie too remote from the supply of debris to be thus covered. The glacier streams, however, floating out to sea on the surface of the salt water, would gradually drop on it the detritus with which they are always charged, and a bed of stratified sand and clay by degrees would overspread the bare rock, till the growing mass of boulder-clay buried it. Now in Arran, as I have mentioned, I found several cases of just such a bed resting directly on the rock, and buried beneath the boulder-clay; and the inference seemed to me a fair one, that these beds indicated a sudden subsidence of the land at that point. I still believe them to indicate the remoteness of the ice-cake at the time of their deposition,

* Such jerks must of course be supposed considerable, since, if minute, the subsidence in this way would practically be slow and steady as in the other.

but I no longer think that remoteness of the ice due to sudden subsidence. Had it been so, the basement-bed of sand and clay would have run round our whole coast pretty much at the same level. But this is not true, even for Arran. The stratified beds, therefore, on the basement rock must, where they exist, be accepted merely as local phenomena, and be therefore explained by local causes, connected probably with those banks of eruptive rock which lie across the valleys, and which must have interfered with the free motion of the ice.

Thus, in the absence of any evidence of a sudden depression we accept the other alternative of steady gradual subsidence, during which each separate level must in turn have formed the shore line and lain at the edge of the ice foot, and have received the glacial detritus—the coarse red clay and stones which the ice and its accompanying streams were bringing down—while the finer detritus would be spread as stratified sands and gravels over the boulder-clay beds already deposited.

10. Was the subsidence rapid? A question I rather ask than answer. If the mass of the boulder-clay suggest long-continued formation, the comparative rarity of interstratified beds, and the merely local development of the overlying beds of sand and clay—though perhaps capable of explanation on other grounds—still seem to point to such a steadiness of climate and of currents as is hardly compatible with a very lengthened duration of the glacial epoch; and in this case the huge mass of the glacial debris would be the result of an enormous development of the ice-cake, which tallies well enough with various known facts. On the whole, this point must be considered doubtful, and its determination is probably to be sought from a careful examination of the larger shell-beds, the layers of life in which may indicate the duration of their development.

11. There has been no general glaciation of the land since its re-emergence. Had glaciers existed on the land after it rose from the sea, they would certainly have cleared the upper valleys at least, of boulder-clay, and left nothing but great transverse moraines, as is the case in Norway. (See Kjerulf on the glacial phenomena of Norway in the *Edinburgh New Phil. Journal* for July 1863, p. 8.) Instead of this the boulder-clay thins out gradually upwards, and fringes the upper valleys as a beach terrace.

12. The existing boulder-clay must represent all ages throughout the whole glacial epoch; for so long as the ice-cake was present on any part of the land the manufacture of boulder-clay went on.

13. There must be drift-beds of sand and clay resting on the boulder-clay, but truly contemporary with the boulder-clay of a higher level than that on which they lie. It may not be easy or possible to determine which of these answer to one another; but it is obvious, that while the coarser debris from the land was forming boulder-clay under the ice-foot, a deposition of the lighter detritus must have been going on from the fresh-water currents that were setting

seaward, and scattering, first the sand and ultimately the mud, with which they were charged, on the boulder-clay already spread over the lower levels. Thus these two beds, the boulder-clay formed under the ice-foot, and the stratified sands and clays deposited in deeper water, though so different in texture, would be really strictly contemporary. Of course the determination in any particular case, of which are the corresponding beds, is now the more difficult task, from the erosion of the beds at certain points, and their *remaniement* or remanufacture in others.

14. We may determine, approximately at least, the relative age of the drift-beds. Superposition, of course, implies subsequence in time, but this principle is of very limited application, and will not avail for the comparison of the boulder-clay in different districts. If, however, the boulder-clay was deposited under the ice-foot as the land was subsiding, the sea-level affords us a standard of comparison for the boulder-clay beds over the whole country. Allowing for possible differences in the thickness of the ice-foot, which would be deeper of course in the valleys than on the hill faces, and deeper also in a mountainous than in a flat region, all boulder-clay beds are contemporary which rest on the basement rock at the same level above the sea; and of two beds at different levels that is the older which lies on the rock nearest the sea-level.

15. Since the boulder-clay period there has been no material change of any kind on the basement rock of the country.

Such changes might have occurred in three ways. In the process of subsidence and re-elevation the whole face of the land might have been remodelled, and hill and valley have changed places. Such a change, or something like it, has been asserted even by so eminent an authority as the great German geologist NAUMANN. (See *Geognosie*, vol. i. p. 249.) But in our drift-beds we have casts made of our valleys as they sank under the sea, and these casts show that what are valleys now were valleys then; in other words, they assure us that the subsidence and re-elevation of the land has not been accompanied by any such protrusion of one part of the coast above another, or of the interior above the coast line, as to affect the relative contours of hill and valley.

Another form of change is that on the river beds, the erosion of which is generally attributed to existing streams; whereas in Arran we find that the burns are only now beginning to lay bare the rock which underlies the boulder-clay. If, then, making all allowance for the slight erosive power of such small streams, they have yet done so little here against sands and clays, how much less elsewhere against solid rock. They seem, in fact, to be only now beginning to occupy the old river-beds formed long ago under the ice, and in part even earlier.

The third change which has been often asserted, consists in the erosion of the older and present coast-lines by the sea, upon which calculations have even

been founded to determine the length of time during which the sea stood at the forty-foot terrace-line, and so on. Now, admitting the obvious fact of the destruction of the rock at many points of our present sea-line, it yet appears that on the whole the influence of the sea in modelling the land in recent times has been very small. In a great many cases it has not so much as penetrated the boulder-clay; and even where that has been washed away, as in the coast of the Little Cumbrae, figured and described by Mr SMITH of Jordan Hill, in his "Newer Pliocene Geology," p. 144, the striations of the rock remain with wonderful freshness. From all this it is obvious that the terrace lines of the basement rock are not due to the action of the present sea, but were given to it previous to its submergence.

In short, we find that all the latest geological changes, with their accompaniment of river and sea-action, have not materially modified the face of the country —the rock skeleton of which was moulded finally under the glacial epoch.*

LIST OF THE SHELLS FOUND IN THE ARRAN BEDS, WITH THE DEPTHS AT WHICH THEY LIVE ON OUR OWN AND THE NORWEGIAN COASTS, AS GIVEN BY FORBES AND HANLEY, AND BY DANIELSEN.

Name.	British.	Norwegian.
Balanus crenatus.	Deep water to 50 fathoms.	.....
Panopœa norwegica.	Deep water to 90 ,,	Not known living.
Tellina balthica, (a brackish variety of Solidula.)	Shore.	20–60 fathoms.
Cyprina islandica.	5–80 ,,	20–40 ,,
Astarte elliptica.	10–40 ,,	20–60 ,,
arctica.	80 (dead) ,,	10–80 ,,
compressa.†	7–70 ,,	10–40 ,,
striata (a variety of compressa.)	.....	.....

* I cannot let this paper go without expressing my obligations for much information and many suggestions to my friend Mr GEIKIE's valuable paper on the Phenomena of the Drift. I am gratified to agree with him on the land origin of the material which constitutes the boulder-clay, and on the subsidence of the land during the formation of the boulder-clay. On some other points, too, I think we are not very far from an agreement, though I fear we differ fundamentally on many of the most important.

† I only found one specimen of this species. The shells were partially crushed, but the two valves were united, and retained both epidermis and ligament. Mr SEARLES WOOD, who was good enough to examine it for me, says it is one he does not recognise. Mr S. P. WOODWARD, who has also taken the trouble of looking at it, considers it an unusually large *A. compressa* of the variety *striata*. It is one-third larger than even the excessively large specimens of the species from the Red Crag, figured by Mr SEARLES WOOD in his Bivalves of the Crag.—(*Pal. Soc. Pub.*, vol. ii. p. 184, pl. 16, fig. 8, *a* and *c*.)

LIST OF THE SHELLS FOUND IN THE ARRAN BEDS, &c.—*Continued.*

Name.	British.	Norwegian
Cryptodon { *Syn.* Lucina. / *Syn.* Axinus. } Sarsii.	.....	20–180 fathoms.
Modiola modiolus.	0–60 fathoms.	Shore.
Leda (*syn.* Yoldia) pygmæa.	25–40 „	30–140 „
pernula.	{ 35–160 *Macandrew*, / *N. E. Atlantic.* } „	10–140 „
Pecten opercularis.	5–100 „	30–50 „
islandicus.	35–100 (dead.) „	10–60 „
Litorina litorea.	Shore. „	Shore.
Turritella communis.	4–100. „	10–30 „
Natica.	Species not determinable.	.....

Description of Plates.

PLATE XXI.

Fig. 1. Lower part of Torlin Water seen from the east, 450 ft. above the sea, to show the contours of the land, the clothing of boulder-clay, and the way in which the burn-course is cut through it. The banks in the burn-course are from 60 to 120 ft. high. (P. 524–528.)

Fig. 2. Bed of various clays, &c., in the Torlin Burn, below the church. (P. 528.)

1. Small gravel and minute angular stones, 3 inches.
2. Fine clay with angular stones in upper corner, 2 inches.
3. Sand, 3 inches.
4. Clay with stones, 3 inches.
5. A nest of gravel, 3 inches.
6. Sand, 4 inches.

} All this is purply and red like the shale-rocks of the district.

7. Fine yellowish sand, 3 inches.
8. Fine reddish sand and clay, without stones, 6 inches.
9. A nest of gravel, 3 inches.
10. Sandy clays distinctly laminated, and slightly varying in texture; dipping under next bed, No. 11 at 75°; shells found in this, 8 inches.
11. Very dense, dark, coarse boulder-clay, with few stones, and slightly stratified; shells found here, 5 or 6 feet.
12. Layer of large stones and coarse gravel, 1 foot.
13. Boulder-clay.

Fig. 3. Great shell-bearing banks in the Cloinoid Burn-course, 80 to 100 feet high. The upper part grass-grown. The centre being obscured by debris is omitted. (P. 529.)

Fig. 4. A bed of sand overlaid by boulder-clay. The whole subjected to a forward thrust under pressure. (P. 530.)

Fig. 5. Immense boulder-clay bank on the Cloinoid Burn, 140 feet high; in many places perpendicular; with shells. In the centre of the bank, the rock crops out. (P. 530.)

Fig. 6. Basin in the Cloinoid Burn-course, to the contours of which the boulder-clay conforms. (P. 531.)

Fig. 7. } These should go together, as they show the same beds, and are nearly continuous, 100 yards
Fig. 8. } only intervening. They both occur at the lower end of the basin, shown in fig. 6.

Fig. 7. Shows at the bottom a bed formed under a glacier. To the left it is overlaid by the boulder-clay, which to the right has been eroded, and the glacier-bed here is overlaid by a layer of large stones. (P. 531.)

Plate XXII.

Fig. 8. At the bottom, the same glacier-formed bed overlaid to the right by boulder-clay; to the left, both the glacier-formed bed and that of boulder-clay seem to have been eroded, and a mass of stones and gravel, like that in fig. 7, but thicker, lies on the hard glacier-bed. (P. 532.)

Fig. 9. Bed, 100 feet high, in Crogcréver Burn-course, just above its junction with the Slaodrigh Burn. (P. 532.)

Fig. 10. Gorge in Crogcréver Burn, the west side being formed by the rock, the east side by the boulder-clay lying down the dip of the strata. (P. 533.)

Fig. 11. Bed of hard yellow clayey gravel, lying in behind a barrier of felstone. The burn has partly cut in between the rock and the bed, but in the distance in the fig., has turned to the right, and cut through the bank. (P. 533.)

Fig. 12. The same bed further up the burn, with overlying beds. (P. 533.)

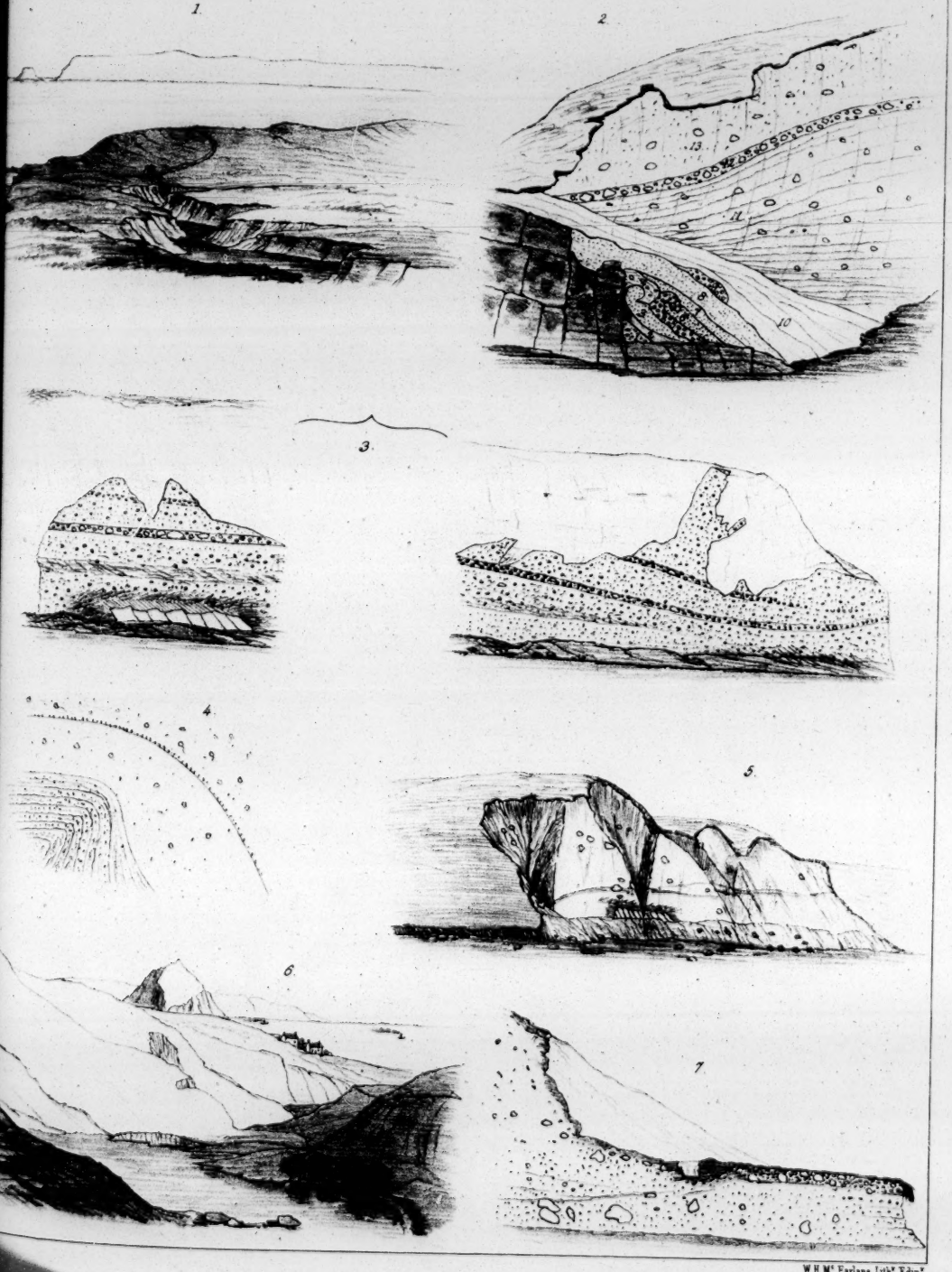
1. The hard yellow clayey sand or gravel, with a huge striated greenstone boulder sticking in it. This bed is laminated, but not stratified on its upper surface to the left.
2. Overlying 1. Red boulder-clay, with a large boulder sticking in it, but projecting above its surface into
3. Coarse sand, 10 feet thick in parts.
4. Very large stones, 3 feet.
5. Fine light-coloured sand, 6 inches.
6. Earth.

Fig. 13. The same bed still further up the burn, and just below the felstone rock. To the left, in the burn-course, is boulder-clay. Underlying this is the hard yellow clayey gravel here much discoloured, and containing heather stalks. Above this is debris, with boulder-clay showing atop. (P. 534.)

Fig. 14. Beds in Clachan Glen, 20 feet high. At the bottom is a talus of debris. Above this, to the left, a face of the shale-rock, 2 to 3 feet high. Lying up on this to the right, 2 feet the dense gravel. Overlying this bed, and to the left lying directly on the shales (which have been stripped there of the dense gravel), 3 feet of large stones; then 2 feet fine sand. Earth atop. (P. 535.)

Fig. 15. Great banks in Clachan Glen. (P. 535.)

1. Hard gravelly clay, 15 feet.
2. Looser stones and gravel, 4 feet.
3. Dense finely laminated clay with fine sand, becoming more sandy towards the top, containing twigs, 15 feet.
4. Dense hard boulder-clay, 8 feet.
5. Big stones and gravel, 5 feet.
6. Boulder-clay or drift, 30 to 40 feet.



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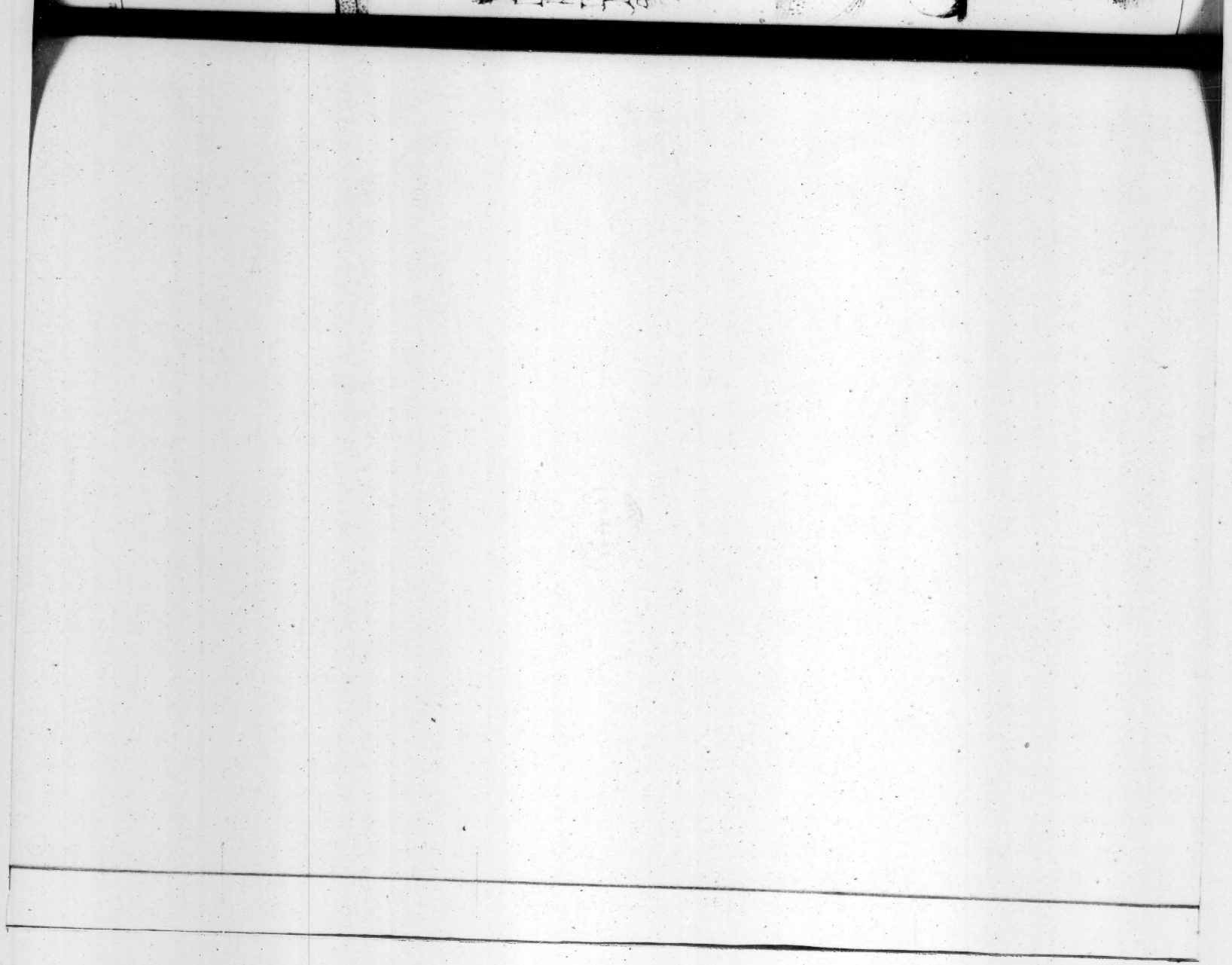

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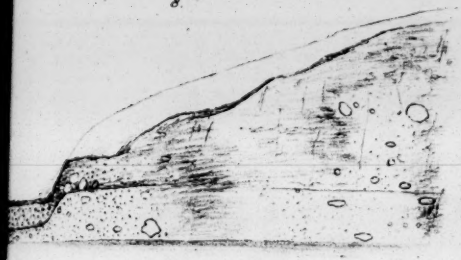

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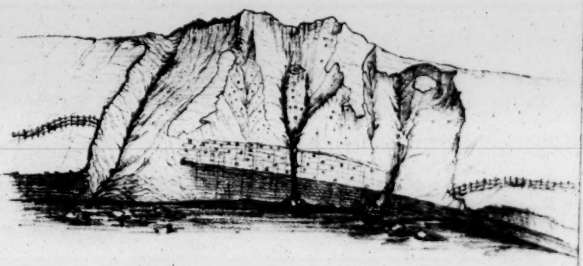

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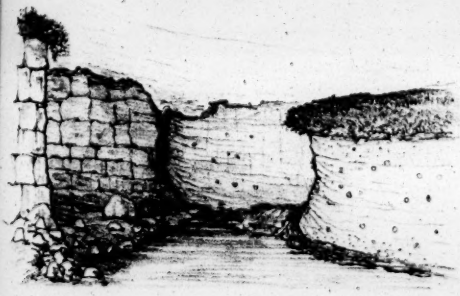

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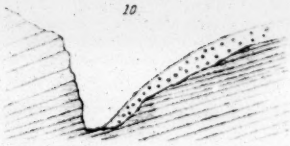

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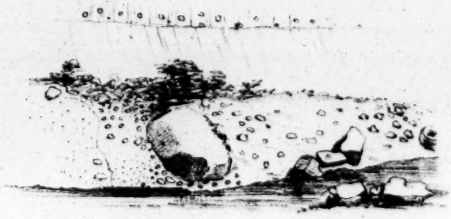

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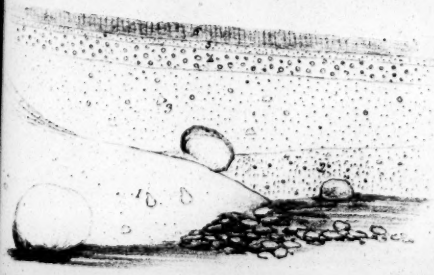

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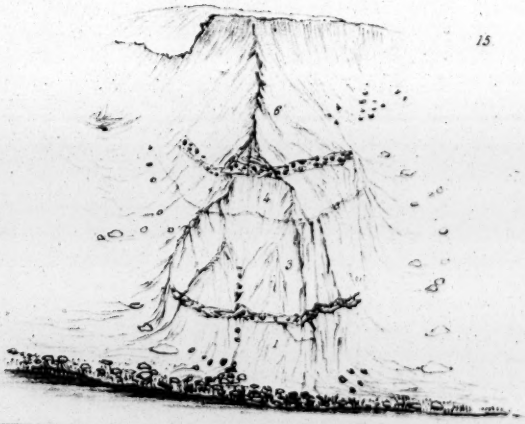

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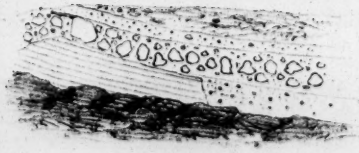

W.H.M.c Farlane Lith.r Edin.r

XL.—*On the Principal Deities of the Rigveda.** By J. Muir, Esq., D.C.L., LL.D.

(Read 7th March 1864.)

In the paper which I had the honour to read before the Society last winter, I stated the reasons, drawn from history and from comparative philology, which exist for concluding that the Brahmanical Indians belong to the same race as the Greek, the Latin, the Teutonic, and other nations of Europe. If this conclusion be well-founded, it is evident that at the time when the several branches of the great Indo-European family separated to commence their migrations in the direction of their future homes, they must have possessed in common a large stock of religious and mythological conceptions. This common mythology would, in the natural course of events, and from the action of various causes, undergo a gradual modification analogous to that undergone by the common language which had originally been spoken by all these tribes during the period of their union; and, in the one case as in the other, this modification would assume in the different races a varying character, corresponding to the diversity of the influences to which they were severally subjected. We shall not, therefore, be surprised to find that even the oldest existing mythology of the Indians differs widely from the oldest known mythology of the Greeks, any more than we are to find that the Sanskrit in its earliest surviving forms is a very different language from the earliest extant Greek, since the Vedic hymns, the most primitive remains of Sanskrit poetry, date from a period when the two kindred races had been separated for perhaps above a thousand years, and the most ancient monuments of Greek literature are still more recent. Yet, notwithstanding this long separation, we might reasonably anticipate that some fragments of the primitive Indo-European mythology should have remained common to both the eastern and the western branches of the family; while, at the same time, we should, of course, expect that such traces of common religious conceptions would be more distinctly

* This essay contains the substance of a series of papers either already communicated, or intended to be communicated, to the Royal Asiatic Society of Great Britain and Ireland, in which the same subject is treated in greater detail, and with numerous references to the original passages of the Rigveda. For further information on the gods of the Veda, reference may also be made to Professor H. H. Wilson's prefaces to the three volumes of his translation of the Rigveda; to Professor Rudolph Roth's papers, "Die Sage von Dschemschid," and "Die höchsten Götter der Arischen Völker," in the Journal of the German Oriental Society, vols. iv. and vi.; to the paper by the same author "On the Morality of the Veda," and the account of the "main Results of the later Vedic Researches in Germany," by Professor Whitney, in the third volume of the Journal of the American Oriental Society; and to Professor Max Müller's "History of Ancient Sanskrit Literature," and his "Essay on Comparative Mythology," in the Oxford Essays for 1856. The sketches of Rudra and Vishnu given in this paper are abridged from the fuller accounts of those gods in my "Sanskrit Texts," vol. iv.

perceptible in the older than in the more recent literary productions of the several peoples. And such, in point of fact, turns out to be the case. The mythology of the Veda does exhibit in some points a certain similarity to that of Homer and Hesiod, and the mutual resemblance between the religious ideas of those ancient works is, upon the whole, greater than that existing between the later Indian and the later Greek pantheons. I say that, *upon the whole*, the older Indian mythology coincides more nearly with the Greek than the later Indian mythology does. But, on the other hand, the later Indian system presents some points of resemblance with the Greek which the Vedic system does not exhibit. I allude to the fact that we find in the Indian epic poems and Purānas a god of the sea, a god of war, and a goddess of love, who are unknown to the oldest parts of the Veda, and yet correspond in a general way to the Poseidon, the Ares, and the Áphrodite of the Greeks. Personifications of this sort may, however, be either the product of an early instinct which leads men to create divine representatives and superintendents of every department of nature, as well as of human life and action; or they may arise in part from a later process of reflection which conducts to the same result, and from a love of systematic completeness which impels a people to fill up any blanks in their earlier mythology, and to be always adding to and modifying it. Resemblances of this last description, though they are by no means accidental, are not necessarily anything more than the results of similar processes going on in nations possessing the same general tendencies and characteristics. But the older points of coincidence between the religious ideas of the Greeks and the Indians, to which reference was first made, are of a different character, and are the undoubted remains of an original mythology which was common to the ancestors of both races. This is shown by the fact that, in the cases to which I allude, it is not only the functions, but the names, of the gods which correspond in both literatures.

But the value of the Vedic mythology to the general scholar does not consist merely in the circumstance that a few religious conceptions, and the names of two or three deities, are common to it with the Greek. It is even more important to observe that the earliest monuments of Indian poetry, consisting, as they do, almost exclusively of hymns in praise of the national deities, and being the productions of an age far anterior to that of Homer and Hesiod, represent a more ancient period of religious development than we discover in the Greek poets, and disclose to us, in the earliest stages of formation, a variety of myths which a few centuries later had assumed a fixed and recognised form.* It is also to be noticed that, from the copiousness of their materials, the hymns of the Rigveda supply us with far more minute illustrations of the natural workings of the human mind, in the period of its infancy, upon matters of religion than we can

* See Professor Max Müller's essay on "Comparative Mythology," in the Oxford Essays for 1856, p. 47.

find in any other literature whatever. From their higher antiquity, these Indian hymns are also fitted to throw light on the meaning of a few points of the Greek system which were before obscure. Thus, as we shall see, the Indian *Dyaus* (sky, or heaven) explains the original meaning of the Greek Zeus, and the Sanskrit Varuṇa gives a clue to the proper signification of Ouranos.

As in my former paper I stated the grounds on which the Vedic hymns are assumed to have been composed at a period considerably more than a thousand years before our era, I shall here take their great antiquity for granted, and proceed to give some account of their cosmogony and mythology.

To a simple mind, reflecting in the early ages of the world with awe and wonder on the origin of all things, various solutions of the mystery might naturally present themselves. Sometimes the production of the existing universe would be ascribed to physical, and at other times to spiritual, powers. On the one hand, the speculator, perceiving light and beauty emerge slowly every morning out of a gloom in which all objects had, shortly before, appeared to be confounded, might conceive that in like manner the brightness and order of the world around him had sprung necessarily out of an antecedent night in which the elements of all things had existed together in undistinguishable chaos. Or, on the other hand, contemplating the results effected by human energy and design, and arguing from the less to the greater, or, rather, impelled by an irresistible instinct to create other beings bearing his own likeness, but endowed with higher powers, he might feel that the well-ordered frame of nature could not possibly have sprung into being from any blind necessity, but must have been the work of a conscious and intelligent will. In this stage of thought, however, before the mind had risen to the conception of one supreme creator and governor of all things, the various departments of nature were apportioned between different divinities, each of whom was imagined to preside over his own special domain. But these domains were imperfectly defined. One blended with another, and might thus be subject in part to the rule of more than one deity. Or, according to the various relations under which they were regarded, these several provinces of the creation might be subdivided amongst a plurality of divinities, or varying forms of the same divinity. These remarks might be illustrated by numerous instances drawn from the Vedic mythology. In considering the literary productions of this same period, we further find that as yet the difference between mind and matter was but imperfectly conceived, and that although in some cases the distinction between some particular province of nature and the deity who was supposed to preside over it was clearly discerned, yet in other instances the two things were confounded, and the same visible object was at different times regarded diversely as being (1) either a portion of the inanimate universe, or as (2) an animated being and a cosmical power. Thus in the Vedic hymns the sun, the sky, and the earth are severally considered sometimes as

natural objects governed by particular gods, and sometimes as themselves deities who generate and control other beings.

The varieties and discrepancies which are in this way incident to all nature-worship are, in the case of the Vedic mythology, augmented by the number of the poets by whom it was moulded, and the length of time during which it continued in process of formation. The Rigveda consists of more than a thousand hymns, composed by successive generations of poets during a period of many centuries. The authors of these hymns give expression not only to the notions of the supernatural world which they had inherited from their ancestors, but also to their own new conceptions. In that early age the imaginations of men were peculiarly open to impressions from without; and in a country like India, where the phenomena of nature are often of the most striking description, such spectators could not fail to be overpowered by their influence. The creative faculties of the poets would thus be stimulated to the highest pitch. In the starry sky in the dawn, in the morning sun scaling the heavens, in the bright clouds floating across the air and assuming all manner of magnificent and fantastic shapes, in the thunder, lightning, rain and tempest, they beheld the presence and agency of different divine powers, propitious or angry. In the hymns composed under any such influences, the authors would naturally ascribe a peculiar or exclusive importance to the deities by whose energy the phenomena appeared to have been produced, and would celebrate their praises with proportionate fervour. Other poets might attribute the same natural appearances to the action of other deities, whose greatness they, in like manner, would extol; while others again would devote themselves to the service of some other god, whose working they seemed to witness in some other department of creation. In this way, while the same traditional divinities were acknowledged by all, the power, dignity, and functions of each several god might be differently estimated by different poets, or, perhaps, by the same poet, according to the external influence by which he was awed or inspired on each occasion. In such circumstances, it need not surprise us if one particular power or deity is in one place put above, and in another place subordinated to, some other god; is sometimes regarded as the creator, sometimes as the created. This is illustrated in the case of the first Vedic divinities, to whom I shall refer—viz., Heaven and Earth.

Dyaus and Prithivī.

It has been observed by a recent French writer, that "the marriage of Heaven and Earth forms the foundation of a hundred mythologies."* According to the Theogony of Hesiod (116 ff.), the first thing that arose out of chaos was "the broad-bosomed Earth, the firm abode of all things." She in her turn "produced the starry Heaven (Ouranos), co-extensive with herself, to envelope her on every

* Albert Réville, Essais de Critique Religieuse, p. 383.

part." From the union of these two powers sprang Oceanos, Kronos, the Cyclopes, Rheia, &c. (132 ff.); and from Kronos and Rheia again were produced Zeus, Here, and other deities (453 ff.).

The Rigveda (which is, as I have already intimated, the earliest source of information regarding the religion of India) contains no uniform or consistent system of theogony or cosmogony. But in numerous passages Heaven and Earth (Dyaus and Prithivī) are spoken of together as the parents of all things; and several separate hymns are dedicated to their honour. They are characterised by a profusion of epithets, not only of such a kind as are suggested by their various physical characteristics, vastness, breadth, profundity, productiveness, but also by others of a moral or spiritual nature, as innocuous, or beneficent, promoters of righteousness, and omniscient. In the Veda we are not told, as we are in the system of Hesiod, which of the two, Heaven or Earth, was the older. On the contrary, one of the ancient poets seems to have been perplexed by the difficulty of this question, as at the beginning of one of the hymns (i. 185) he exclaims, "which of these twain was the first, and which the last? How were they produced? Sages, who knows?" Besides being described together in the dual as the "parents," Heaven and Earth are separately spoken of in various passages, the one as the father, the other as the mother, as in vi. 51, 5—"O father Heaven, benignant mother Earth, brother Agni, and ye Vasus, be gracious to us."

I must here remark, by the way, that the words which stand in the original of this verse for father Heaven, or rather Heaven father, viz., *Dyaush pitar*, answer exactly to the Ζεύς πατήρ of the Greeks, and the Diespiter of the Latins, though, as is well known, Zeus is not in the Greek mythology, as he is in the Indian, identical with the primeval Heaven, the father of all things, but is his grandson; while again, the Indian god, who corresponds in name, and also in some points in character, with the Greek Οὐρανός, is Varuna, who, however, as we shall by and by see, differs from Οὐρανός in various respects.

The word PRITHIVĪ, on the other hand, which in most parts of the Rigveda is used for Earth, has no connection with any Greek word of the same meaning. It seems, however, originally to have been merely an epithet, meaning "broad;" and may have supplanted the older word *go*, which stands at the head of the earliest Indian vocabulary, as one of the synonymes of Prithivī (earth), and which closely resembles the Greek Γαῖα or Γῆ. In this way *Gaur mātar* may have once corresponded to the Γῆ μήτηρ or Δημήτηρ of the Greeks.

This designation of the Earth—the prolific source of all vegetable products, and the home of all living creatures—by the epithet of mother, is perfectly natural, as is proved by common usage. This is remarked by Lucretius in various passages,* (referred to by Professor SELLAR in his "Roman Poets of the

* De Rerum Naturâ, ii. 991 ff.; 998 ff.; v. 793 ff.; 799 ff.; 821 ff.

Republic," pp. 236, 247, 276), in which the poet says, that the Earth "has deservedly received and retains the name of mother." The Greek poets also, as Hesiod, Æschylus, and Euripides, speak in like manner of the Earth as the universal mother and nurse.* In like manner Tacitus (Germania, 40) tells us that certain of the German tribes "worshipped, in common, Ertha,† that is, mother earth, and imagined her to interfere in the affairs of men, and to move about among them in a covered car." And the conception of the Heaven as the father of all things is also, though in a less degree, a natural one, and is noticed by Lucretius where he says (ii. 992), that "We have all the same father, the Heaven, from whom the bounteous Earth receives that moisture by which she is rendered fruitful." The same idea may be obscurely implied by Diodorus Siculus (i. 7), where he says, that, in the opinion of some speculators, "heaven and earth had, according to the original constitution of things, but one form, the natural properties of the two being blended; but that afterwards, when the body of the one had become separated from that of the other, the world assumed that regular order which we now witness." And further on he adds: "And in regard to the nature of the universe, Euripides, who was a disciple of Anaxagoras, the physical philosopher, does not appear to have differed from the views which have been stated. For in his Melanippe he lays it down that 'The heaven and the earth were of one form; but when they became separated from each other, they produced all things and introduced them into the light,—trees, birds, beasts, the offspring of the deep, and the race of mortals.'"

But the Rigveda regards Heaven and Earth as the parents not only of men, but also of the gods, as appears from the epithet *deva-putre*, viz. "the twain who have gods for their children," which is applied to them in various passages.

On the other hand, however, these two divinities, Heaven and Earth (as I have above intimated), exemplify the general remark already made, that the Vedic deities are constantly appearing in opposite characters,—sometimes as supreme and as creators, at other times as subordinate and created. In many places Heaven and Earth are said to owe their existence and support to the gods, sometimes to one, and sometimes to another.‡ In one passage (i. 160, 4) it is said

* Hesiod Opp., 561, γῆ πάντων μήτηρ. Æschylus, Prom. 90 παμμῆτέρ τε γῆ; Sept. cont. Thebas, 16, γῆ τε μητρὶ, φιλτάτῃ τροφῷ. Euripides, Hippol., 601, ὦ γαῖα μῆτερ ἠλίου τ' ἀναπτυχαί. Compare also the name of the goddess Demeter, an old form of Ge meter. (See "Liddell and Scott's Lexicon," *s. v.*) Diodorus, i. 12, says that the Egyptians, "conceiving the earth as a sort of receptable of things in course of production, had designated her as mother; and that the Greeks had, in like manner, called her Demeter, the form of the word being slightly changed through time; since she was in ancient times named Gê Mêtêr (Earth Mother), as Orpheus testifies when he says: 'Earth (Gê) is the mother of all, Demeter, the wealth-bestowing.'"

† *Ertham* is Ritter's emendation, the common reading being *Nerthun*. (Compare Ritter's note on section 9 of the Germania.)

‡ In Rigveda, x. 54. 3, Indra is said to have created the father and the mother (Heaven and Earth) from his own body.

(and though the words seem to be meant as eulogistic of Heaven and Earth, they also affirm their creation), "He was the most skilful of all the skilful gods, who produced, who meted out, Heaven and Earth, and established them with undecaying supports." In other places, Heaven and Earth are said to bow down, to tremble, to be disturbed, at the presence of particular deities. In several hymns we find various speculations about their origin. One as to their respective priority has been quoted above. In another passage (x. 31, 7), the poet asks—"What was the forest, what was the tree, from which they fashioned Heaven and Earth?" In another hymn (x. 81, 3), the creation of the worlds is ascribed to the sole agency of the great Architect Visvakarman, who is also (x. 82, 3) called the father, the generator, the disposer, who knows all spheres and worlds, and gave names to the gods; but here, too, the same question is asked, as to whence the wood came of which Heaven and Earth were constructed, and other questions are put, which show the sense of awe and mystery with which the poet was oppressed.

Elsewhere (in a hymn, x. 129, quoted in the paper which I read before the Society last year), it is said that formerly there was neither non-existence nor existence, neither death nor immortality, neither night nor day. Nothing existed but the One, in whom love or desire arose, which was the first germ of mind,* and led to all further development. "Who can tell" (the poet proceeds) "whence this creation arose? The gods are subsequent to its production: Who then knows whence it sprang? He who in the highest heaven is its ruler, he knows, or perhaps not even he."

The Vedic Gods in general.

The gods (to whom I now pass) are sometimes said to be thirty-three in number, eleven belonging to each of the three spheres into which the universe is usually divided in the Rigveda, Heaven, Earth, and the region intermediate between the two. As we have already seen, these deities are occasionally described as being the progeny of Heaven and Earth; and in the passage just quoted, they are expressly affirmed to have come into existence subsequently to the creation of the world. In the Rigveda they are constantly spoken of as immortal; but in the later mythology, at least, their immortality is regarded as merely relative, since

* The part here assigned to love or desire (*Kāma*), in the creation, corresponds, as the classical scholar will have noticed, to the position of Eros in the Greek mythology. Hesiod (Theog. 120) makes this deity coeval with Gaia and Tartarus, and prior to Ouranos. (See "Smith's Dict. of Greek and Roman Biogr. and Myth." under the art. *Eros*, and the passages of Aristotle, Plato, and Aristophanes, there referred to.) In the Satapatha Brāhmana, and other similar works, the creative acts of Prajāpati are constantly said to have been preceded by desire. In the Atharva Veda, Kāma is distinctly personified as the god of desire in general, and as of love in particular; and his darts are there spoken of (iii. 25, 1 ff.) just as they might be by a Greek, or by a modern, poet: "I pierce thee in the heart with the terrible arrow of love (Kâma). May Love pierce thee in the heart, having bent his shaft winged with anxiety, pointed with desire," &c.

they are considered to perish, as far as regards their corporeal organisation, at every periodical dissolution of the universe. Their souls, however, like those which animate all other living creatures, from Brahmā to a plant, are, according to later theories, imperishable. In the Rigveda a specific origin is ascribed to many of the deities, as, for instance, to the important class called the Adityas, including Varuna, Mitra, and others, who are all regarded as the sons of Aditi. Indra, too, is in several places spoken of as having both a father and a mother.

In the Brāhmanas, the gods are regarded as having been originally mortal, and many discrepant stories are told of the way in which they acquired the prerogative of immortality.

Aditi.

It is not very easy to define the character of Aditi, the goddess whom I have just alluded to as the mother of Mitra, Varuna, and the other Adityas. In the old Indian vocabulary, the Nighantu, she is identified with *Prithivi*, the earth; and some of the epithets assigned to her in the Rigveda, such as "the widely-extended," "the supporter of creatures," "the friend of all men," would agree with this supposition. Some others of her designations, however, as "the luminous," appear to be more appropriate to the sky; and in various passages Aditi seems to be distinguished from the earth. Perhaps she may best be considered as a personification of universal nature, with which, in the following remarkable verse (i. 89, 10), she is in fact identified: "Aditi is the heaven; Aditi is the intermediate firmament; Aditi is mother, and father, and son; Aditi is all the gods, and the five tribes of men; Aditi is whatever has been born; Aditi is whatever shall be born." In another verse (v. 62, 8), she is thus mentioned, along with another goddess, Diti:—

"Ye, Mitra and Varuna, ascend your car, and from thence ye behold Aditi and Diti."

From her name, and the manner in which she is introduced, the latter goddess must be held to stand for something antithetical or supplementary to Aditi. The two together are meant to represent the whole creation, though it is not very clear what is the separate idea which each is intended to convey.

In a hymn of the tenth book of the Rigveda, supposed, from its position in the collection, and from its contents, to be of comparatively late date, the process of creation is described with greater minuteness than in most other passages, and the share which Aditi took in it is declared, though not in a very intelligible way—Rigveda, x. 72. 1, "Let us in chanted hymns celebrate with praise the births of the gods, any one of us who in this later age may behold them. 2. Brahmanaspati blew forth these births like a blacksmith. In the earliest age of the gods, the existent sprang from the non-existent. 3. In the first age of the

gods the existent sprang from the non-existent. Afterwards the regions sprang from Uttānapad. 4. The earth sprang from Uttānapad; from the earth sprang the regions. Daksha sprang from Aditi; Aditi sprang from Daksha. 5. Aditi was verily produced, she who is thy daughter, oh Daksha. After her the gods were born, blessed, partakers of immortality. 6. When, oh gods, ye moved, in agitation, upon those waters, then a violent dust issued from you, as from dancers. 7. When, oh gods, ye, like heroes, replenished the worlds, ye drew forth the sun, which was hidden in the (æthereal?) ocean. 8. Of the eight sons of Aditi, who were born of her body, she approached the gods with seven, and cast out Mārttānda, the eighth. 9. With seven sons Aditi approached the former generation. She again produced Mārttānda for birth as well as for death."

It will have been observed, that in the fourth verse of this hymn, Dakshais said to have sprung from Aditi, and reciprocally, Aditi from Daksha. The old Indian expositor Yāska (Nirukta, x. 23), thus attempts to explain this circumstance, which had struck him as very strange:—"Daksha is, they say, a son of Aditi, and is celebrated among the sons of Aditi. And yet Aditi, on the other hand, is the daughter of Daksha, according to the text, 'Daksha sprang from Aditi, and Aditi sprang from Daksha." How can this be possible? In this way, viz., that they may have had the same origin; or, in conformity with the nature of the gods, they may have been born from each other, and have derived their substance from each other."

Varuna and Mitra,

The most famous of the sons of Aditi are Varuna and Mitra, who are very frequently associated with each other in the Rigveda. I have already stated above, that Varuna corresponds in name to the Οὐρανός of the Greeks. "Uranos," as Professor MAX MÜLLER remarks,* "in the language of Hesiod, is used as a name for the sky; he is made or born that 'he should be a firm place for the blessed gods.'† It is said twice that Uranos covers every thing (v. 127), and that when he brings the night he is stretched out everywhere, embracing the earth.‡ This sounds almost as if the Greek mythe had still preserved a recollection of the etymological power of Uranos. For Uranos is the Sanskrit Varuna, and is derived from a root *var* to cover, &c." I repeat, however, what I have said above, that the parallel between the Greek Uranos and the India Varuna does not hold in all

* Oxford Essays for 1856, p. 41.

† Hesiod. Theog. 126,—Γαῖα δέ τοι πρῶτον μὲν ἐγείνατο Ἶσον ἑαυτῇ
Οὐρανὸν ἀστερόεντ', ἵνα μιν περὶ πάντα καλύπτοι,
Ὄφρ' εἴη μακάρεσσί θεοῖς ἕδος ἀσφαλὲς αἰεί.

‡ Ibid. v. 176,—ἦλθε δὲ Νύκτ' ἐπάγων μέγας Οὐρανός, ἀμφι δὲ Γαίῃ
ἱμείρων φιλότητος ἐπέσχετο καί ῥ' ἐτανύσθη
πάντῃ.

points. Not to insist on the fact, that Varuna is a far more important deity in the mythology of the Veda than Uranos is in that of Hesiod, there is also this special difference between the two, that in the Indian mythology there is no relation between Varuna and Prithivī, the earth; as husband and wife, as there is between Uranos and Gaia in Hesiod; nor is Varuna represented like Uranos as the progenitor of Dyaus or Zeus, except in the general way in which he is said (like many of the other Indian deities) to have formed and to preserve heaven and earth. The original identity of the two gods, however, appears to be not the less undoubted.

Varuna is also, in the opinion of certain writers,* connected, at least, indirectly, with the Ahura Mazda of the old Persian mythology; and in support of this it may be alleged,—(1.) That the name of Asura, the divine being,† is frequently applied to Varuna, as an epithet; (2.) That the class of Indian gods, called Adityas, of whom Varuna is the most distinguished, bears a certain analogy to the Amshaspands of the Zend mythology, of whom Ahura Mazda is the highest; and, (3.) That a close connection exists between Varuna and Mitra, just as Ahura and Mithra are frequently associated in the Zendavesta, though the position of the two has otherwise become altered, and Mithra is not even reckoned among the Amshaspands. Other scholars, however, think that there is no sufficient proof of Varuna and Ahura Mazda being connected with one another.

The common origin of the Mitra of the Indian and the Mithra of the Persian mythology is, however, placed beyond a doubt by the identity of their names. Accordingly, the late Dr F. Windischmann, in his dissertation on the Persian Mithra,‡ regards it as proved that this god was common to the whole primitive Aryan race before the separation of its Iranian (or Persian) from its Indian branch; though the conception of his character was afterwards modified by Zoroastrian ideas. That Mithra was worshipped in Persia in the age of Herodotus is, as Windischmann remarks, established by the currency of such Persian names as Mitradates and Mitrobates. Herodotus himself (i. 131) speaks of Mitra not as a god but as a goddess. But Xenophon describes the Persians as swearing by the god Mitra. And Plutarch, in his treatise on Isis and Osiris, chapter xlvi., tells us that Zoroaster conceived of Mithra as standing between the deities Oromazes, the representative of light, and Areimanius the representative of darkness and ignorance. I need not further refer to the Persian Mithra, the ultimate introduction of whose worship into the west, in the time of the Roman Emperors, is matter of history.

* Roth in the Journal of the German Oriental Society, vi. 69 f.; Whitney in the Journal of the American Oriental Society, vol. iii. p. 327.

† A name identical with the Zend Ahura, as the letter *s* of Sanskrit words is always represented by *h* in Zend.

‡ Abhandlungen für die kunde des Morgenlandes. Mithra, ein Beitrag zur Mythengeschichte des Orients. Leipzig, 1857.

I return to the Mitra and Varuna of the Rigveda. The frequent association of these two gods is easily explained, if the Indian commentators are right in determining that Mitra is the sun, or the deity who presides over the day, while Varuna is the god who envelops everything in darkness, and rules over the night. In one text of the Rigveda, it is said of the latter that he "embraces the nights, and by his wisdom establishes the day, and does every thing perfectly." On this Indian interpretation, Professor Roth makes the following ingenious remarks:*—"Though such representations, as expressed in Indian exegesis, are far too narrow and one-sided, they nevertheless contain a certain amount of truth, and we may guess by what process they are to be developed. If Varuna is, as his name shows, the Aditya whose abode and whose sphere of authority is the bright heaven, in whose bosom is embraced all that lives; and if, therefore, he forms the remotest boundary, beyond which human thought can seek nothing further, then is he also one who can hardly be attained either by the eye or the imagination. By day the visual power cannot discover this remotest limit; the bright heaven presents to it no resting place. But at night this curtain of the world in which Varuna is enthroned, appears to approach nearer, and becomes perceptible as the eye finds a limit. Varuna is closer to men. Besides, the other divine forms which, in the clouds, in the atmosphere, and in the rays of light, filled up the space between the earth and yonder immeasurable outermost sphere, have vanished. No other god now stands betwixt Varuna and the mortal beholder."

Varuna is, notwithstanding, represented in the Veda as being sometimes visible in a bodily shape. He then assumes a luminous aspect, or is clad in golden armour. He sits in his abode exercising sovereignty, surrounded by his spies (or angels); and in two passages he is described as the joint occupant with Mitra of a vast palace, supported by a thousand columns.† Again, these two deities are described as ascending their chariot, which shines with a golden radiance at the break of day, and at sunset assumes the colour of iron. Seated in this car, and soaring in the empyrean, they behold all things in heaven and earth. The sun is in one passage denominated the golden-winged messenger of Varuna; in other places he is said to be the eye of Mitra and Varuna. Both of these deities, but in particular Varuna, are celebrated by a variety of epithets, as exercising sovereign authority and universal sway, as possessing a spiritual nature, and divine wisdom. The grandest cosmical functions are ascribed to Varuna. Possessed of illimitable resources, this great being has meted out, created, and upholds heaven and earth. He dwells in all worlds as sovereign: indeed the three worlds are embraced within him. The wind which resounds through the firmament is his breath. He has placed the sun in the heaven, and opened up

* Journal of the German Oriental Society, vi. 70 f.

† Compare Ovid. Met. ii. ff.: "Regia Solis erat sublimibus alta columnis," &c.

a boundless path for it to traverse. He has hollowed out the channels of the rivers. It is by his wise contrivance that, though all the rivers pour their waters into the sea, the sea is never filled.* By his ordinance the moon shines in the sky, and the stars which are visible by night disappear on the approach of daylight. Neither the birds flying in the air, nor the rivers in their sleepless flow, can attain a knowledge of his power or his wrath. His spies (or angels) behold both worlds. He himself has a thousand eyes. He knows the flight of birds in the sky, the path of ships on the sea, the course of the far-sweeping wind, and perceives all the hidden things that have been or that shall be done. No creature can even wink without him. He is a witness of men's truth and falsehood. His power and his omniscience are thus celebrated in the Atharva Veda (iv. 16, 1–6):

(1.) "The Great Ruler of these (worlds) beholds, as if he were close at hand. When any man thinks to do aught by stealth, the gods know it all; (2.) and (they perceive) every one who stands, or walks, or glides along secretly, or withdraws into his house, or into any lurking place. Whatever two persons, sitting together, devise, is known to Varuna the king (present there as) a third. (3.) This earth, too (belongs) to King Varuna, and that vast sky, with its far distant limits. The two oceans (aërial and terrestrial), are Varuna's loins; and he dwells in this small pool of water. (4.) He who should flee far beyond the sky, would not there escape from Varuna the king. His spies (or angels), descending from heaven, traverse this world; thousand-eyed they look across the whole earth. (5.) King Varuna perceives all that is within, and all that is beyond, heaven and earth. The winkings of men's eyes are numbered by him. He handles (all) these (things) as a gamester his dice. (6.) May thy destructive nooses, which are cast sevenfold and threefold, ensnare the man who speaks lies, and pass by the man who speaks truth!"†

* Compare Ecclesiastes i. 7.—"All the rivers run into the sea; yet the sea is not full; unto the place from whence the rivers come, thither they return again."

† Then follow two verses containing imprecations. After giving a German translation of this hymn in his "Dissertation on the Atharva Veda" (Tübingen, 1856), Professor Roth remarks:—"There is no hymn in the whole Vedic literature which expresses the Divine omniscience in such forcible terms as this; and yet this beautiful description has been degraded into an introduction to an imprecation. But in this case, as in many other passages of this Veda, it is natural to conjecture that existing fragments of older hymns have been used to deck out magical formulas. The first five, or even six, verses of this hymn might be regarded as a fragment of this sort."

I have attempted to transfer this hymn into English verse as follows:—

"The mighty Lord on high our deeds, as if at hand, espies:
The gods know all men do, though men would fain their sins disguise.
Whoever stands, whoever moves, or steals from place to place,
Or hides him in his secret den,—the gods his movements trace.
Wherever two together plot, and deem they are alone,
King Varuna is there, a third, and all their schemes are known.
This earth is Varuna's, and his those vast and boundless skies;
These oceans are his loins, and yet in that small pool he lies.
Whoever far beyond the sky should think his way to wing,
Yet could not there escape the hand of Varuna the king.

The attributes and functions assigned to Varuna impart to his character a moral grandeur and sanctity far surpassing that ascribed to any other Indian deity. He is supposed to have unlimited control over the destinies of mankind. He is continually supplicated to drive away evil and sin. He is entreated not to steal away, but to prolong life; and to spare the suppliant who daily transgresses his laws. With his bonds or nooses he seizes and afflicts transgressors. Mitra and Varuna conjointly are said to be armed with many nooses for entangling liars. And Indra and Varuna are described as binding men with bonds not formed of cords. On the other hand, Varuna is said to be gracious even to him who has committed sin. He is the wise guardian of immortality, and a hope is held out to the good that they shall behold him reigning together with Yama in blessedness in the world to come.

I shall add some specimens of a hymn (Rigveda vii. 86) already translated by Professor MAX MÜLLER, in which Vasishtha, the *rishi*, or seer, who appears to be the author, expresses his sense of Varuna's displeasure, and implores the restoration of his favour. I begin with the third verse:—"Seeking to know that sin, O Varuna, I inquire; I resort to the wise to ask. The sages all tell me the same; it is Varuna who is angry with thee. 4. What great sin is it, Varuna, for which thou seekest to slay thy worshipper and friend?* Tell me, O unassailable and self-existent god; and, freed from sin, I shall speedily come to thee with adoration. 5. Release us from the sins of our fathers, and from those which we have committed in our own persons. O king, release Vasishtha like a robber who has fed upon cattle; release him like a calf from its tether. 6. It was not our will, Varuna, but some seduction, which led us astray,—wine, anger, dice, or thoughtlessness. The stronger perverts the weaker. Even sleep occasions sin."

The following touching hymn (vii. 89) has also been already translated by

> His spies descending from on high glide all this world around,
> And thousand-eyed their gaze they cast to earth's remotest bound.
> Whate'er beyond the heaven and earth, whate'er exists between,
> That too by Varuna the king is all distinctly seen.
> The ceaseless winkings all he counts of every mortal's eyes:
> He wields this universal frame, as gamester holds his dice.
> Those knotted nooses which thou fling'st, O god, the bad to snare,—
> All liars let them overtake, but all the truthful spare."

With this hymn compare Psalm cxxxix. 1–10, *passim;* with verse 2, compare St Matthew xviii. 20; and with verse 5, St Matthew x. 30.

* In another place (vii. 88, 4, ff.) the same seer alludes to his previous friendship with Varuna, and to the favours formerly conferred on him by that deity, and inquires the reason of their cessation. "Varuna placed Vasishtha on his boat; by his power the wise and mighty god made him a rishi, to offer praise in an auspicious period of his life, that his days and dawns might be prolonged. 5. Where are those friendships of us two?* Let us seek the harmony which we enjoyed of old. I have gone, O self-existing Varuna, to thy vast and spacious house with a thousand gates. He who was thy friend, intimate, constant, and beloved, has, O Varuna, committed offences against thee. Let not us who are guilty reap the fruits of our sin. Do thou, O wise god, grant protection to him who praises thee."

* Compare Psalms lxxxix. 49, and xxv. 6.

Professor MAX MÜLLER:—"Let me not, O king Varuna, go to the house of earth. Be gracious, O mighty god, be gracious. 2. I go along, O thunderer, quivering like an inflated skin; be gracious, &c. 3. O bright and mighty god, I have transgressed through want of power, be gracious, &c. 4. Thirst has overwhelmed thy worshipper when standing even in the midst of the waters; be gracious, &c. 5. Whatever offence this be, O Varuna, that we mortals commit against the people of the sky (the gods); in whatever way we have broken thy laws by thoughtlessness, be gracious, O mighty god, be gracious."

Indra.

Professor ROTH* is of opinion that Varuna belongs to an older dynasty of the gods than Indra, and that during the Vedic period the high consideration which originally attached to the former god, was in course of being transferred to his rival. However this may be, there is no doubt that Indra is, as ROTH remarks,† the favourite deity of the Aryan Indians. More hymns of the Rigveda are dedicated to his honour than to the praise of any other divinity. Although, however, his greatness is celebrated in the most magnificent terms, he is not, as I have already noticed, regarded as an uncreated being, but is described in numerous passages as having a father and a mother. Thus it is said of him (Rigveda iv. 17, 4) "Thy father was the parent of a most heroic son: the maker of Indra, he who produced the celestial and invincible thunderer, was a most skilful workman." And again (x. 134, 1): "A divine mother bore thee; a blessed mother bore thee." In one place only is his mother's name mentioned, and she is there called Nishtigrî. This word is treated by the commentator as a synonyme of Aditi; but though Indra is regarded as an Aditya in the later mythology, and appears to be addressed as such, along with Varuna, in one passage of the Rigveda (vii. 85, 4), he is not, as far as I am aware, described as such in the other parts of that collection.

Indra is the regent of the atmosphere or intermediate region, the Jupiter Tonans and Jupiter Pluvius‡ of the Vedic Pantheon. He is the most martial of all the deities. Even as an infant he is said to have manifested his warlike disposition. "As soon as he was born," says one text (viii. 45, 4, 5) the slayer of Vrittra seized his weapon and asked his mother, 'Who are they that are renowned as fierce warriors?'" He leads the armies of the gods in their assaults on the Asuras or Titans, destroys all the superhuman enemies of his worshippers, and grants them victory over their mortal foes.

A great variety of laudatory epithets are lavished upon Indra. He is styled

* "Jour. Germ. Orient. Society," vi. 73; "Sanskrit and German Lexicon," *s.v.* Indra.

† "Sanskrit Lexicon," *s.v.*

‡ See STRABO, xv. 1, 69, p. 718; quoted by LASSEN, Indische Alterthumsk. ii. 698; Λέγεται δὲ καὶ ταῦτα παρὰ τῶν συγγραφέων, ὅτι σέβονται μὲν τον ὄμβριον Δία οἱ Ινδοὶ, καὶ τον Τάγγην ποταμὸν, καὶ τοὺς ἐγχωρίους δαίμονας.

youthful as well as ancient, undecaying, strong, agile, heroic, martial, all-conquering, lord of unbounded wisdom and irresistible power, wielder of the lightnings, &c., &c. He has vigour in his body, strength in his arms, a thunderbolt in his hand, and wisdom in his head. He assumes the most beautiful forms, and is invested with all the splendour of the sun. The Vedic poets have described a few of the features of his personal appearance. The epithet which is most constantly applied to him is *Susipra* or *Siprin*,—in the interpretation of which the Indian commentator wavers "between god with the handsome cheeks or nose," and "the god with the shining helmet or turban." He is also called "the ruddy-cheeked," the "ruddy-haired," the "ruddy, or golden-hued." He wears a ruddy, or golden beard, which is violently agitated when he puts himself in motion. He is also called the "iron god," which the commentator explains to mean that he wears a coat of iron mail. But his forms are endless; he can assume any shape he pleases. Holding in his hand a golden whip, he is mounted on a golden car, which moves more swiftly than thought, drawn by two ruddy or tawny steeds, snorting, neighing, and irresistible, with flowing golden manes, hair like peacock's feathers, and tails like peacocks. He is also said to be borne along by the horses of the sun; or, by a natural and obvious image, by the horses of the wind (Vâta). He is armed with a thunderbolt, forged by Tvashtri, the Indian Vulcan, which is variously described as of gold and of iron, as four-angled, as ending in a hundred, and a thousand points. He is elsewhere said to carry a bow, and to discharge arrows with a hundred points, and winged with a thousand feathers. Invoked by his mortal worshippers, he speedily obeys their summons, and arrives in his chariot to receive their offerings. He finds food prepared for his horses, and large libations of the juice of the soma plant (*Asclepias acida*, or *Sarcostemma viminale*) are poured out for himself to quaff. All the gods, we are told, hasten eagerly, when invited, to partake of this beverage, but Indra is particularly addicted to the indulgence, and seems to be dependent upon it for all his valour and energy. His mother gave him this juice to drink on the very day of his birth. Exhilarated by copious draughts of this elixir, and fortified by the encouragement both of gods and men (who are even said to place the thunderbolt in his hand), Indra hurries off, escorted by troops of Maruts or Winds, and sometimes attended by his faithful comrade Vishnu, or by Agni, or by Vâyu, to encounter the hostile powers in the atmosphere, who malevolently shut up the liquid treasures of the clouds. These demons of drought, who are called by a great variety of names, such as Ahi, Vrittra, Sushna, Namuchi, &c., &c., and who, on their side, also, are armed with every variety of celestial artillery, vainly attempt to resist the onset of the god. Heaven and earth quake with affright at the crash of Indra's thunder, and even Tvashtri himself, the forger of that thunder, trembles at the noise of his own handiwork, and at the fury of the impetuous deity by whom it is wielded. The enemies of Indra are speedily

pierced and shattered by the very sound of his iron bolts. The waters, released from their imprisonment, descend in copious streams to the earth, fill all the rivers, and roll downward in torrents to the ocean. The gloom which had overspread the sky is dispersed, and the sun is restored to his position in the heavens. Constant allusions to these conflicts between the opposing powers of the atmosphere occur in nearly every part of the Rigveda, and the descriptions are sometimes embellished with a certain variety of imagery. The clouds are represented as mountains, or are variously characterised as the autumnal, moving, iron, or stone-built cities of the demons of the atmosphere, which Indra overthrows. He destroys his enemies when he discovers them on the aërial mountains, or hurls them back when they attempt to take the sky by escalade. One is a monster with ninety-nine arms: a second has three heads and six eyes; a third he pierces with ice, or crushes with his foot; the head of a fourth he strikes off with the foam of the waters.

The growth of much of the imagery just described is perfectly natural and easily intelligible, especially to persons who have lived in India and witnessed the phenomena of the seasons in that country. At the close of the long hot weather, when every one is crying aloud for rain to moisten the earth and cool the atmosphere, it is often extremely tantalizing to see the clouds collecting and floating across the sky, day after day, without discharging their contents. And in the early ages, when the Vedic hymns were composed, it was quite in consonance with the other ideas which their authors entertained, to imagine that some malignant influence was at work in the atmosphere to prevent the fall of the fertilizing showers of which the parched fields stood so much in need. It was but a step further to personify both the hostile power and the beneficent agency by which it was at length overcome. Indra is thus at once a terrible warrior and a gracious friend, whose shafts deal destruction to his enemies, while they are the instruments of deliverance and prosperity to his worshippers. The phenomena of thunder and lightning almost inevitably suggest the idea of a conflict between opposing forces: even we ourselves, in our more prosaic age, often speak of the war, or strife, of the elements. The other appearances of the sky, too, afforded abundant materials for poetical imagery. The worshipper would at one time transform the clouds into the chariots* and horses of his god, and, at another time, would seem to perceive in their piled-up masses the cities and castles which he was advancing to overthrow.

The power and glory of Indra are characterised by the grandest epithets. Thus, it is said of him in different texts: "His greatness transcends the sky, the earth, and the atmosphere. He who fixed the quivering earth, who gave stability to the agitated mountains, who meted out the vast atmosphere, who propped up

* Compare Psalm civ. 3.

the sky, he, O men, is Indra. Not all the gods are able to frustrate the counsels of Indra, who established the earth and this sky, and, wonder-working, produced the sun and the dawn. Through fear of thee, Indra, all the mundane regions, however steady, begin to totter; heaven and earth, mountains, forests, all that is fixed, is afraid at thy coming. Indra is not to be overcome, Sakra (another name of Indra) is not to be overpowered; he hears and sees all things. At the birth of thee, the glorious one, the heavens trembled, and the earth, through fear of thy wrath. Heaven and earth are not sufficient for his girdle. All the gods yield to him in power and force. The two worlds are equal to but the half of him. Which of the poets who were before us have found out the end of all thy greatness? seeing that thou didst produce at once the father and the mother (*i.e.*, heaven and earth) from thine own body."

These passages afford a fair specimen of the strains in which Indra is most commonly celebrated in the hymns. It will be observed that the attributes which are assigned to this deity are chiefly those of physical superiority and dominion over the external world. In fact, he is not generally represented as possessing the spiritual elevation and moral grandeur which are so strikingly characteristic of Varuna. There are, however, many texts in which his close relations with his worshippers are described, and a few in which an ethical character is attributed to him. Faith in him is confessed or enjoined in various passages; the reality of his existence and power is asserted in opposition to sceptical or faithless doubts. He is the friend, and even the brother, of his present worshippers, as he was the friend of their forefathers; but he desires no friendship with the man who offers no oblations. He is reminded that he himself has friends, while his adorers are friendless. His friend is never slain or conquered. It is he almost exclusively who is invoked as the patron of the Arian Indians, or civilized invaders of Hindostan, and their protector against their barbarous aboriginal foes, or their unseen and supernatural enemies. He is invoked by men as a father; he is embraced by the hymns of his votaries as a husband is embraced by his wives; his right hand is seized by suppliants for riches; his powerful arms are resorted to for protection, and he is a deliverer easy to be entreated. He is implored not to slay for one, two, three, or even for many sins. Destruction falls upon the man who offers him no libations, while he richly rewards his faithful servants. Yet he is sometimes naively importuned to be more prompt in his generosity, and is even given to understand that his worshipper, if in his place, and possessed of his means, would be more liberal. He is supplicated for all sorts of temporal blessings, and, among the rest, for victory in battle. As a man, in walking, puts first one foot forward and then the other, so Indra, by his power, changes men's relative positions; he subdues the fierce, and puts others in the foremost ranks; he is the enemy of the

prosperous and ungodly man, while he protects his own servants, and leads them into a "large room,"* into celestial light and security.

Vâyu and the Maruts.

Vâyu, or the Wind, who, as we have seen, is often associated with Indra, does not occupy a very prominent position in the Rigveda. But few epithets are applied to him. He is called beautiful or conspicuous, and handsome in form. He is thousand-eyed, and swift as thought. Like other deities who have been already passed under review, he rides in a shining golden chariot, drawn by ruddy or purple steeds, and his team is sometimes said to consist of as many as a hundred, or even a thousand horses. Indra frequently rides by his side. In one of the latest hymns (x. 168), the phenomena of the wind are picturesquely described. His chariot rolls along, resounding, and rending all it encounters. He drives before him, as he advances, the dust of the earth. He never ceases to move along the paths of the atmosphere. The poet then asks, in phrases some of which are almost those of St John (iii. 8), "In what place was he born? whence has he sprung? Soul of the deities, source of the universe, this god wanders where he lists; his sound is heard, but his form is not (seen)."

The Maruts or Rudras, deities of the storm, receive in the Rigveda a much more frequent and enthusiastic celebration than Vāyu. They are the sons of Rudra (who will be noticed below) and Prisni. Numerous hymns are dedicated to their honour, in which they are described by a great variety of picturesque epithets. They are compared to blazing fires; they are free from soil, and of sun-like brilliancy. In one place they are thus apostrophised:—"Spears rest upon your shoulders, ye Maruts; ye have anklets on your feet, golden ornaments on your breasts, fiery lightnings in your hands, and golden helmets on your heads." They shatter the demon of drought into fragments; they are clothed with rain; they distribute showers over all the world, and alleviate the burning heat; they shake the mountains, the earth, and both the worlds; they overturn trees, and like wild elephants, they consume the forests; they have iron teeth; they roar like lions, and all creatures are afraid of them; they are swift as thought; they ride, with whips in their hands, in golden cars, with golden wheels, drawn by ruddy, tawny, or speckled horses, with which their chariots are said to be winged.

The Maruts, as we have seen, are frequently described as the attendants of Indra; but they are the subjects of celebration in many separate hymns in which that deity is never mentioned.

Rudra.

Rudra, who, under the name of Siva, or Mahâdeva, and as the third person

* This is a phrase of frequent occurrence in the Rigveda. Compare the very similar expressions in Psalms xviii. 19, xxxi. 8, and cxviii. 5.

of the Indian triad, the Destroyer, is one of the three great gods in the later Indian mythology, is a deity of but subordinate importance in the hymns of the Rigveda. Like most of the other gods, however, he is designated in those hymns by a variety of magnificent epithets. He is self-dependent, the strongest and most glorious of beings, the father of the world, cognisant of all the doings of gods and men. He is described as seated in a chariot; as being himself brilliant as the sun; as arrayed in golden ornaments, and wearing braided hair; as wielding a thunderbolt; as armed with a bow and arrows, a strong bow and fleet arrows. His shafts are discharged from the sky, and traverse the earth. He is called the slayer of men. His anger and his destructive bolts are frequently deprecated; but he is also represented as benevolent, gracious, easily entreated, as the source of health and prosperity to man and beast. He is often described as the possessor of healing remedies, and is characterised as the greatest of physicians. Rudra is also designated in various texts as the father of the Rudras or Maruts, the class of deities last described, who rule the winds; and from this relation we might expect that he would be represented as still more eminently than they, the generator of tempests and chaser of clouds. Except, however, in a small number of texts, there are few distinct traces of any such agency being assigned to him. The numerous vague epithets which he constantly receives would not suffice to fix the particular sphere of his operation, or even to define his personality, as most of them are applied to other deities. While, however, the cosmical function of Rudra is thus but obscurely indicated, he is, as we have seen, described as possessing other marked and peculiar characteristics. There can be little doubt, though he is frequently supplicated to bestow prosperity, and addressed as the possessor of healing remedies, that he is principally regarded as a malevolent deity, whose destructive shafts—the source of disease and death—the worshipper strives by his entreaties to avert. If this view be correct, the remedies which Rudra dispenses, may signify little more than the cessation of his destroying agency.*

Vishnu.

Vishnu, who, at a later period, was considered as one of the class of gods mentioned above, the Adityas, and who, as the second deity in the great Indian triad, has cast all the other gods except Rudra, or Siva, into the shade, was not, as compared with Indra or Varuna, or perhaps even with Savitri, a very prominent object of adoration in the Vedic age. There are, however, a few hymns in which he is celebrated, sometimes singly, but mostly in conjunction with Indra, and also a good many detached verses in which he is mentioned. The characteristic function by which he is repeatedly distinguished from every other god,†

* See Sanskrit Texts, vol. iv. p. 339, f.

† Only Indra is associated with him in two passages (vi. 69. 5; and vii. 99. 6) as taking vast strides.

is that of striding across the heavens in three paces. These three steps are explained by one of the ancient interpreters as denoting the triple manifestation of light—as fire on earth, as lightning in the atmosphere, and as the sun in the sky; and by another, as designating the three stages of the sun's daily movement—his rising, culmination, and setting. From this difference of view prevailing between two of the oldest expounders of the Veda, it appears that these three steps of Vishnu must, from a very ancient period, have been regarded as something enigmatical. Some of the highest divine functions and attributes are also assigned to Vishnu in the hymns. Thus he is said to support alone the sky and the earth, and to comprehend all the worlds with his three vast strides. No one, even the soaring birds, can attempt to follow his third and loftiest step. He alone is acquainted with the highest sphere. No one born, or yet to be born, can in thought attain to the furthest limit of his greatness. The pious enjoy celestial bliss in his abode. Varuna and the Asvins do homage to his power. But we are not, therefore, to imagine that he was regarded as superior to the other deities; for Indra is associated with Vishnu even in some of the hymns in which the latter is most magnified. Nay, in one place, the power through which Vishnu takes his three strides is said to be derived from Indra; in two other texts Vishnu is represented as celebrating Indra's praises; and, as described in various other passages, the former appears to play a secondary part as compared with the latter. Besides, the same high functions and awful attributes which are ascribed to Vishnu, are in other and far more numerous texts assigned to Indra, Varuna, and other deities.*

Sūrya and Savitri.

The great powers presiding over day and night are, as we have seen, supposed by the Indian commentators to be personified in Mitra and Varuna. But these two deities, and especially Varuna, are far more than the mere representatives of day and night. They are recognised as moral governors, as well as superintendents of physical phenomena. There are two other deities who are far more direct representatives of the solar orb—viz., Sūrya and Savitri, who are in a few passages described as belonging to the same class of gods as Mitra and Varuna—viz., the Adityas. It is under these two different names that the sun is chiefly celebrated in the Veda, according, perhaps, to the different aspects in which he is viewed, or the functions which he is conceived as fulfilling. Different sets of hymns are devoted to his worship under each of these appellations; and the epithets which are applied to him under each of these characters are for the most part different.

Sūrya is described as moving through the heavens on a car, which is some-

* See my Sanskrit Texts, vol. iv. Preface iv. ff., and pp. 54–101.

times said to be drawn by one, sometimes by several, sometimes by seven, horses. His path is prepared by the Adityas, Mitra, Aryaman, and Varuna. The god Pûshan goes as his messenger, with his golden ships, which sail in the aerial ocean. Sûrya is the preserver, the soul, of all things, moving or stationary; the vivifier of men; the upholder of the sky. He rolls up darkness like a hide. He is far-seeing, and by an image common, I suppose, to most literatures (and which we find in Homer and Æschylus),* he is said to be all-seeing, beholding all worlds, and the good and bad deeds of men. He is the eye† of Mitra and Varuna. In many passages, however, his dependent position is asserted. Thus, he is said to have been produced, or placed in the sky, or caused to shine, by Indra, or by some other deity. Ushas, the Dawn, is in one place said to be his wife. In another passage the Dawns are, by a natural figure, said to produce him.

The name Savitri is derived from a Sanskrit root *su*, to propel, stimulate, or inspire; and may therefore be taken to denote the sun in his character of stimulator or inspirer. This signification of the name is frequently referred to in the Veda, and is coupled with the constant use, in various other forms, of the verb from which it is derived, to denote the functions attributed to this god. As described in the Rigveda, Savitri is pre-eminently the brilliant and golden deity. He is golden-eyed, golden-handed, golden-tongued; he is surrounded by a golden lustre; he mounts a golden car, drawn by radiant horses; he stretches out his golden arms, which infuse energy into all creatures, and reach to the utmost ends of the sky. He is also called broad-handed, beautiful-handed, beautiful-tongued. He beholds all things; he illuminates the atmosphere and all the regions of the earth. His ancient paths in the sky are said to be free from dust. He is called a divine spirit (*asura*). His will and independent dominion cannot be resisted, even by Indra, Varuna, Mitra, Aryaman, Rudra, or any other being. The waters fall and the winds blow by his ordinance. His praises are celebrated by the Vasus, by Aditi, by the royal Varuna, by Mitra, and by Aryaman. He is the lord of creatures, the supporter of the world and of the sky. In one place he is even said, whether literally or figuratively, to bestow on the gods the gift of immortality. It would appear to result from all this, that Savitri was at one time the object of a very enthusiastic adoration in India; and in fact the holiest text in the Veda (iii. 62. 10), that which is called par excellence *the gâyatrî*, is addressed to him. This verse is thus rendered by Mr Colebrooke (Misc. Ess. i. 30)—"Let us meditate on the adorable light of that divine ruler (*Savitri*): may it guide our intellects." Professor H. H. Wilson translates it thus: "We meditate on that desirable light of the divine Savitri, who influences our pious rites." Professor Benfey, in his

* Iliad, iii. 277, xiv. 344 f.; Odyssey, viii. 270; and Æsch. Prom. 91. Compare Ovid. Met., iv. 171 f.; 195 ff.

† Compare Hesiod, Opp. et dies: πάντα ἰδὼν Διὸς ὀφθαλμὸς καὶ πάντα νοήσας. κ τ λ.

"Sāma Veda" (ii. 812), gives yet a different rendering: "We receive this glorious brightness of the generator, of the god who will prosper our works."

In the hymns Savitri is sometimes expressly distinguished from Sûrya. To explain this circumstance, the Indian commentator asserts, that before his rising, the sun is called Savitri, and at his rising and setting Sûrya; and in another place he says, that though the godhead of the two deities is identical, they may yet, from the diversity of their forms, be spoken of as separate agents. Yāska, a much older writer, says, that "the time of Savitri's appearance is when darkness has been removed, and the rays of light have become diffused over the sky." But it is scarcely consistent with this explanation, that in one text Savitri is said to exercise his influence after the rising of the sun.

In other passages of the Rigveda, the two names appear to denote the same deity.

Tvashtri.

Another god who, in the later mythology, is regarded as one of the Adityas, but who does not yet bear that character in the Rigveda, is Tvashtri, the Vulcan of the Indian pantheon. He is represented as the most skilful of all artizans, and as versed in all admirable contrivances. He sharpens the iron axe of Brahmanaspati, and forges the thunderbolts of Indra. It is his peculiar function to fabricate forms: he gives shape to heaven and earth; he bestows generative power, moulds all structures, human and animal, out of the seminal germ; forms husband and wife for each other in the womb. He gave his daughter Saranyu in marriage to Vivasvat, the sun, and is thus the grandfather of Yama; and he is also described as the father-in-law of Vāyu, the god of the wind.

Agni.

Agni is the god of fire, the Ignis of the Latins. The word, as all scholars know, has been lost in Greek. He is one of the most prominent deities of the Rigveda, since nearly as many entire hymns are addressed to him as to Indra, and more than are assigned to any other divinity. Agni is not, like the Greek Hephæstus, or the Latin Vulcan, the artificer of the gods (an office which, as we have just seen, is in the Veda allotted to Tvashtri), but derives his importance almost exclusively from his connection with the ceremonial of sacrifice. He is an immortal, who has taken up his abode among mortals, as their guest, their friend, and their domestic priest. He is a sage, intimately acquainted with all the forms of worship, and qualified, by his wisdom and power, to bring them all to a successful termination. Every oblation which he superintends goes straight to the gods. He concentrates in himself, and exercises in a superior sense, all the various priestly offices distributed by the Indian ritual among a number of separate human functionaries. He is a messenger moving between heaven and earth, and commissioned both by gods and men to maintain their mutual communications, to announce to the immortals the hymns, and to convey to them the offer-

ings, of their worshippers. On the part of men, he invokes the gods, invites them to attend the ceremonies instituted in their honour, arrives with them seated in the same chariot, and receives them with reverence and adoration. In other places, he is somewhat differently described as the mouth, and the tongue, through which both gods and men participate in the sacrifices. He is the banner, symbol, or outward manifestation, the father, guardian, protector, lord, and king, of sacrifice; the lord of the house; the lord and king of the people; and the father, mother, brother, and son, of his worshippers, some of whom claim with him a hereditary friendship.

In the descriptions of the Rigveda, the element of fire is frequently, and almost inevitably, confounded with the deity who is supposed to be its representative,—many of the epithets applied to the latter being quite as appropriate, if not more appropriate, to the former. Thus, although Agni is often said to have been generated by the gods, or to be the offspring of heaven and earth, or to have been brought from heaven by Mātarisvan; he is also constantly alluded to as having been first kindled by Manu, or some other ancient sage, and his birth is described as resulting from the homely process of rubbing together two pieces of stick; which are spoken of as his parents,—parents, whom their infant offspring afterward unnaturally devours. This infant is, however, like the wriggling brood of a serpent, very difficult to catch; but when seized, he is nourished with oblations of clarified butter. This nourishment is alluded to in the epithets "butter-haired," and "butter-formed." The following are some of his other appellations; "smoke-bannered," "black-pathed" (this alludes, of course, to the way in which fire chars the wood which it consumes); "brilliant-coloured," "brilliant-flamed," "flaming-haired," "golden-haired," "golden-bearded," "golden-formed," "sharp-weaponed," "sharp-toothed," "golden-toothed," "four-eyed," "thousand-eyed," and "thousand-horned." His flames roar like the winds, like the waves of the sea, like a lion, like a bull; he envelopes the woods, and blackens them with his tongue; he shears the hair of the earth; he shaves the ground, as a barber a beard. He rides on a chariot of light, or of lightning, or of a brilliant or golden colour, drawn by fleet horses, of a ruddy or tawny hue.

In some passages, Agni appears to be identified with light in general, as where it is said (Rigveda, x. 88, 6, 10, ff.) "Agni is by night the head of the earth; from him is produced the sun which rises in the morning . . . . With a hymn the gods, through their power, produced in the heaven Agni, who fills the world. They made him to exist in a threefold character. . . . . When the adorable gods placed him in the sky as the solar orb, the son of Aditi, then they beheld all the worlds." The "threefold character" here alluded to, means, according to an old commentator, the three forms of fire upon earth, lightning in the atmosphere, and the sun in the heavens.*

* See Nirukta, vii. 23, and xii. 19; and my Sanskrit Texts, vol. iv. p. 55 f.

Although Agni is not in general celebrated in the same lofty strains as Indra and Varuna, there are yet a few passages in which the attributes of creator and preserver of the universe are assigned to him by the Vedic poets. Thus, he is said to have produced the two worlds; to have meted out the regions of the air, and the heavenly luminaries; to have spread out heaven and earth, like two skins; to have propped up the sky; and to exceed the universe in greatness. All the deities fear and reverence him. He delivers them from evil. His ordinances are not violated. He knows the races of the gods, the recesses of heaven, and the secrets of men. He beholds all worlds. He is celebrated and worshipped by Mitra, Varuna, the Maruts, and the 3339 divinities. Through him Varuna, Mitra, and Aryaman triumph. He is sometimes identified with other gods, such as Indra, Vishnu, Varuna, Mitra, Aryaman, Tvashtri, Rudra, and is said to comprehend them all within himself, as the circumference of a wheel surrounds the spokes. He is also at times associated with some of the other deities, especially with Indra, in several of whose functions, such as that of thunderer, slayer of Vrittra, and destroyer of cities, he is said to participate, and of whom he is in one place said to be the twin brother.

The votaries of Agni prosper. He is the friend of the man who entertains him as a guest, and he bestows protection and wealth on the worshipper who sweats to bring him fuel, and wearies his head to serve him. He has it in his power to bestow many kinds of blessings, and to avert many species of misfortunes, and is therefore supplicated to grant his favour and protection, and to be an iron wall with a hundred ramparts to shield his votaries. He is master of all the treasures in the earth, the atmosphere, and the sky. All blessings proceed from him, as branches from a tree.

In one passage, the worshipper naively says to Agni,—"If I were thou, and thou wert I, thy aspirations should be fulfilled on the spot;" and again,—"If, Agni, thou wert a mortal, and I an immortal, I would not abandon thee to wrong, or to penury. My worshipper should not be poor, nor distressed, nor miserable."

The blessings which this god is solicited to bestow are almost entirely of a physical character; but in one or two places, he is asked to forgive sin. He appears occasionally to be regarded as the arbiter of immortality; and in a funeral hymn he is besought to convey the "unborn part" of the deceased to the world of the righteous.

The Asvins.

The Asvins seem to have been a puzzle even to the oldest Indian commentators, one of whom, YÂSKA, refers to them in the following terms (Nirukta, xii. 1):—"Now come the deities whose sphere is the heaven. Of these the two Asvins are the first in order. They are called Asvins (from a root *as*), because

they pervade everything—the one of them with moisture, the other with light. AURNABHÂVA says they are denominated Asvins, because they have horses (*asvaih*). But who are these Asvins? 'Heaven and Earth,' say some. 'Day and Night,' say others. 'Two kings, performers of holy acts,' say the legendary writers. Their time is subsequent to midnight, whilst the break of day is delayed."

It may seem singular that two gods of a character so little defined as that of the Asvins should have been the objects of so enthusiastic a worship as appears, from numerous hymns in the Rigveda, to have been paid to them in ancient times. But the reason may have been, that they were regarded and hailed as precursors* of the return of day, after the darkness and dangers of the night.

According to one of the hymns in the tenth book of the Rigveda (xvii. 1, 2), they appear to have been regarded as the twin sons of Vivasvat and Saranyu. They are also called the grandsons of the Sky, and the offspring of the Sea—whether this means the terrestrial or the atmospheric ocean. The time of their appearance is the early dawn, when they yoke their horses to their car, and descend from heaven to receive the adorations and offerings of their votaries. In one place their sister is mentioned; and the Indian commentator considers that Ushas or Aurora is meant.† Their chariot is of a singular formation, being three-wheeled, triangular, and triple in some other parts of its construction, which are not very easy to explain. They are also fancifully requested to bestow a number of different sorts of blessings thrice. Their chariot is further described (like those of other gods) as golden, as swifter than thought, or than the twinkling of an eye, as thousand-formed, and decorated with a thousand banners. They are sometimes said to be drawn by a single ass,‡ but more frequently by fleet and winged horses.

The Asvins themselves are represented as young, beautiful, radiant, of golden form, wearing many shapes, and decked with lotus garlands, agile, fleet as thought, skilful, profound in wisdom, strong, awful, overthrowers of pride, armed with terrible and golden spears. They are also described as physicians, and restore the blind, sick, and emaciated to sight, health, and strength. In several hymns the numerous succours of various kinds which they had in former times granted

* ROTH says of them (Journal Germ. Orient. Society, iv. 225), "The two Asvins, though, like the ancient interpreters of the Veda, we are by no means at one about the conception of their character, hold yet, according to their signification, a perfectly distinct position in the entire body of the Vedic deities of light. They are the first bringers of light in the morning sky, who in their chariot rapidly precede the Dawn, and prepare the way for her." Compare Professor MAX MÜLLER's "Lectures on the Science of Language," 2d Series (which have just been published as this paper is passing through the press), pp. 489, ff.

† The passage alluded to is Rigveda, i. 180, 2. In another text, i. 123, 5, Ushas is said to be the sister of Bhaga and Varuna.

‡ See HAUG's "Aitareya Brâhmana," vol. ii. p. 273. "The Asvins were winners of the race with a carriage drawn by donkeys; they obtained (the prize). Thence (on account of the excessive efforts to arrive at the goal) the donkey lost its (original) velocity, became devoid of milk, and the slowest among all animals used for drawing carriages," &c. The race alluded to is one which the gods ran to settle a point in dispute between them. See p. 270 of the work just quoted.

to their worshippers are enumerated, and among them a few cures are specified. They are frequently supplicated for various kinds of blessings; they are implored to prolong life, and even to forgive sin; and their hereditary friendship with the worshipper is sometimes appealed to.

Soma.

I have already alluded to the important share which the exhilarating juice of the soma plant (*Asclepias acida* or *Sarcostemma viminale*) assumes in bracing Indra for his conflict with the hostile powers in the atmosphere. This juice, or rather the plant from which it is extracted, is personified in a god Soma, who is, or rather at one time was, the Indian Bacchus. The whole of the hymns in the ninth book of the Rigveda, 114 in number, besides a few in the other books, are dedicated to his honour. It is clear, therefore, as Professor Whitney remarks,* that his worship must have been at one time remarkably popular. The soma sacrifice, in fact, formed an important part of the old Brahmanical ritual, as well as that of the ancient Persian worship.† But with the decline of the Vedic rites, and the transformation of the old, or the introduction of new, deities, the early popularity of Soma has long since passed away, and his name is familiar to those learned Brahmans only who, in a few places, maintain the old tradition of the Vedic observances. The hymns addressed to Soma were intended to be sung while the juice of the plant from which he takes his name was being pressed out and purified.‡ They describe enthusiastically the flowing forth and filtration of the divine liquid, and the effects produced on the worshippers,§ and supposed to be produced on the gods, by partaking of the

* Journal of the American Oriental Society, iii. 299.

† See Dr Haug's Aitareya Brâhmana, i. 59, ff.; and Windischmann's "Somacultus der Arier;" as well as my "Sanskrit Texts," vol. ii. pp. 469, ff., where the most important parts of this dissertation are translated or abstracted. See also the extract there given from "Plutarch de Isid. et Osir." 46, where the *soma* plant, which in Zend is called *haoma*, is mentioned under the name of ὅμωμι.

‡ Whitney, Journal of the American Oriental Society, as above: "Sanskrit Texts," vol. ii. p. 470.

§ These effects are thus described in a verse (Rigveda, viii. 48, 3) which may be freely translated as follows:—

"We've quaffed the Soma bright, and are immortal grown:
We've entered into light, and all the gods have known.

"What mortal now can harm, or foeman vex us more?
Through thee beyond alarm, immortal god, we soar."

Compare Euripides, "Cyclops," 578, ff.,—

ὁ δ' οὐρανός μοι συμμεμιγμένος δοκεῖ
τῇ γῇ φέρεσθαι. τοῦ Διός τε τὸν θρόνον
λεύσσω τὸ πᾶν τε δαιμόνων ἀγνὸν σέβας.

I subjoin a free translation of the 119th Hymn of the Tenth Book, in which Indra himself is supposed to express his sensations when in a state of exhilaration:—

"1. Yes, yes, I will be generous now; and grant the bard a horse and cow.
I've quaffed the soma draught.

beverage. The juice itself is called an immortal draught, and a medicine for the sick. The god, too, is said to cover whatever is naked, to heal whatever is diseased; through him the blind sees, and the lame walks. The most magnificent attributes and functions are assigned to him. He is the friend, the ally, and the soul of Indra, whose vigour he stimulates, and whom he renders triumphant in his conflicts with Vrittra. He rides in Indra's chariot; he is armed with sharp and terrible weapons; and he is, like Indra, the destroyer of hostile demons, and the overthrower of their cities. And not only so, but he is also declared to be the father of the gods, the creator of the sky and the earth, of Agni, of Sûrya, of Indra, and of Vishnu. All creatures are in his hand; he is the king of gods and men, the upholder of the heavens, and the sustainer of the earth; he destroys darkness, and causes the sun to rise. He is thousand-eyed, beholds all worlds, and strikes down the impious into the abyss. He is the possessor of all resources. He bestows immortality on gods and men; and it is worthy of remark that in a passage (ix. 113, 7 ff.) where the joys of paradise are more distinctly anticipated and described than in most other parts of the Rigveda, the deity from whom this future felicity is asked is Soma. Two of the verses of this hymn are as follows:—
"Place me, O purified god, in that everlasting and imperishable world, where there is eternal light and glory. O Indu (Soma), flow for Indra. Make me immortal in that world where King Vaivasvata (Yama) lives, where is the innermost sphere of the heaven, where those great waters flow. O Indu, flow for Indra," &c.

"2. These draughts impel me with the force of tempests in their furious course.
I've quaffed the soma draught.

"3. They drive me like a car that speeds when whirled along by flying steeds.
I've quaffed, &c.

"4. Not fonder to her calf the cow than that fond hymn which seeks me now.
I've quaffed, &c.

"5. I turn it over while I muse, as carpenter the log he hews.
I've quaffed, &c.

"6. The tribes of men, the nations all, I count as something very small.
I've quaffed, &c.

"7. The sky and earth, though vast they be, don't equal even the half of me.
I've quaffed, &c.

"8. The heavens in greatness I surpass, and this broad earth, though huge her mass.
I've quaffed, &c.

"9. Come, let me as a plaything seize, and put her wheresoe'er I please.
I've quaffed, &c.

"10. Come, let me smite with vigorous blow, and send her flying to and fro.
I've quaffed, &c.

"11. My half is in the heavenly sphere; I've dragged the other half down here.
I've quaffed, &c.

"12. How great my glory and my power! Aloft into the skies I tower.
I've quaffed, &c.

"13. I'm ready now to mount in air, oblations for the gods to bear.
I've quaffed the soma draught."

Yama.

Yama, the son of Vivasvat and Saranyu, is the ruler of the world to come. In the later mythology he becomes distinctly the Indian Pluto, the judge of the dead, who in a future state recompenses the good and bad according to their deserts; but he is there depicted principally as an object of terror. The awful side of his character is not altogether unrecognised even in the Rigveda, where he is said to have two insatiable dogs with four eyes and wide nostrils, which guard the road to his abode, and wander about among men. They are evidently regarded with dread, and the spirits of the departed are advised to hurry past them. The bonds or nooses of Yama are also mentioned in one place along with those of Varuna, and he is, in another passage, identified with death, and described as sending a bird as a forewarner of doom. In a text of the Atharva-veda, death is said to be his messenger. But in the Vedic hymns he is most commonly represented as the sovereign and guardian of the blessed. In a text already quoted, the worshipper prays to be admitted to the abode of Yama, in the innermost sphere of heaven. In another place the souls of the departed are desired to proceed by the path which their fathers had trodden before, and which would introduce them to the vision of Yama and Varuna dwelling together in blessedness. Professor Roth has pointed out that Yama was sometimes regarded by the Indians as the first man, the first who departed to the other world and became its ruler.* Thus, in one text of the Rigveda, he appears to be spoken of as the sole existing mortal; and in another place he is described as having been the first to discover the way to heaven. This is still more distinctly expressed in a verse of the Atharvaveda, which runs thus: "Worship with an oblation that King Yama, the son of Vivasvat, the gatherer together of men, who was the first of mortals that died, the first who departed to this [heavenly] world." In another verse of the Rigveda he is described as carousing with the gods under a leafy tree.

It is quite clear from all this that towards the end, at least, of the Vedic age, the Indians had a distinct belief in a future state of rewards. The following are some of their other ideas regarding the future destinies of men. When the remains of the deceased have been placed upon the funeral pile, and the process of cremation has commenced, Agni, the god of fire, is besought not to scorch or consume the departed, not to rend asunder his skin or his limbs, but after the flames have done their proper work, to convey to the fathers the mortal who has been presented to him as an offering. The eye of the dead is bidden to go to the sun, his breath to the wind, and his other members to the sky, the earth, the waters, or the plants, according to their several affinities. As for his "unborn part," Agni is supplicated to kindle it with his heat, and putting on his most

* Manu, Yama's twin brother, is, however, far more frequently mentioned in the Rigveda as the first man or the progenitor of the Indians. See my paper on Manu, in the 20th vol. of the Journal of the Royal Asiatic Society, pp. 406, ff.

auspicious form, to convey it to the world of the righteous. Leaving behind it here below all that is evil and imperfect, and traversing the vast abyss of darkness which separates this world and the third heaven, the spirit soars in a car, or on wings, on the undecaying pinions of Agni, wafted by the Maruts, and fanned by delightful zephyrs, to the realms of eternal light, recovers there its ancient body, now invested with celestial radiance, meets with the forefathers who are dwelling in festivity with Yama, is recognised by that god as one of his own, obtains from him a delectable abode, and enters on a new existence and more perfect life, which is passed in the presence of the gods, and employed in the fulfilment of their pleasure. In a passage of the Atharvaveda, an expectation that the family relations will be maintained in the next world is expressed in these words: "Conduct us to heaven; let us be with our wives* and children." In the verses which follow those I have already quoted from a hymn addressed to Soma, the happiness of heaven is said to consist in the fulfilment of all desires, and the unrestrained enjoyment of a variety of gratifications, the nature of which is not explained, but which yield complete satisfaction. These pleasures are probably to be understood as being of a sensual character, as in a passage of the Atharvaveda (iv. 32; 2, 4) a promise is held out to those who offer up a particular oblation that their sexual appetites shall be abundantly gratified in paradise, and that they shall there be able to revel in milk, curds, butter, honey, and wine. Virtuous men of different classes,—the faithful worshippers of the gods, the performers of austerities, the brave who have fallen in battle,† the bestowers of liberal gifts,—are all said to be dwellers in that higher sphere. These glorified saints, who ride in the same chariots with the gods, are supposed to be capable of exercising an influence over the destinies of their descendants, to have the power of hearing their prayers, and of granting them protection and riches. They therefore share in the honours paid to the deities, are worshipped, like them, with oblations, are petitioned for temporal blessings, and are supplicated not to injure any one who has committed any offence against them.

The Rigveda contains no very explicit reference to future punishment; but the Atharvaveda speaks in one place of the nethermost darkness, and in another text of hell.

In regard to Yama, see Professor Roth's paper in the "Journal of the German Oriental Society," iv. 426 ff., which refers to all the principal texts on the subject; and the same author's article, in the third volume of the "Journal of the American Oriental Society," on the "Morality of the Veda," pp. 334 ff. See also Professor

* The later Indian writings hold out to the widow who burns herself on her husband's funeral pile, the hope of rejoining him in heaven. See Colebrooke's "Misc. Essays," i. 116, f.

† In the Mahâbhârata (xii. 3657) it is declared that "thousands of beautiful nymphs (*Apsarases*) hasten to meet the hero who has been slain in battle, exclaiming, 'Be my husband.'" Again, at v. 3667, it is said: "Behold, these shining worlds, filled with daughters of the Gandharvas, and yielding all manner of delights, belong to the brave."

MAX MÜLLER's article, on "The Funeral Ceremonies of the Brahmans," in the volume of the first-named journal for 1855, pp. xiv. ff., where many of the texts which I have referred to are translated into German. I may now add the same author's new volume of "Lectures on the Science of Language," 2d Series, pp. 513, ff., where he combats some of ROTH's conclusions.

Goddesses of the Rigveda.

Of the female divinities occurring in the Rigveda, some have been already noticed—viz., Prithivī, the Earth, the wife of Dyaus; Aditi, the mother of the Adityas, with Diti her counterpart; Nishtigrī, the mother of Indra; Pris'ni, the mother of the Maruts; and Saranyu, the mother of Yama and the Asvins.

Of the other goddesses, the most important is Ushas, the Ἠως, Ἀὼς, or Αὐώς of the Greeks, and the Aurora of the Latins,* to whom twenty separate hymns, and numerous detached verses, are dedicated. Of one of these hymns, some of which are very beautiful and imaginative, a specimen was given in the paper which I read before the Society last year.

Sarasvatī also is a goddess of some, though not of any very great, importance in the Rigveda. She is primarily, if not throughout, a river deity,† as her name, "the watery," clearly denotes; and in this capacity she is celebrated in a few separate hymns, as well as in a number of detached passages. In one of these texts, as well as in later works, allusion is made to sacrifices being performed on the banks of this river, and of the Drishadvatī; and the Sarasvatī, in particular, seems to have participated in the reputation of sanctity which, according to a passage in the Institutes of Manu (iii. 17 ff.), attached to the whole region lying between these two small streams, and situated to the westward of the Jumna. The Sarasvatī thus appears to have been (though in a less degree) to the early Indians what the Ganges (which is only twice named in the Rigveda, and was not then regarded with any special veneration) became at a later period to their descendants. When once the river had acquired a divine character, it was perhaps not unnatural that she should be regarded as the patroness of the ceremonies which were celebrated on the margin of her holy waters, and that her direction and blessing should be invoked as essential to their due performance and success. The connection into which she was thus brought with sacred rites may have led to the further step of supposing her to exercise an influence on the composition of the hymns which formed an important part of the proceedings, and of identifying her with Vāch (the Latin Vox), the goddess of speech.

Sarasvatī is frequently invited to the sacrifices, along with several other goddesses, Ilā, Bhāratī, Mahī, Hotrā, Varūtrī, and Dhishanā, who were not, like her,

* See BENFEY's "Griechisches Wurzellexicon, i. 27, and ii. 334.

† In the Brahmavaivartta Purana she is said to have been changed into a river by an imprecation of the Gangā. See Professor AUFRECHT's "Catalogue of the Bodleian Sanskrit MSS." p. 23.

river nymphs, but (most of them, at least) personifications of some department of religious worship, or sacred science. In many, or in most, of the passages where Sarasvatī is praised, her original character is distinctly preserved,—as, where she is mentioned along with other streams, or characterised as the divinest of rivers, or as one of the seven rivers, or as the mother of rivers, or as flowing pure from the mountains to the sea, or as wearing away the hills on her banks with her impetuous and resounding current. But she is also described as a purifier, as unctuous with butter, as stimulating, directing, and favouring the prayers of the worshippers, as riding on the same chariot with the oblations and with the sainted forefathers, as bringing prosperity and riches, as the source of life, as affording perfect protection, as sheltering her votaries like a tree, as conquering enemies, or delivering from them, as the wife of a heroic husband, as carrying a golden spear, as a slayer of foes or of Vrittra, and as filling the terrestrial and intermediate regions.

In the later mythology, Sarasvatī became the wife of the god Brahmā, and is regarded as the goddess of eloquence, in which capacity she is frequently invoked, much in the same fashion as the Muses were by the Greeks.

The other goddesses of the Veda are not of much consequence. We have, indeed, a Varunânî, an Indrānī, and an Agnāyī, who were regarded as the consorts of Varuna, Indra, and Agni, and a Rodasī, who is said by Yāska to be the wife of Rudra, all of whom might therefore have been expected to occupy positions corresponding to the rank of their respective husbands. Such, however, is not the case. They play no such important parts as Juno and Minerva perform in classical mythology. They are rarely mentioned; except Indrânî, they are never associated with their husbands,* and no distinct functions are assigned to them.

We meet also with a few personifications, such as Sraddhā, of religious faith; Nirriti, of evil; Aranyānī, of sylvan solitude, &c. &c.

Though it thus appears that in the Vedic age there was no female divinity of much importance, the case has been far otherwise in later times. Passing over the consorts of Vishnu, and of his incarnate representatives, Rāma and Krishna, I need only refer to the spouse of Mahādeva, who, under the names of Umā, Pārvati, Durgā, Kālī, &c., has held a prominent place in Indian mythology ever since the age of the great Epic poems, who still continues to be one of the principal objects of popular terror and adoration in all parts of India, and who is identified by some of the Brahmanical sectaries with the great divine Energy from which the creation of the universe is declared to have proceeded.

* In iii. 53, 4 ff. Indra is thus addressed:—"A wife, Indra, is one's home; she is a man's dwelling: therefore let thy horses be yoked, and carry thee thither. But whenever we pour forth a libation of soma, then may Agni hasten to call thee. Depart, Indra; come hither, brother Indra; in both quarters thou hast inducements. Whenever thy great chariot halts, thy steed is unharnessed. Depart, Indra, to thy home; thou hast drunk the soma; thou hast a handsome wife, and pleasure in thy house. Wherever thy great chariot halts, thy steed should be unharnessed."

From the above review, it is clear that there are but few of the Vedic gods who can be certainly identified with any deities of the Greek and Roman mythologies, by the double correspondence of names and functions. Of the numerous divinities who were originally common to these three branches of the Indo-Germanic family, the greater part became soon so extensively modified by one or other, or all, of these races, after their separation from each other, that at the dawn of history only two or three survived in such a form that we can without hesitation affirm them to have preserved, in some measure at least, their original character from the earliest times. These, as we have seen, are the Dyaus or Dyaushpitar, the Varuna, and the Ushas of the Veda, corresponding with the Zeus and Diespiter, the Ouranos, and the Eôs or Auôs, and Aurora of the Greeks and Latins. The Indian Agni, too, is evidently the Latin Ignis; but I am not aware that any trace exists in Latin literature of the element of fire having ever been worshipped under this name; and the adoration of Agni may, perhaps, have originated with the Indians and Persians after they parted from their kindred tribes. I need scarcely allude to Mitra or Mithra, who, though common to the Indian and Iranian mythologies, was unknown in the West till his worship was introduced from Persia.

Several of the remaining deities of the Veda, such as Indra, the thunderer, Sūrya or Savitri, the sun, Vāyu, the wind, Yama,* the god of the dead, correspond in their functions with the Jupiter, Apollo, Æolus, and Pluto of the classical writers; but as the names of these parallel divinities do not coincide in the different literatures, the resemblances in their offices scarcely suffice, perhaps, to establish any traditional connection between them, or to prove anything more than a similarity in the mental processes by which these gods were severally created. Between different systems of nature-worship, especially between the systems prevailing among cognate races (even though they may have been long separated), we may reasonably expect a general resemblance, as the great physical objects and phenomena which are common to all countries, are also those to which the process of personification is most naturally applied.

But it is not merely the primitive deities of the earliest Indo-European race which have undergone modification. The gods of the Veda themselves were soon subjected to a similar process, the most eminent among their number being, in the course of time, reduced to a subordinate rank, while others, originally less distinguished, were raised to the highest position. In the later mythology Varuna, the noblest of the Vedic divinities, was stripped of his attribute of supreme dominion, as well as of all his moral grandeur, and was only regarded as the god

* Yama's brother, Manu (who, as I have mentioned above, p. 573, note, is most commonly represented in the Rigveda as the first man, or progenitor of the Aryan race), resembles in name the Greek Minos, and the Mannus of the early Germans. See my paper on this subject in the "Journal of the Royal Asiatic Society," vol. xx. pp. 429 f.

of the ocean. Indra continued, indeed, to be looked upon as the chief of the Indian Olympus, but he became the monarch of the subaltern gods and goddesses alone. Vishnu, who in the Rigveda is far less prominent than Indra, soon began to eclipse his ancient comrade. In the systematic mythology of the Puranas, he is the preserver, the second of the three persons in whom the divine nature is represented, while his own special votaries identify him with the Eternal Spirit. In like manner Rudra, who plays a subordinate part in the ancient hymns, became, in later ages, the third person in the Hindu triad, and was exalted by his own sectaries to the same supreme dignity which they allege is erroneously claimed for his rival, Vishnu.

Note.—The passage regarding the original union, subsequent separation, and consequent fecundity of heaven and earth, which I have quoted from DIODORUS SICULUS, i. 7, in p. 552 of the preceding paper, finds a curious illustration in the following extract from the "Aitareya Brâhmana," which I cite according to Dr HAUG's translation, p. 308:—"These two worlds (heaven and earth) were (once) joined. (Subsequently) they separated. (After their separation) there fell neither rain, nor was there sunshine. The five classes of beings (gods, men, &c.) then did not keep peace with one another. (Thereupon) the gods brought about a reconciliation of both these worlds. Both contracted with one another a marriage according to the rites observed by the gods."

XLI.—*The Law of the Volumes of Aeriforms extended to Dense Bodies.* By Rev. J. G. Macvicar, M.A., D.D., Moffat.

(Read 18th April 1864.)

It is certain that the unities which constitute aeriform media, when they have been fully separated from each other by an adequate temperature, and relieved from excessive pressure, are all equal to each other in volume, whatever the aeriform may be, either singly or in couples, or in pairs of couples, double pairs of couples, &c., giving such ratios as $1 : \frac{1}{2} : 4 : 8$, &c.

More than this cannot be affirmed, except by hypothesis or convention. But this is a great deal. By this, many formulæ justly entitled to the name of rational (for they are representative of the molecules of bodies, both as to quality and quantity), become probable. But the construction of such formulæ is limited to such molecules as can be raised into the aeriform state. And the discovery of some such law in reference to such bodies as are permanently dense, is at this moment a great desideratum in science; for the doctrine of atomic volume, as insisted upon by some distinguished chemists, is beset with endless embarrassments, and is, besides, plainly wanting in that simplicity and breadth which belong to all laws which are really those of nature.

What the author here proposes is to show, that the same law which has been discovered in reference to aeriforms holds good also in reference to dense substances—viz., that the molecules of which they consist, whatever the dense body may be, are all equal to each other in volume, either singly or in couples, &c.

The method which he here adopts to prove this is, *first*, To construct out of the least chemical units, or the aeriform elements of bodies, such molecules as have the highest intrinsic verisimilitude in their favour; and, *secondly*, To show that these molecules, as measured by their atomic weights, give the specific gravities as found by the balance,—the argument being the same in form as that which has settled the question as to the volume of aeriforms.

By intrinsic verisimilitude, is meant the probable reality arising from that fact which forms the basis of all demonstrated science—viz., that nature is a dynamical system, or system of applied or concrete mathematics.

But to begin: In having to do with molecules, we have obviously to do with structures in which the constitutive forces are in a statical condition. This suggests at once, as the forms of molecules, the regular polyhedra of geometry, towards an adequate discussion of which all Euclid's labours and the ancient geometry aspired, and may I not say the modern geometry now again aspires. They are only five in number, which, beginning with the most perfect, stand thus:—

The icosahedron,	20 trigonal elements,	or facets.
The dodecahedron,	12 pentagonal elements,	
The octohedron,	8 trigonal elements,	
The hexahedron,	6 square elements,	
The tetrahedron,	4 trigonal elements,	

But it is with the more perfect, the two first named, almost exclusively, that we shall have to do when treating of the primary molecules of bodies. When, indeed, the elements forming into molecules are very volatile, then, instead of the icosahedron (or icosatom), which is the form of culmination, but which demands the simultaneous concurrence of 20 elements to construct one molecule, we have the octohedron and tetrahedron, whose elements are the same as those of the icosahedron, but which demand the concurrence of only 8 or 4. As to the hexahedron, its sphere of development is not among the primary molecules of bodies, but among those highly composite molecules, where crystallisation begins.

It must be added also, that the cosmical element of *aq* or HO, which is here (as is usual) taken as unity for specific gravities, is assumed to be (as indeed *aq* and HO indicate) dimorphous; as is also its molecule of culmination, or that which constitutes a particle (or unit-volume) of water; the architectural element of the latter, when it adopts the form of one of the regular polyhedra (the dodecahedron), being not one but three atoms of HO. In other words, our unit-volume of water consists of 36 atoms of steam or vapour, while most other substances consist of 12 only, or 20 at the most. Hence, our constant divisor for specific gravities is aq_{36}, which we may write AQ. and whose weight on the hydrogen scale is therefore $9 \times 36 = 324$.*

Adopting the letter X, then, to represent any chemical unit whatever, and Y and Z to represent dissimilar units, we immediately obtain as the molecules of dense bodies X_{20} and X_{12}, and occasionally, among volatile substances, X_8 and X_4.

But here the grand law of chemistry, the law of differentiation, presents itself to our regard. Dissimilars only unite chemically; and the condition of molecular stability or repose in the interior of a mass or medium is, that it be duly differentiated internally. Hence, given X as material for molecular constructions, we shall not only have X_{20} or X_{12}, but as often as possible, and that in the same

* That an unit-volume or particle of water consists of three times the number of chemical units or elements or minims, which other molecules in general consist of, is shown by the numbers of aeriform volumes which it and they respectively give. Thus, a volume of cold water gives from 1700 to 1728 volumes of steam, according to estimate, of which the third part is from 567 to 576. A volume of alcohol, when its vapour is heated up to the temperature of steam, gives 570 volumes. A volume of ether, supposing its aeriform units of volume to be dedoubled so as to assimilate it to water and alcohol, gives, at the same temperature, $2 \times 285{\cdot}9 = 572$ volumes. And so in other cases, where at first sight there seems no relation. Thus, oil of turpentine gives, of the same temperature, only 193 volumes. But the formula of the unit of that liquid is $C_{20}H_{16} = 4(C_5H_4)$. It requires, therefore, to be multiplied by 3 to bring it up to the dodecatom, and render its vapour-volume comparable with the others. Now, $3 \times 193 = 579$, near enough the others. Good experiments in this field would be very valuable.

mass or medium, both of them either adjusted to each other binarily and most stably as $X_{20}X_{12}$, or inscribing and circumscribing each other as X_{32}. For the same reason, X_{12} or X_{20}, when they are forbidden to differentiate each other, may both of them be differentiated by the addition of some dissimilar unit or units on two opposite points of their forms to develope poles, and thus to impart unity of direction in the functioning of the molecules; for both the dodecahedron and the icosahedron of geometry are in themselves isometrical. We thus obtain a series of molecules of which we may tabulate the leading members (for the sake of subsequent reference) as follows, adding the corresponding numbers in the authorised chemistry, which always draws, and often quarters molecules, as our forefathers did traitors, till one of the molecular constituents is reduced to a single element.

		Authorised Numbers.	Examples.
Type (α)	X_{12} . . . 12X	X	Cl
„ (β)	X_{20} . . . 20X	X	S
„ (γ)	$X\,X_{12}X$ . . 14X	X	K
„ (δ)	$X\,X_{20}X$ . . 22X	X	Hg
„ (ϵ)	$Y\,X_{10}Y$ . . 10X + 2Y	5X + Y	ClO_5
„ (ζ)	$XY\,X_{12}YX$ . . 14X + 2Y	7X + Y	ClO_7
„ (η)	$YXY\,X_{10}YXY$. 12X + 4Y	3X + Y	ClO_3
„ (θ)	$YXZ\,X_{10}ZXY$. 12X + 2Y + 2Z	3XY + 3XZ	$\dot{K}\dot{S}i^3 + \dddot{A}l\ddot{S}i^3$

These are the principal members of the first or lowest series of molecules. But of these, others are composed according to the same laws. Thus we have $(X_{12})_{12}$, &c.

Such views will not be thought altogether strange with regard to organic molecules, the established formulæ of many of which already give as many, nay very many, more atoms as their constituents. But they will be thought very strange in reference to merely chemical, metallic, and mineral bodies. I know of no reason, however, to call for a break in the economy of nature in this respect between one class of molecules and another, or to explain such a break, if it really existed. And as the view here advanced is in favour of the unity of nature and the simplicity of science, it can have nothing against it, except its novelty or its inadequacy to explain phenomena.

To see how it stands in this respect, therefore, let us give a specimen of specific gravities, deduced *a priori* by its aid, and compare them with the experimental results obtained by the balance. And to anticipate the charge of special selection of such as alone would serve my purpose, let us take the most familiar and notable substances, whether of the laboratory of nature or of the chemist, some undecomposable bodies non-metallic and metallic, some acids, some salts, and some stones. The reader is only requested to bestow his critical eye specially upon those that are abundant in nature in the concrete state, and respecting whose specific gravity there is no doubt.

Undecompo sable Bodies.

Oxygen, 16; *Sulphur*, 32; *Selen.* 80; *Tellur.* 128.*

Of these the three last, as is well known, are most closely allied, and to all appearance isomorphous. But they differ remarkably in weight. Hence, of the three, S, the lightest, alone can attain completely to the icosatom. The heaviest, Te, can attain only to the octatom. While that which is intermediate, Se, give both.

$$\text{Sulphur} \quad . \quad G=\frac{S_{20}}{AQ}=\frac{32\times 20}{324}=1{\cdot}975. \quad \text{Expt. } 1{\cdot}98.$$

$$\text{Tellurium} \quad . \quad G=\frac{Te_8}{\frac{1}{2}AQ}=\frac{128\times 8}{162}=6{\cdot}32. \quad \text{Expt. } 6{\cdot}3.$$

$$\text{Selenium} \quad . \quad G=\left\{\begin{array}{l}\frac{Se_{20}}{2AQ}=\frac{80\times 20}{648}=4{\cdot}94.\\ \frac{Se_8}{\frac{1}{2}AQ}=\frac{80\times 8}{162}=3{\cdot}94.\\ Se_8\ Se_{20} \quad =4{\cdot}44.\\ Se_8\ Se_{20}\ Se_8 \quad =4{\cdot}21.\end{array}\right. \quad \text{Expt. } 4{\cdot}3 \ldots 4{\cdot}8.$$

Oxygen does not belong to the same category as these three substances. Oxygen is, in reference to sulphur, what fluorine is in reference to selenium.

Ozone, which is a name for oxygen in every state except that in which it exists in the atmosphere (and probably for more), shows by the disappearance of volume when it is generated, that oxygen is capable of the molecular or dense state, though it cannot be thrown into this state by mechanical pressure. From carbonic acid we may infer what the molecular form and specific gravity of similarly condensed oxygen would be; for all the habits of fixed air, compared with those of oxygen, and all the relations between oxides and carbonates in nature, lead us to infer that carbonic acid is not only isovoluminous with atmospherical oxygen, and, like the latter, contains a couple of atoms of oxygen in one normal or atmospherical unit, but differs from it only by carrying an atom of carbon as a nucleus between the two atoms of oxygen (whence, of course, its physiological relations must be completely changed). Now, with regard to carbonic acid which has been condensed, the dodecatom gives—

$$\text{Carbonic Acid} \quad . \quad G=\frac{(CO_2)_{12}}{AQ}=\frac{22\times 12}{324}=\cdot 815. \quad \text{Expt. at zero, } \cdot 803.$$

* I have here taken the atomic weights of this group of bodies as it stands in the unitary notation. But I must here add, that our molecular theory gives these numbers, not as atomic, but as molecular weights, proper to the smallest regular polyhedron, viz., the tetrahedron, so that the S, Se and Te of the unitary system are in ours, S_4, Se_4 and Te_4, while the O of the unitary system, which is truly an unit of oxygen gas, is in ours a coupled molecule = ⓞ, or O_2. Sulphur as S_4 exists free, and unites with hydrogen and metals; and, in a word, functions as O. As S, on the other hand (being the alternate or reciprocal form of O), it unites with O, and when by itself forms an icosatom S_{20}, which is of course also the nucleus of $(S_4)_{20}$; but S_{20} differs from $(S_4)_{20}$, at least when secularly consolidated, in possessing metallic lustre, and in being very stable, if not undecomposable. Its existence as a separate substance has only been obscurely detected. But for the sake of reference, it may be called Sulphurium; its symbol $S_{20}=8\times 20=160$.

Similarly condensed oxygen would therefore be only about ·6. The molecule of ozone appears to be the dodecatom and icosatom one $=O_{32}=4$ vols.

Fluorine, 19; *Chlorine*, 35·5; *Bromine*, 80; *Iodine*, 127.

Our theory of molecular structures is here at war with current atomic weights. In our theory every atomic weight consists of five times as many units of weight as there are units of number in the hydrogen scale; and the elements of the same substance from different regions of nature, and probably after different manipulations in the arts, are apt to vary somewhat in the number of units of weight of which they consist, their full weights being generally somewhat heavier than those most recently adopted in chemistry. Thus the full weight of Cl is believed to be $=36$, and of $1, =128$. But that by the way. This difference tells but very slightly on specific gravities.

$$\text{Chlorine. } G=\frac{Cl_{12}}{AQ}=\frac{36\times 12}{324}=1\cdot 33. \quad \text{Expt. } 1\cdot 33.$$

$$\text{Bromine. } G=\frac{Br_{12}}{AQ}=\frac{80\times 12}{324}=2\cdot 99. \quad \text{Expt. } 2\cdot 99.$$

$$\text{Iodine . } G=\frac{I_{12}}{AQ}=\frac{128\times 12}{328}=4\cdot 74. \quad \text{Expt. } 4\cdot 94\ ?$$

If the experimental determination in the case of iodine be correct, it indicates that some of the molecules are differentiated. (See type (γ), p. 583.)

Phosphorus, 31; *Arsenic*, 75; *Antimony*, 120.

We may here take phosphorus last, in order to follow it by carbon, with which it has many both physical and physiological relations.

$$\text{Arsenic } \quad G=\frac{As_{12}}{\frac{1}{2}AQ}=\frac{75\times 12}{162}=5\cdot 6. \quad \text{Expt. } 5\cdot 6 \ldots 5\cdot 9.$$

$$\text{Antimony } G=\frac{Sb_{14}Sb_{22}}{2AQ}=\frac{120\times\overline{14+22}}{648}=6\cdot 6. \quad \text{Expt. } 6\cdot 6 \ldots 6\cdot 7.*$$

It may be shown that when both an icosatom, either single, such as X_{20}, or with poles, as X_{22}, forms from the same element, such as P or C, which element also forms dodecatoms, then the icosatom is much more open (as for instance to the attacks of oxygen, and therefore far more combustible) than the dodecatom. Hence, with regard to

Phosphorus

$$\text{Most combustible . } G=\frac{P_{20}}{AQ}=\frac{31\times 20}{324}=1\cdot 913. \quad \text{Expt. } 1\cdot 9 \ldots 2.$$

$$\text{Least combustible . } G=\frac{P_{12}}{\frac{1}{2}AQ}=\frac{31\times 12}{162}=2\cdot 29. \quad \text{Expt. } 2\cdot 14. \ldots 2\cdot 23.$$

$$\text{White (natural) . } G=\frac{P_{12}P_{20}}{2AQ}=\frac{31\times\overline{12+20}}{648}=1\cdot 53. \quad \text{Expt. } 1\cdot 52.$$

* The dodecatom and icosatom, when both are differentiated, that is $X\,X_{12}X$, $X\,X_{20}X=36X$ (the number of elements in AQ), gives, when divided by 2AQ, the same specific gravity as the compound dodecatom $(X_{12})_{12}=4\times 36X$ when divided by 8AQ.

Carbon.

Like phosphorus carbon is an eminently allotropic substance, giving specially these three forms (independently of those which are proper to organisation and charcoal, which I shall not touch upon); *first*, the diamond; *secondly*, an incombustible or less combustible residuum when diamond is burned; and, *thirdly*, coke and graphite. According to what has been stated, the diamond being a mature product of nature, eminently stable, ought to have for its molecule a structure differentiating itself, such as $C_{12}C_{20}$ or C_{32}. The more combustible part ought to be the open or circumscribing icosatom, C_{20}, leaving C_{12} as the residuum. The icosatom ought also to be the molecule of coke and perhaps graphite.

Diamond . $G=\frac{(C_{12}C_{20})_{12}}{2AQ}=\frac{12\times\overline{72+120}}{648}=3{\cdot}55$. Expt. 3·55.

Unburnt Residuum $G=\frac{(C_{12})_{12}}{AQ}=\frac{12\times72}{324}=2{\cdot}67$. Expt. 2·67 (Jacquelain.)

Coke . . $G=\frac{(C_{20})_{12}}{2AQ}=\frac{12\times120}{648}=2{\cdot}22$. Expt. 1·8 . . . 2·3.

Liebig has proposed as the formula of Newcastle coal $C_{24}H_{13}O$. This is obviously a dodecatom of $CHC=C_2H$, moistened all over by an atom of HO on each of the twelve points of the periphery. It is a good molecular formula; but coal is not an individualised substance. Detach the moisture, however, to obtain a pure hydrocarbon, and then impart to it stability in the current of chemical action by bestowing an atom of oxygen gas,—*i.e.*, 2O, on each pole, or O merely when O = 16, and we obtain two of the most eminent products of the distillation of coal, our formulæ being as usual the doubles of those usually given.

Benzene $\frac{C_{24}H_{12}}{\frac{1}{2}AQ}=\frac{144+12}{162}=\cdot963$. Expt. ·956 (solid.)

Carbolic acid $\frac{C_{24}H_{12}O_4}{\frac{1}{2}AQ}=\frac{144+12+32}{162}=1{\cdot}16$. Expt. 1·065.

But these calculations are on the supposition that the entire mass or liquid consists of homogeneously constructed molecules, of which there is but little chance.

By simple additions of its own elements, $CHC=C_2H$, instead of oxygens on the poles, Benzene gives Toluole, &c., . . . Cymole. And it dedoubles into naphtha $(CH)_{12}=C_{12}H_{12}$, or with H omitted on the poles, $C_{12}H_{10}$.

The element CH being the same in all, and the liquid differentiated to the full, there may be,—

Naphtha $G=\left\{\begin{array}{l}\frac{(CH)_{20}}{\frac{1}{2}AQ}=\frac{140}{162}=\cdot86\\ \frac{(CH)_{12}(CH)_{20}}{AQ}=\frac{84+140}{324}=\cdot69\\ \frac{(CH)_{12}}{\frac{1}{4}AQ}=\frac{84}{80}=1{\cdot}04\end{array}\right\}$ Mean ·86. Expt. ·86 . . . ·90.

Every sort will soon proceed to differentiate itself, in spite of every process which aims at obtaining it in a homogeneous state. The products of the most careful fractional distillation probably continue to be what they are found to be only for a short time. Some of these give exquisitely constructed molecules, as for instance the distillate lately obtained by MM. CAHOURS and PELOUSE from the rock-oil of America. Its formula is $C_{12}H_{14}$, which doubling, we obtain $C_{24}H_{28}$, plainly indicating, a dodecatom whose body is composed of naphtha with double walls, and whose poles are atoms of marsh gas. Whence we can understand the permanency of rock-oil in the presence of moisture and oxygen; for it is on the poles that chemical action mostly takes place; and marsh gas (for a reason which may be shown) is in all natural conditions secure from the attack of oxygen. Moreover, this beautiful hydrocarbon seems to have succeeded in the hands of its discoverers in gaining its way up to icosatomic molecules, each occupying no less than 16 normal volumes. Thus,—

$$\text{Distillate of Rock-oil . } G=\frac{(C_2H_4(C_2H_2)^{10}C_2H_4)_{20}}{16AQ}=\frac{2880+560}{16\times 324}=\cdot 664. \quad \text{Expt. } \cdot 669.$$

Many may be the molecular types developed by the reduction of the naphthas, especially by the departure of hydrogen in greater quantity than carbon, and the introduction of nitrogen and oxygen instead. The most highly partitioned, and the last which admits of being raised into the aeriform state, and thus separated from the carbonaceous residuum and brought before the chemist, is an element consisting of a single atom of H surrounded by five of C with an atom of H on one pole, to carry it up. Its elementary formula is consequently C_5H_2. But it is dissymmetrical. And in the aeriform state, with a normal or atmospherical volume it must be $C_{10}H_4$; and with a double volume, like most compounds, it must be $C_{20}H_8$. It is therefore the naphthalinic element. As the terminal of one pole is H and of the other C, we may well infer that it is capable both of the icosatom and the dodecatom. These give,—

$$\text{Naphthaline . } G=\begin{cases}\dfrac{(C_5H_2)_{12}}{AQ}=\dfrac{360+24}{324}=1\cdot 183. & \text{Expt. } 1\cdot 153 \text{ (Reichenbach.)}\\ \text{Mean } 1\cdot 035. & \text{Expt. } 1\cdot 048 \text{ (Ure.)}\\ \dfrac{(C_5H_2)_{20}}{2AQ}=\dfrac{600+40}{2\times 324}=\cdot 987.\end{cases}$$

If heat were gently applied and sufficient time given, and means of purification existed, it looks as if, between naphtha and naphthaline, there might be obtained the essential oils; for their element is a combination of the type of marsh gas with that of naphthaline with omission of the atom of H on the pole of the latter, which causes its dissymmetry. Or, the essential oil element is marsh gas $CHH_3C = C_2H_4$ with $(CH)_3$, substituting H_3. But 3CH may be variously applied so as

to give five elementary types, all different from each other, and the compounds may be numberless. The first molecule is the tetrahedral $(C_5H_4)_4 = C_{20}H_{16}$. This, with half a normal or aqueous volume, or the octohedral molecule with one such volume, or the icosatom with two, gives,—

$$\text{Essential Oils } G = \begin{cases} \dfrac{C_{20}H_{16}}{\frac{1}{2}AQ} = \dfrac{120+16}{162} = \cdot 84. & \text{Expt. } \cdot 84 \ldots 86. \\ \dfrac{(C_5H_4)_{20}}{2AQ} = \dfrac{600+80}{2\times 324} = 1\cdot 05. & \text{Mean } \cdot 945. \quad \text{Expt. } \cdot 94. \end{cases}$$

But when, in order to continued existence, greater stability is required (as in drenching with chlorhydric acid) the dodecatom in some cases is constructed. Thus, of cubeb camphor, the formula is $C_{15}H_{12}ClH = (C_5H_4)_3ClH$, a very usual formula. When expanded, it gives type η (p. 583). And calling $C_5H_4 = X$, and $ClH = Y$ we obtain,—

$$YXY\,(X)_{10}YXY. = X_{12}Y_4 \text{ or } X_6Y_2 \text{ or } X_3Y.$$

$$\text{Camphor } G = \left\{ \frac{C_{20}H_{16}O_2}{\frac{1}{2}AQ} = \frac{120+16+16}{162} = \cdot 94. \right. \quad \text{Expt. } \cdot 99.$$

$$\text{Sugar } G = \left\{ \frac{(H_{12}H_{11}O_{11})_{12}}{4AQ} = \frac{12\times(72+11+88)}{4\times 324} = 1\cdot 58. \right. \quad \text{Expt. } 1\cdot 56 \ldots 1\cdot 6.$$

$$\text{Acetic Acid } G = \begin{cases} \dfrac{(C_4H_4O_4)_{20}}{4AQ} = \dfrac{60\times 20}{4\times 324} \\ \dfrac{(C_4H_4O_4)_{12}}{2AQ} = \dfrac{60\times 12}{2\times 324} \end{cases} \begin{matrix} = \cdot 926. \\ = 1\cdot 111. \end{matrix} \quad \begin{matrix} \text{Mean } 1\cdot 068. \quad \text{Expt. } 1\cdot 063 \ldots 1\cdot 065. \quad \text{Liquid.} \\ \text{Expt. } 1\cdot 1. \quad \text{Soli}^{\text{d}}. \end{matrix}$$

This implies that the melted crystals of acetic acid consist of dodecatoms and icosatoms in equal numbers—a complete differentiation, which also appears in

$$\text{Alcohol } G = \begin{cases} \dfrac{(C_4H_6O_2)_{20}}{4AQ} = \dfrac{46\times 20}{4\times 324} \\ \dfrac{(C_4H_6O_2)_{12}}{2AQ} = \dfrac{46\times 12}{2\times 324} \end{cases} \begin{matrix} = \cdot 741. \\ = \cdot 882. \end{matrix} \quad \begin{matrix} \\ \text{Mean } \cdot 796. \quad \text{Expt. } \cdot 795. \end{matrix}$$

Ether is an eminently mobile liquid, and, when alone, seems to be constructed like the essential oils.

$$\text{Ether } G = \begin{cases} \dfrac{(C_4H_5O)_4}{\frac{1}{2}AQ} = \dfrac{37\times 4}{162} = .913. \\ \dfrac{(C_4H_5O)_{20}}{4AQ} = \dfrac{37\times 20}{4\times 324} = \cdot 571. \end{cases} \quad \begin{matrix} \\ \text{Mean } \cdot 742. \quad \text{Expt. } \cdot 736. \end{matrix}$$

Metals.

We may here take the leading metals in the order of the simplicity of their molecules.

I. *Metals.*—Molecule the dodecahedron, either without or with the poles marked

$$X_{12} \text{ or } X\,(X)_{12}X.$$

the latter giving the specific gravity uniformly $\frac{1}{6}$th heavier than the former.

Lithium $\dfrac{Li\,(Li)_{12}Li}{\frac{1}{2}AQ} = \dfrac{7+84+7}{162} = \cdot 60.$ Exp. ·59.

Sodium $\dfrac{Na\,(Na)_{12}Na}{AQ} = \dfrac{23+276+23}{324} = \cdot 99.$ Exp. ·97.

Potassium $\dfrac{K\,(K)_{12}K}{2AQ} = \dfrac{39+468+39}{2\times 324} = \cdot 84.$ Exp. 86.

Magnesium $\dfrac{(Mg)_{12}}{\frac{1}{4}AQ} = \dfrac{12\times 12}{81} = 1\cdot 76.$ Exp. 1·74.

* Calcium $\left\{\begin{array}{l} \dfrac{(Ca)_{12}}{\frac{1}{2}AQ} = \dfrac{20\times 12}{162} = 1\cdot 45 \\ \dfrac{Ca\,(Ca)_{12}Ca}{\frac{1}{2}AQ} = \dfrac{20\times 14}{162} = 1\cdot 69 \end{array}\right\}$ Mean 1·57. Exp. 1·57.

Mercury.

One of our two icosatoms, though marked as differentiated, and consisting of twenty-two elements, is completely isometrical, and in its atmosphere spherical. It is therefore suitable for the liquid state at any temperature, as might be shown, but for the necessity of avoiding here all morphological exposition.

$$\text{Mercury } G = \frac{Hg\,(Hg)_{20}Hg}{\frac{1}{2}AQ} = \frac{100\times 22}{162} = 13\cdot 58. \quad \text{Expt. } 13\cdot 59.$$

* The relation between the atomic weights of these eminently active or basilous metals to each other and those of the principal constituents of the crust of the earth is very interesting. Thus, estimating their functioning by their respective attractions to the earth (their weights), the first, Lithium, simply by doubling and doubling again, gives nitrogen, aluminium, and iron, disregarding fractions in atomic weights, and taking the integers immediately above them,—

$$2Li = 2\times 7 = 14 = N, \text{ Si and Al}$$
$$4Li = 4\times 7 = 28 = Fe, \text{ and its companions.}$$

Then the sulphide or deutoxide of each gives the metal in the series immediately above,—

$$Li \quad = Li = 7.$$
$$LiO_2 = Na = 7+16 = 23.$$
$$NaO_2 = K \ = 23+16 = 39.$$

Further, if we take Na at 24, and K at 40, which may be the primæval unreduced or full weights of these elemental bodies, then, by dedoubling, we obtain weights to complete the series,—

$$\frac{Na}{2} = \frac{24}{2} = 12 = Mg.$$

$$\frac{K}{2} = \frac{40}{2} = 20 = Ca.$$

Thallium, in our theory, comes out a composite metal, of the type $X\ X_{20}X$. The body being sulphurium, the poles sodium, all locked together.

$$Tl = Na\ S_{20}Na = 23+160+23 = 206, \text{ or } 208.$$

Let cold be urged so as to reduce the open icosatoms to the more compact dodecatoms. Then,—

$$\text{Solid Mercury. } G=\frac{Hg_{12}}{\frac{1}{4}AQ}=\frac{100\times 12}{81}=14{\cdot}8. \quad \text{Exp. } 14{\cdot}4.$$

II. *Metals.*—Molecule the icosahedron without or with the poles marked

$$X_{20} \text{ and } X\,(X_{20})X.$$

$$\text{Tin } \frac{(Sn)_{20}}{\frac{1}{2}AQ}=\frac{69\times 20}{162}=7{\cdot}28. \quad \text{Expt. } 7{\cdot}29.$$

$$\text{Copper } \frac{Cu\,(Cu)_{20}Cu}{\frac{1}{4}AQ}=\frac{32\times 22}{81}=8{\cdot}6. \quad \text{Expt. } 8{\cdot}9.$$

$$\text{Iron } \frac{Fe\,(Fe)_{20}Fe}{\frac{1}{4}AQ}=\frac{28\times 22}{81}=7{\cdot}6. \quad \text{Exp. } 7{\cdot}7.$$

The formulæ here, however, are equivocal, because the fine symmetrical combination $X_{12}X_{20}X_{12}=2\,X\,X_{20}X$.

$$\text{Lead } \frac{(Pb_{12})_{12}}{4AQ}=\frac{104\times 144}{4\times 324}=11{\cdot}5. \quad \text{Exp. } 11{\cdot}4.$$

$$\text{Zinc } \frac{(Zn_{12})_{12}}{2AQ}=\frac{32{\cdot}5\times 144}{2\times 324}=7{\cdot}22. \quad \text{Exp. } 6{\cdot}9. \ldots 7{\cdot}2.$$

Here, too, they are equivocal, thus $(X_{12})_{12}=4\,(X\,X_{12}X, X\,X_{22}X)$; that is, a combination of dodecatom and icosatom both differentiated.

$$\text{Bismuth } \frac{Bi_{12}Bi_{20}}{2AQ}=\frac{208\times\overline{12+20}}{2\times 324}=10{\cdot}3. \quad \text{Expt. } 9{\cdot}8.$$

But this metal, in our molecular synthesis, comes out as composite. Also some others; the metallic atoms in symmetrical relation, and locked into each other by that structure which renders so many metals in the mass tenacious and ductile. Our synthesis also presents some fine metals, which have not yet been found in the separate state,—as, for instance, one bearing to selenium the same relation that zinc does to sulphur. The metallic molecules are of the type $X\,X_{20}X$, or rather $Y\,X_{20}Y$.

Cadmium, ... Mg(Zn)Mg=12+32+12=Cd 56.
Tin, ... Al (Zn) Al=14+32+14=Sn 60.
Lead, ... Mg (Se)Mg=12+80+12=104 Pb.
Silver, ... Al (Se) Al=14+80+14=108 Ag.

avoiding fractions in atomic weights.

Noble Metals.

III. *Metals.*—Already differentiated by their molecular structure, and therefore not urgently in need of oxygen or other element:—

Type $X_{12}X_{20}$.

Silver $G=\frac{Ag_{12}Ag_{20}}{AQ}=\frac{108\times\overline{12+20}}{324}=10{\cdot}6$. Exp. 10·5.

Gold $G=\frac{Au_{12}Au_{20}}{\frac{1}{2}AQ}=\frac{98{\cdot}5\times\overline{12+20}}{162}=19{\cdot}1$. Exp. 19·2.

Platinum $G=\frac{Pt_{12}Pt_{22}}{\frac{1}{2}AQ}=\frac{98{\cdot}5\times\overline{12+22}}{162}=20{\cdot}66$. Expt. 20·8.

Aluminium $G=\frac{Al_{12}Al_{20}}{\frac{1}{2}AQ}=\frac{13{\cdot}75\times\overline{12+20}}{162}=2{\cdot}7$. Expt. 2·56 . . . 2·67.

The construction of molecules might often be simplified by introducing the doctrine of substitution supposed to take place in the act of reducing the metal from its ore. Thus the element of ore in reference to the last is Al_2O_3. Suppose that as fast as every atom of O (or Cl, or be what it may) is expelled, one of Al takes its place, and we have as the metallic unit Al_5.

$$G=\frac{(Al_5)_{12}}{AQ}=\frac{13{\cdot}75\times60}{324}=2{\cdot}54.$$

The Three Eminent Acids.

The glacial sulphuric acid shows its individualised character by its aptitude for crystallizing.

Glacial Sulphuric acid, $\frac{(SO_3 2HO)_{20}}{2AQ}=\frac{\overline{40+18}\times20}{23\times24}=1{\cdot}790$. Expt. 1·785 at 15°C. Playfair.

In the strongest oil of vitriol of commerce, MM. Gay Lussac, Marignac, and Bineau found always $\frac{1}{12}$th more than one atom of water. This indicates

Strongest Sulph. Acid, $G=\frac{(SO_3HO)_{12}+HO}{AQ}=\frac{\overline{12\times40+9}+9}{324}=1{\cdot}843$. Expt. 1·842.

The number of true hydrates, or symmetrically constructed molecules of oil of vitriol SO_3HO, or, as our notation gives it, $\dddot{Saq}S$ (the least element of sulphur being S=8, and the unitary equivalent a tetratom S_4) is very great, the most dilute being similar in structure to a particle of sea water or animal water, viz., one atom if symmetrical as $C_2H_4N_2O_2$, or $\dddot{Saq}S$, or a couple of atoms if dissym-

metrical as ClNa as a nucleus, in a molecule of water of the type $X\,X_{12}X$, viz., AQ $AQ_{12}AQ$ which gives about 1 per cent. of oil of vitriol. The next hydrate is when we have a particle of water on each pole only, AQS$\overset{\cdots}{\text{a}}$qSAQ, which gives about 7 per cent. of oil of vitriol. And this forms spontaneously in a damp atmosphere.

In hydrosulphuric acid we have S_4 the tetratom, with H on each pole; condensable under great pressure.

$$\text{Sulphydric acid, } \frac{(H_2S_4)_{20}}{2AQ} = \frac{34 \times 20}{2 \times 324} = 1{\cdot}09. \quad \text{Forced Expt. about } {\cdot}9.$$

In sulphurous acid we have S_4 partitioned as in sulphuric acid, giving as the product of combustion of sulphur the element SO, heteropolar, and capable both of the dodecatomic and icosatomic molecule; also condensable only under pressure.

$$\text{Sulphurous acid, } \frac{(SO)_{12}(SO)_{20}}{AQ} = \frac{16 \times 12 + 16 \times 20}{324} = 1{\cdot}58. \quad \text{Forced Exp. } 1{\cdot}45.$$

Resembling sulphurous acid in the dissymmetry of its element is chlorhydric acid, HCl. But its weight is greater and its volume larger, the icosatom as is most usual having double the volume of the dodecatom.

$$\text{Chlorhydric acid, } G = \left\{ \frac{(HCl)_{12}}{AQ} + \frac{(HCl)_{20}}{2AQ} = \frac{36{\cdot}5 \times 12}{324} + \frac{36{\cdot}5 \times 20}{2 \times 324} = 1{\cdot}369 + 1{\cdot}126. \right. \quad \text{Mean } 1{\cdot}247. \quad \text{Expt. } 1{\cdot}247.$$

The most stable hydrate, that into which both weaker and stronger acids resolve themselves by boiling, has for its authorised formula ClH + 16aq. Doubling as usual, we obtain 2ClH + 32aq, or 2ClHaq + 30aq, *i.e.*, 10(OHaqHO), leading to the construction of the fine dodecatom,

$$G = \frac{aqHCl\,(OHaqHO)_{10}ClHaq}{AQ} = \frac{9 + 37 + (27 \times 10) + 37 + 9}{324} = 1{\cdot}111. \quad \text{Expt. } 1{\cdot}111.$$

The formula of nitric acid NO_5HO when doubled, gives also a dodecatom. But all its constituents are so bent upon the aeriform state, that the icosatom cannot be constructed till the molecule is loaded with aq, as by continued boiling. And even then it is so tender, that the discordant results of chemists indicate a differentiated liquid, or a liquid struggling for stability by differentiation. But the tetrahedron or octohedron is decided. Thus, taking the usual formula,—

$$\text{Nitric Acid } \frac{(NO_5HO)_8}{AQ} = \frac{\overline{54 + 9} \times 8}{324} = 1{\cdot}55. \quad \text{Expt. } 1{\cdot}55.$$

A Few Common Salts.

Table Salt $\frac{(NaCl)_{12}}{AQ} = \frac{\overline{23+35\cdot5}\times 12}{324} = 2\cdot16$. Expt. 2·15. Kopp.

with full atomic weights, it comes out 2·22. Expt. 2·22.

Sulphate of Soda $\frac{(SO_3NaO)_{12}}{AQ} = \frac{\overline{40+31}\times 12}{324} = 2\cdot62$. Expt. 2·63.

Glauber's Salt is a fine molecule, the body composed of $(aqHO)_{10}$, the poles SO_3NaO; formed into a dodecatom occupying 8 volumes, G = 1·49. Expt. 1·47.

Sulphate of Potass $G = \left\{ \frac{(SO_3KO)_{20}}{2AQ} = \frac{\overline{40+47}\times 20}{2\times 324} = 2\cdot67. \right.$ Expt. 2·66.

Nitre . . . $G = \begin{cases} \frac{(NO_5KO)_{12}}{2AQ} = 1\cdot87. \\ \text{with poles marked} = 2\cdot18 \end{cases}$ Mean 2·08 Expt. 2·07.

Nitrum Flammans . $G = \begin{cases} \frac{(NO_5HO,NH_3)_{12}}{2AQ} = 1\cdot48 \\ \text{with poles marked } 1\cdot75. \end{cases}$ Mean 1·62. Expt. 1·7.

Sal ammoniac . $G = \left\{ \frac{(ClH_4N)_{36}}{4AQ} = 1\cdot5, \right.$ Expt. 1·5.

Ammonia, in our theory, in one of its forms is isomorphous with aqaq, or double vapour, and affects the molecule am_{36}, like aq_{36}.

A Few Common Minerals.

As the attempt to show the structure of highly compounded molecules in common writing is necessarily difficult, let us here take the shortest symbols possible for the common mineral constituents, as for instance, $\ddot{S}$ for SiO_2, $\ddot{\ddot{A}}$ for Al_2O_3, $\dot{F}$ and $\ddot{F}$ for FeO and Fe_2O_3, and $\dot{H}$ for HO.

Quartz $G = \frac{\ddot{S}(\ddot{S})_{12}\ddot{S}}{\frac{1}{2}AQ} = \frac{14\times 30}{162} = 2\cdot59$. Expt. 2·5 . . . 2·8.

The dodecatom with $\ddot{\ddot{A}}$ instead of $\ddot{S}$, gives G = 3·82. Expt. 3·87.

But as developed by nature, it may be,—

Sapphire $G = \frac{(\ddot{\ddot{A}}_{12}) + \dddot{A}(\ddot{\ddot{A}})_{12}\dddot{A}}{AQ} = 3\cdot82 + 4\cdot44$ Mean = 4·12. Expt. 4·08.

But the same specific gravities are obtained by the finer molecule of the form $\frac{X_{20}X_{12}X_{20}}{2AQ}$. And such molecules we are not to regard as too large, as is clearly

shown by the very small percentages sometimes occurring in the analysis of minerals. It is indeed usual to regard these small percentages as accidental ingredients. But that will not do. Thus, prase is represented as quartz, with about 2·9 per cent. of protoxide of iron as an accidental ingredient. But on constructing that molecule, which seems to be the most prevalent in the crystalline world, viz., the compound dodecatom with marked poles, which, to show all its constituents, may be expanded on the paper thus:—

$$\mathrm{X\,(X)_{12}X\big(X\,(X)_{12}X\big)_{10}X\,(X)_{12}X},$$

2·9 per cent. of protoxide of iron just gives four atoms, one to differentiate each of the two poles of the two polar dodecatoms, that is, to take the place of the separate atoms of X in the above formula,

$$\dot{\mathrm{F}}\,(\ddot{\mathrm{S}})_{12}\dot{\mathrm{F}}\ \big(\ddot{\mathrm{S}}\,(\ddot{\mathrm{S}})_{12}\,\ddot{\mathrm{S}}\big)_{10}\dot{\mathrm{F}}\,(\ddot{\mathrm{S}})_{12}\mathrm{F}.$$

Nay, half this quantity, or 1·5 per cent. might enter into the symmetry of the molecule. And so it may possibly be with $\dot{\mathrm{T}}\mathrm{i}$ in rose quartz, as found by FUCHS. But let us look to the grand products of nature.

The quartz molecule will tend to be differentiated more effectually than by an element of its own kind on each pole of the dodecatom. And more especially, we may expect $\dddot{\mathbb{A}}$ and $\dot{\mathrm{K}}$, or $\dot{\mathrm{N}}\mathrm{a}$, or both performing this function and giving,

$$\text{Felspar}\left\{\begin{array}{l}\dot{\mathrm{K}}\,\dddot{\mathbb{A}}(\ddot{\mathrm{S}})_{12}\dddot{\mathbb{A}}\dot{\mathrm{K}},\ \text{or (See Type } \zeta)\\ \dot{\mathrm{K}}\,\ddot{\mathrm{S}}\,\dddot{\mathbb{A}}(\ddot{\mathrm{S}})_{10}\dddot{\mathbb{A}}\,\ddot{\mathrm{S}}\,\dot{\mathrm{K}}.\quad \text{(See Type } \theta)\end{array}\right\} = \mathrm{K_2O_2.Al_4O_6.12SiO_2}.$$

which last is the formula for orthoclase, in WATT'S Chemical Dictionary (the atomic weights of O and of Si being here halved.) (See Art. *Felspar*). But both the ratios of ingredients and the specific gravities show that, in order to explain nature, we must rise from the simple dodecatom to the compound dodecatom. Thus, the purest felspar or adularia gives,

$$\mathrm{G} = \frac{(\dot{\mathrm{K}}\,\dddot{\mathbb{A}}(\ddot{\mathrm{S}})_{12}\dddot{\mathbb{A}}\,\dot{\mathrm{K}})_{12}}{8\mathrm{AQ}} = 2{\cdot}59.\quad \text{Expt. } 2{\cdot}53\ \ldots\ 2{\cdot}58.$$

			Theory.	Berthier.	
Adularia	$144\mathrm{SiO_2}$	4320	64·5	64·20	from St Gothard.
	$24\mathrm{Al_2O_3}$	1248	18·6	18·40	
	$24\mathrm{KO}$	1148	16·8	16·95	

$\mathrm{G} = 8 \times 324\ \big)6716\big(\ 2{\cdot}59.$ Expt. 2·53 . . . 2·58.

But in order to show the substitutions which take place during further differentiation and development, the above formula requires to be written out,

$$\dot{\mathrm{K}}\,\dddot{\mathbb{A}}(\ddot{\mathrm{S}})_{12}\dddot{\mathbb{A}}\mathrm{K}\big(\dot{\mathrm{K}}\,\dddot{\mathbb{A}}(\ddot{\mathrm{S}})_{12}\dddot{\mathbb{A}}\,\mathrm{K}\big)_{10}\dot{\mathrm{K}}\,\ddot{\mathbb{A}}(\mathrm{S})_{12}\dddot{\mathbb{A}}\mathrm{K}.$$

Thus, let us substitute $\dot{\mathrm{N}}\mathrm{a}$ for $\dot{\mathrm{K}}$, in the polar dodecatoms, and we obtain,—

$$\dot{\mathrm{N}}\mathrm{a}\,\dddot{\mathbb{A}}(\ddot{\mathrm{S}})_{12}\dddot{\mathbb{A}}\dot{\mathrm{N}}\mathrm{a}\big(\dot{\mathrm{K}}\,\dddot{\mathbb{A}}(\ddot{\mathrm{S}})_{12}\dddot{\mathbb{A}}\,\dot{\mathrm{K}}\big)_{10}\dot{\mathrm{N}}\mathrm{a}\,\dddot{\mathbb{A}}(\ddot{\mathrm{S}})_{12}\,\dddot{\mathbb{A}}\,\dot{\mathrm{N}}\mathrm{a}.$$

			Theory.	Abich.	
Felspar	$144SiO_2$	4320	65·14	65·72	from Baveno.
	$24Al_2O_3$	1248	18·82	18·57	
	$20K\dot{O}$	940	14·17	14·02	
	$4NaO$	124	1·85	1·25	
	$G=8\times324$	)6632(	2·55.	Expt. 2·5552.	

(See *Millar's Mineralogy*, p. 367.)

On the other hand, the smallest amount of differentiation that is possible on the felspathic, is when we have $\ddot{S}$ instead of $\dot{K}$ or $\dot{N}a$, with only one atom of $\dot{L}i$ on each of the twelve parts which constitute the compound dodecatom. This gives,—

			Theory.	Plattner.	
Petalite	$156SiO_2$	4680	76·6	78·01	from Elba. (Castor.)
	$24Al_2O_3$	1248	20·4	18·86	
	$12LiO$	180	2·9	2·76	
	$G=8\times324$	)6108(	2·39	Expt. 23·8 . . . 2·4.	

But in all the felspars, the silica greatly overmatches the alumina. Instead of a molecule of silica with an atom of alumina on each pole, the balance of nature is rather the reverse of this, or at least, instead of $\ddot{\mathbb{A}}(\ddot{S})_{12}\ddot{\mathbb{A}}$, it is $\ddot{S}\,\ddot{\mathbb{A}}\,\ddot{S}$. Thus, the clay which results from the unremitting application of the atmosphere to the hard compounded rocks, and advances nature towards a vegetable kingdom, has, for its element $\dot{H}\ddot{S}\ddot{\mathbb{A}}\ddot{S}\dot{H}=Al_2O_3 2SiO_2+2HO$. We may, therefore, look in nature for some abundant mineral in which the element of the body of the molecule, instead of being $\ddot{S}$ merely, shall be $\ddot{S}\ddot{\mathbb{A}}\ddot{S}$. But such a molecule over all its periphery must be wholly non-metallic, like quartz itself. Let us, therefore, differentiate it by introducing more fully metallic matter of the same form. We thus obtain the molecule,—

$$\dot{K}\ddot{S}\dot{K}\,(\ddot{S}\,\ddot{\mathbb{A}}\dot{S})_{12}\,\dot{K}\,\ddot{S}\,\dot{K}$$

			Theory.	H. Rose.	
Common Mica	$26SiO_2$		49·0	49·2	from Kimito.
	$12Al_2O_3$		39·2	39·0	
	$4K\dot{O}$		11·8	9·8	

But here small quantities of Fe_2O_3, HO, and F, are neglected, and in this region, differentiation is so active, that molecular construction becomes very difficult, and indeed as variable as analyses.

Stability, simplicity, and facility of construction return, when magnesia and lime are introduced.

$$\text{Arragonite } G=\frac{(CaOCO_2)_{20}}{AQ}=\frac{\overline{28+22}\times20}{324}=3\cdot09.\quad \text{Expt. } 2\cdot9\ .\ .\ .\ \cdot0\ .$$

But as greater stability is urged, or spontaneously acquired, the icosatom must tend to the dodecatom. Hence, a series of structures, giving $G=2\cdot47$ to $2\cdot77$, of

which the most complete is, as usual, the compound dodecatom, which closely written is,—

Calcite or Marble . $G=\dfrac{(CaOCO_2)_{12})_{12}}{8AQ}=\dfrac{\overline{28+22}\times 144}{8\times 324}$ $\Big\}=2{\cdot}77$. Expt. 2·6 . . . 2·8.

Limestone, as chemically conceived, seems a singular combination. But if we could dedouble alumina and peroxide of iron, as we can dedouble limestone into a protoxide, and a deutoxide, that singularity would disappear. Thus, written according to the law of symmetry or union by difference, carbonate of lime is OCaOCO similar in structure to OAlOAlO, and OFeOFeO, &c.; and for mineralogical notation it might be written $\ddot{C}\dot{a}\ddot{C}$ like $\ddot{\mathbb{A}}$ and $\ddot{\mathbb{F}}$.

The fully differentiated phosphate (see type θ) gives,

$$\text{Bone Phosphate } \ddot{\dot{P}}\,\dot{C}a\,\ddot{\dot{P}}\,(\dot{C}a)_{10}\,\ddot{\dot{P}}\,\dot{C}a\,\ddot{\dot{P}}=4\,(PO_5 3CaO).$$

the usual compound dodecatom occupying 8 volumes. Written with the usual formula, we obtain for this substance,

$G=\dfrac{(PO_5 3CaO)_{12}}{2AQ}=\dfrac{\overline{71+84}\times 12}{2\times 324}=2{\cdot}87$. And with poles marked 3·35. Mean 3·11. Expt. 3·1 . 3·2.

Purely magnesia minerals are rare, but as a mineral constituent, none is more interesting or important than magnesia.

			Theory.	Walmsted.	
Olivine $\dot{F}\,\ddot{S}\,\dot{F}\,(\dot{M}g\,\ddot{S}\,\dot{M}g)_{12}\,\dot{F}\,\ddot{S}\,\dot{F}=$	$14SiO_2$	420	40·2	40·1	from Somma.
	$24MgO$	480	45·9	44·2	
	$4FeO$	144	13·8	15·3	
	324	)1044(	3·22.	Expt. 3·3 . . . 3·4.	

It is not the symmetrical or septary element $\dot{M}g\ddot{S}i\dot{M}g$, however, but the quinary combinations $\dot{M}g\ddot{S}i$ and $\dot{C}a\ddot{S}i$, *i.e.* the sesquiform, so great a favourite with nature, that are the great mineral constituents. They seem generally to form into dodecatoms, the poles marked by the application of molecule to molecule, or by the omission of an atom of the basic element, or by substitution of $\dot{F}e$ for $\dot{M}g$ or $\dot{C}a$. Of these, the simplest symmetrical structure is that in which $2\dot{F}e$ serve to unite 3 dodecatoms, the central consisting of $\dot{C}a\ddot{S}$, and the lateral of $\dot{M}g\ddot{S}$.

$(\dot{M}g\ddot{S})_{12}\dot{F}\,(Ca\,\ddot{S})_{12}\dot{F}\,(\dot{M}g\,\ddot{S})_{12}$ omitting from the six poles $(\dot{M}g+\dot{M}g)\ (\dot{C}a+\dot{C}a)\ (\dot{M}g+\dot{M}g)$;

or written out,—

$$\ddot{S}\,(\dot{M}g\,S)_{10}\,\ddot{S},F,\ddot{S}\,(\dot{C}a\,\ddot{S})_{10}\,\ddot{S},F,\ddot{S}\,(\dot{M}g\,\ddot{S})_{10}\ddot{S}.$$

			Theory.	Bonsdorf.	
Hornblende	$36SiO_2$	1080	59·0	59·7	from Taberg.
	$20MgO$	400	21·8	21·1	
	$10CaO$	280	15·3	14·3	
	$2FeO$	72	3·9	3·9	
	$G=2\times 324$	)1832(	2·82	Expt. 2·9 . . . 3·4.	

In talc, when most simple, the sole genetic element is $\dot{M}g\ddot{S}i$: molecules differentiated by $\dot{F}e$. But here differentiation is too active for easy study, until the elements settle into serpentine, to represent which a very interesting molecule presents itself; the body, a sesquicombination of magnesia and silica, and the whole surface hydrated magnesia, except the two poles, which are iron oxide.

$$\dot{F}\ (aq\dot{H}\ \dot{M}g\ddot{S}\ \dot{M}g\ \ddot{S}\ \dot{M}g)_{12}\ \dot{F}.$$

			Theory.	Kerstin.	
Serpentine	36MgO	720	41·7	41·5	from Swartzenberg.
	$24SiO_2$	720	41·7	40·3	
	24HO	216	12·6	12·9	
	2FeO	72	4·2	4·1	
	G = 2 × 324	)1728(	2·66.	Expt. 2·47 . . .	2·6.

And here I may close, conscious that these constructions are but the merest approximations, and the rudest representations of the exquisite structures of nature.

Still, it will not be denied, that to any one who has given his mind to the preceding pages, they supply much for men of science to think about; for in this paper the specific gravities of between sixty and seventy of the most eminent substances of nature and the laboratory have been deduced *a priori*, have been shown to be proportional to certain molecular numbers which are suggested by pure geometry (and the law of differentiation), the same being supposed to prevail all through nature. It is true that our molecular numbers imply that the formulæ of mineral chemistry, and indeed of chemistry generally, are submultiples (halves, quarters, or lesser elements) of the molecules of nature, as nature is here conceived And this will necessarily be against the study of them for a time; as also the too short statement of them in this paper. But let the thing be seriously looked into, and there can be no doubt as to the ultimate issue.

XLII.—*Biographical Sketch of Adam Ferguson, LL.D., F.R.S.E., Professor of Moral Philosophy in the University of Edinburgh.* By JOHN SMALL, M.A., Librarian to the University.

(Read 18th April 1864.)

The Memoir now submitted to the Society, while it details the chief events in the life of a man who occupied a distinguished place in the literature of Scotland, at a period when it had attained a high reputation, cannot claim to be so complete as might be desired. His life was prolonged for several years after nearly all of his early friends had passed away; and since his death many papers have been destroyed or have fallen aside, which would now be of the greatest interest.

Whilst in this way much has been lost that might have given greater completeness to these pages, still, the recent publication of the Diary of his friend Dr CARLYLE of Inveresk, has furnished many additional details, and afforded further evidence of the estimation in which he was held by his literary associates.

Several letters selected from the lives of his distinguished friends, and from the Manuscript Collection of the University, in addition to information derived from the short notices of his life already printed, have afforded the materials for preparing this sketch of one, whose career was more varied, while his public labours and literary connections were not less important and extensive, than those of any of his contemporaries.

Dr ADAM FERGUSON, son of the Rev. ADAM FERGUSON, minister of the parish of Logierait, Perthshire, was born in the manse of that parish on the 20th of June 1723. His father was descended from an old and respectable family in Athole, to whom the estate of Dunfallandy yet pertains; and his mother was the daughter of Mr GORDON of Hallhead, in the county of Aberdeen. In the female line FERGUSON traced a connection with the noble family of ARGYLL, thus referred to in a letter addressed to him by Dr CARLYLE of Inveresk: "I am descended from the Queensberry family by two great-grandmothers—much at the same distance as you are from that of ARGYLL."*

ADAM was the youngest son of a numerous family. His father had been minister of Crathie and Braemar from 1700 to 1714, and was long remembered with gratitude for having sheltered in his manse of Crathie some of the unfortunate Macdonalds on their flight from the treacherous massacre of Glencoe. Just before the Rebellion of 1715 he was translated to Logierait, where he passed the

* MSS., University of Edinburgh.

remainder of a long life, discharging his ministerial duties with exemplary piety and firmness. Although the parishioners were at the period of his induction almost universally hostile to Presbyterian principles, he speedily secured general respect, which he retained till his death in 1754.

FERGUSON received his earlier education partly at home under the tuition of his father, who had soon discovered his son's superior abilities, partly at the parish school of Logierait. He was afterwards sent to Perth, where he attended the classes of Mr JAMES MARTIN, rector of the grammar school, a distinguished teacher, who had numbered amongst his pupils the great Lord MANSFIELD. There he was committed to the charge of his relation, WILLIAM FERGUSON, a merchant, and at one time chief magistrate of that city. At the Grammar School of Perth FERGUSON excelled in classical literature, and especially in the composition of essays; and we learn that his themes were not only praised at the time, but were long preserved, and shown with pride by Mr MARTIN, who declared that none of his pupils had ever surpassed the writer.

In 1738, when he had just entered on his sixteenth year, FERGUSON was enrolled at the University of St Andrews, where he studied Latin under Professor YOUNG, and Greek under Professor PRINGLE. The classes were then ably superintended by Principal TULLIDELPH, to whom FERGUSON had the advantage of being recommended by his father's friend and namesake, the minister of Moulin.

At the commencement of the session, FERGUSON gained by competition one of the foundation bursaries, which are tenable during the curriculum in the Faculty of Arts, and which entitled him to maintenance at the College table. This he owed to his previous excellent training in Latin. His attention was now given to the study of Greek, of which, hitherto, he seems to have had little knowledge; and that so successfully, that at the end of his first session he read Homer with considerable ease. During the summer recess he resolved to read one hundred lines of the Iliad daily, and in this way perused the whole poem. He obtained his degree of M.A. on the 4th May 1742, when he had nearly completed his nineteenth year; and thus finished his curriculum in arts with the reputation of being one of the best classical scholars, and perhaps the ablest mathematician and metaphysician of his time at the University.

Having been intended by his father for the church, FERGUSON entered the Divinity Hall at St Andrews in 1742, under Principal MURISON and Professors SHAW and CAMPBELL; but shortly afterwards he removed to Edinburgh, and continued his course under Professors GOWDIE and CUMMING. There he joined a number of young men who afterwards attained to eminence—amongst whom were JOHN HOME, author of 'Douglas;' WILLIAM ROBERTSON, afterwards Principal of the University; HUGH BLAIR; Mr WEDDERBURN, afterwards Lord LOUGHBOROUGH; and Dr CARLYLE—in forming a debating society. This club after-

wards became merged in the Speculative Society, which still exists in unimpaired efficiency.

FERGUSON, while not neglecting the study of divinity, applied himself less to it than to those subjects of philosophy for which he showed special aptitude, and in which he was afterwards to become so celebrated. In 1745, when he had attended divinity classes for two only, out of the full period of six, years—the time then required before obtaining license to preach—he was offered the appointment of deputy chaplain to the 42d regiment, or "Black Watch," by Mr MURRAY (brother of Lord ELIBANK), who was principal chaplain. For this appointment his knowledge of the Gaelic language was an important qualification. The rules of the Church allowed Gaelic students to be taken on trials after four years' attendance at the Divinity Hall; but it was necessary, in FERGUSON'S case, to obtain from the General Assembly a still farther dispensation. The Assembly, in consideration of his good character and high testimonials, granted special authority for his ordination, and he was ordained by the Presbytery of Dunkeld on the 2d of July 1745. A few days after this he joined his regiment, then serving in Flanders; and in a short time he obtained, on the retirement of Mr MURRAY, the rank of principal chaplain.

We are informed by Dr CARLYLE, that it was through the influence of the Duchess Dowager of ATHOLE that FERGUSON obtained his appointment as chaplain to the 42d Regiment. "Her son, Lord JOHN MURRAY,* had obtained the colonelcy of that regiment when he was not more than twenty-two years of age; and the Duchess had imposed the very difficult task upon FERGUSON, to be a kind of tutor or guardian to Lord JOHN,—that is to say, to gain his confidence, and keep him in peace with his officers, which it was difficult to do. This, however, he actually accomplished, by adding all the decorum belonging to the clerical character to the manners of a gentleman; the effect of which was, that he was highly respected by all the officers, and adored by his countrymen, the common soldiers."

Shortly after FERGUSON joined his regiment, the battle of Fontenoy took place, in which he behaved with the greatest bravery. In that battle, according to the account of the French themselves, "the Highland furies rushed in upon them with more violence than ever did a sea driven by a tempest." FERGUSON went into action at the head of the attacking column, with a drawn broad-sword in his hand, and could with difficulty be persuaded to retire to the rear.† Colonel DAVID STEWART, author of the "History of the Highlanders," remarks, that he continued with his regiment during the whole of the action, in the hottest of the

* Lord JOHN MURRAY—son of JOHN Duke of ATHOLE by his second marriage—was appointed colonel of the Royal Highlanders on April 25, 1745; major-general in 1753; lieut.-general in 1754; and general in 1770.

† Sir W. SCOTT'S Miscellaneous Prose Works, vol. xix. p. 331.

fire, praying with the dying, attending to the wounded, and directing them to be carried to a place of safety. The Colonel further remarks, that FERGUSON, "by his fearless zeal, his intrepidity, and his friendship towards the soldiers (several of whom had been his schoolfellows at Dunkeld); his amiable and cheerful manners, checking with severity when necessary, mixing among them with ease and familiarity, and being as ready as any of them with a poem or a heroic tale, acquired an unbounded ascendancy over them; and while he was chaplain of the corps he held an equal, if not in some respects a greater, influence over the minds of the men than the commanding officer."*

While he was connected with this regiment, he published a sermon, which was his first contribution to literature. It is entitled— *A Sermon preached, in the Ersh Language, to His Majesty's First Highland Regiment of Foot, commanded by* Lord JOHN MURRAY, *at their Cantonment at Camberwell, on the 18th day of December* 1745, *being appointed as a solemn Fast. Translated into English for the Use of a Lady of Quality in Scotland, at whose desire it is now published.*†

This sermon, printed at the request of the Duchess Dowager of ATHOLE, with whom FERGUSON was a particular favourite, is more remarkable for the vigour of its patriotic exhortations than for the elegance of its language, and contains strong denunciations of the attempt made in the year 1745 to seat Prince CHARLES on the throne of Britain.

With this gallant regiment FERGUSON served during the whole of the campaign in Flanders; and on the peace of Aix-la-Chapelle, he obtained leave of absence, and visited the scenes of his youth, where he spent much of his time in wandering amongst the Perthshire mountains. Writing to an intimate friend at a subsequent period, he says, "If I had not been in the Highlands of Scotland, I might be of their mind who think the inhabitants of Paris and Versailles the only polite people in the world It is truly wonderful to see persons of every sex and age, who never travelled beyond the nearest mountain, possess themselves perfectly, perform acts of kindness with an aspect of dignity, and a perfect discernment of what is proper to oblige. This is seldom to be seen in our cities, or in our capital; but a person among the mountains, who thinks himself nobly born, considers courtesy as the test of his rank. He never saw a superior, and does not know what it is to be embarrassed. He has an ingenuous deference for those who have seen more of the world than himself; but never saw the neglect of others assumed as a mark of superiority." ‡

With a desire to obtain a more permanent and more congenial sphere of usefulness, FERGUSON applied for the living of Caputh, a beautiful parish near Dunkeld, in the patronage of the Duke of ATHOLE. He was not, however, in all

* Hist. of the Highlanders, vol. i. p. 292. † Lond., 1746. 8vo. ‡ MSS. Univ. of Edin.

respects qualified for discharging the duties of a Scottish clergyman. Although, by his polished manners and his great abilities, he took a prominent part in private society, he was deficient in the gifts necessary for the popular preacher. His sermons were elaborate disquisitions, showing more acquaintance with systems of philosophy than with the wants of common hearers.* He was unsuccessful in his application for this living; and when the death of his father (whom he had hoped to succeed) took place shortly after this disappointment, he abandoned all intention of undertaking the duties of a parochial charge. He continued to remain attached to his regiment, during its service in Ireland, till about the year 1754, when he resigned his commission.

The knowledge of military affairs thus acquired by his service in the army enabled him to give so much distinctness and liveliness to his descriptions of war in his 'History of the Roman Republic,' that it is remarked by CARLYLE, that he was excelled, in this respect, by no historian but POLYBIUS, who was an eye-witness of so many battles. His military service also proved beneficial to him by opening up a wide field for the observation of human character, and gave him enlarged opportunities of studying the political phenomena of the period.

After resigning the chaplaincy of the 42d Regiment, FERGUSON spent some time in Holland with his friend Mr GORDON, and resolved to give up all thoughts of further exercising the clerical profession. Writing to ADAM SMITH from Groningen, in October 1754, he concludes by requesting a reply to be addressed to him at Rotterdam, "without any clerical titles, for I am a downright layman."†

Shortly after this, FERGUSON returned to Edinburgh, where he renewed his acquaintance with the friends of his youth. As DAVID HUME had at this time given up his appointment of Keeper of the Advocates' Library, he became a candidate for the office, and was appointed HUME'S successor as Librarian and Clerk to the Faculty on the 8th of January 1757.

While he was connected with that Library, FERGUSON became a member of the Select Society, which had been instituted in 1754 by Mr ALLAN RAMSAY, the eminent artist. The meetings of the Society were held weekly in one of the inner apartments of the Library, and were for the purpose of literary discussion,

* The following anecdote illustrates their character:—"Sometimes he lent or presented a sermon to his friends. One of them one day preached a very profound discourse on the superiority of personal qualities to external circumstances, that showed a very thorough acquaintance with the doctrines of Plato and Aristotle. Mr BISSET (his father's successor), in whose church the gentleman delivered this sermon, was at first greatly surprised at hearing such observations and arguments from a worthy neighbour, whom he well knew to be totally unacquainted with the philosophy of Plato, or any other, ancient or modern. When service was over, he paid the young man very high encomiums on his discourse—that it very much exceeded the highest expectations he had ever entertained of the talents of the preacher; who told him very honestly that he knew very little about these things himself, but that *he had borrowed the discourse from his friend* ADAM FERGUSON."—*Histor. Mag.* (1799) vol. i. p. 44.

† This interesting letter is in the possession of the Rev. Mr CUNNINGHAM, Prestonpans.

philosophical inquiry, and improvement in public speaking. This Society exercised an important influence in diffusing a taste for letters in Scotland, and it has been remarked, that "the classical compositions of HUME, ROBERTSON, SMITH, and FERGUSON, the writings of JOHN HOME, of Professor WILKIE, of Lord HAILES, Lord MONBODDO, Sir JOHN DALRYMPLE, the elder Mr TYTLER, all members of the Select Society of Edinburgh, have thrown a lustre on that institution, as marking the commencement of a literary era, which it is doubtful if the succeeding times have yet seen surpassed."*

FERGUSON, shortly after entering on his duties in the Library,† was solicited to undertake the education of the sons of Lord BUTE. HUME, in a letter to GILBERT ELLIOT, of Minto, states, that he had some scruples with regard to accepting this appointment, as he was to have the charge of more than one boy, and adds, "I hope Lord BUTE will conform himself to his delicacy, at least if he wants to have a man of sense, knowledge, taste, elegance, and morals, for a tutor to his son."

Having arranged satisfactorily the terms of his engagement with Lord BUTE, FERGUSON seems to have left his office of Librarian rather abruptly, for, according to the Minutes of the Faculty of 3d January, 1758, "it was represented to the Dean and Faculty that Mr ADAM FERGUSON, who in January last had been constituted Library-keeper and Clerk to the Faculty, had gone from this place some time ago, and had left behind him a letter demitting said offices, so that the Faculty had now neither a Librarian nor a Clerk; by whatever omission or neglect, it happened that the said letter of demission had neither been presented to the Dean nor laid before them. And the same thing being affirmed by divers members, after reasoning on the matter at good length, the Dean and Faculty declared the said offices to be vacant, notwithstanding that the said demission had never been presented."

FERGUSON was succeeded in this office by Mr WILLIAM WALLACE, advocate, elected at the meeting above referred to.

Before giving up his connection with the Advocates' Library, FERGUSON attracted considerable attention by a pamphlet which he wrote in defence of JOHN HOME, author of 'Douglas,' who had incurred the censure of the Presbytery of Edinburgh by the publication of his celebrated tragedy. This pamphlet, entitled *The Morality of Stage Plays seriously considered*,‡ was published anonymously, and was admitted on all hands "to be the only piece on that side that was written with any tolerable degree of discretion." HOME was one of FERGUSON'S most intimate friends, and it is not surprising that FERGUSON was led into taking an active part in the controversy which the publication of 'Douglas' occasioned. HOME had at this time resigned his parochial charge, to avoid the persecution to

* Life of Lord KAMES, vol. i, p. 175.

† Like his predecessor HUME, FERGUSON enjoyed the moderate salary of L.40 per annum.

‡ Edinburgh, 1757, 8vo.

which he was subjected by the Church, while his tragedy was received on the Edinburgh stage with the most enthusiastic applause. Along with Principal ROBERTSON, DAVID HUME, and Dr. BLAIR, FERGUSON had taken a deep interest in the attempts of HOME to have his 'Douglas' properly brought before the public, and it has been stated that it was privately rehearsed by these gentlemen, in the lodgings of Mrs SARAH WARD (one of DIGGES' company), in presence of some of the most distinguished literary men of Scotland.*

From his friendship with DAVID HUME, and ADAM SMITH, then Professor of Moral Philosophy in the University of Glasgow, who were well aware of his extraordinary accomplishments, it was now proposed that FERGUSON should be promoted to a Chair in one of the Scottish Universities. At this time the influence of Lord MILTON,† the political agent of ARCHIBALD Duke of Argyll, was paramount in the patronage of almost every office of emolument and dignity in Scotland. Even in the exercise of the patronage of the Chairs in the University of Edinburgh, so jealously guarded by the Town Council, the influence of Lord MILTON was so strong that Provost DRUMMOND, one of the most meritorious and public-spirited benefactors of the community over which he presided, did not find himself at liberty to promise any preferment at the disposal of the Town Council, without Lord MILTON'S consent being obtained.‡ Under such a system, it is not surprising that Professorships might not only become matters of private arrangement, but, as it would appear by the following letters, even attainable by the payment of considerable sums of money. From the terms of these letters, preserved in the Royal Society,§ it was accordingly contemplated to transfer ADAM SMITH to the Chair of the Law of Nature and Nations in the University of Edinburgh, then expected to become vacant by the retirement of Professor ABERCROMBY, and to appoint FERGUSON to the Chair occupied by SMITH at Glasgow.

"HUME *to* ADAM SMITH."

"*8th June* 1758.

"DEAR SMITH,—I sit down to write to you, along with JOHNSTONE;‖ and as we have been talking over the matter, it is probable we shall employ the same arguments. As he is the younger lawyer, I leave him to open the case, and

* The following was the cast of the piece on that occasion:—"*Lord Randolph*, Dr Robertson (Principal); *Glenalvon*, David Hume (Historian); *Old Norval*, Dr Carlyle (Minister of Musselburgh); *Douglas*, John Home (the Author); *Lady Randolph*, Dr Ferguson; *Anna* (the Maid), Dr Blair (Minister, High Church)."

"The audience that day, besides Mr DIGGES and Mrs WARD, were the Right Hon. Patrick Lord ELIBANK, Lord MILTON, Lord KAMES, Lord MONBODDO (the two last were then only lawyers), the Rev. JOHN STEELE and WILLIAM HOME, ministers."—*Edinburgh Weekly Chronicle*, 21st January, 1829. Dr CARLYLE corroborates this statement so far in his Diary, p. 311.

† It has been stated that in 1742 FERGUSON was Secretary to Lord MILTON, and lived with him in that capacity for some time, at Brunstain House, near Edinburgh.—*Chambers's Journal*, No. 60, 1855.

‡ Sommervill's Life and Times, p. 380.

§ Hume's Life by BURTON, vol. ii. p. 45.

‖ Afterwards Sir WILLIAM PULTENEY.

suppose that you have read his letter first. We are certain that the settlement of you here, and of FERGUSON at Glasgow, would be perfectly easy by Lord MILTON'S interest. The prospect of prevailing with ABERCROMBY is also very good; for the same statesman, by his influence over the Town Council, could oblige him either to attend, which he never would do, or dispose of the office for the money which he gave for it. The only real difficulty is then with you. Pray, then, consider that this is perhaps the only opportunity we shall ever have of getting you to town. I dare swear that you think the difference of place is worth paying something for, and yet it will really cost you nothing. You made above L.100 a-year by your class* when in this place, though you had not the character of Professor. We cannot suppose that it will be less than L.130 after you are settled. JOHN STEVENSON;† and it is JOHN STEVENSON makes near L.150, as we are informed upon inquiry. Here is L.100 a-year for eight years' purchase; which is a cheap purchase, even considered as the way of a bargain. We flatter ourselves that you rate our company at something; and the prospect of settling FERGUSON will be an additional inducement. For, though we think of making him take up the project if you refuse it, yet it is uncertain whether he will consent; and it is attended, in his case, with many very obvious objections. I beseech you, therefore, to weigh all these motives over again. The alteration of these circumstances merit that you should put the matter again in deliberation. I had a letter from Miss HEPBURN, where she regrets very much that you are settled at Glasgow, and that we had the chance of seeing you so seldom. I am, &c."

"*P.S.*—Lord MILTON can with his finger stop the foul mouths of all the roarers against heresy."

"HUME *to the* Rev. JOHN JARDINE."

Without date.

"REV. SIR,—I am informed by the late Rev. Mr JOHN HOME that the still Rev. ADAM FERGUSON'S affair is so far on a good footing, that it is agreed to refer the matter to the Justice Clerk, whether more shall be paid to Mr ABERCROMBY than he himself gave for that Professorship. Now, as it is obvious that in these kind of references, where the question is not of law and justice, the circumstances of the person are to be considered, I beg of you to inform my Lord of the true state of the case. FERGUSON must borrow almost the whole sum which he pays for this office. If any more, therefore, be asked than L.1000, it would be the most ruinous thing in the world for him to accept of the office. I am even of opinion, that if any other method of subsistence offered, it were preferable to this scheme of paying the length of L.1000; at least such would be my sentiments if the case were mine.

* SMITH had lectured on Belles Lettres in Edinburgh in 1748.
† Professor of Logic.

"If the Justice Clerk considers the matter aright, he will never agree to so unreasonable a demand as that of paying more; and I hope you will second these arguments with all your usual eloquence, by which you so successfully confound the devices of Satan, and bring sinners to repentance.—I am, Rev. Sir, your most obsequious humble servant."*

The negotiations above referred to were unsuccessful, and Mr ABERCROMBY was succeeded in the Chair of the Law of Nature and Nations by Mr ROBERT BRUCE in 1759.†

The long wished for opportunity of obtaining for FERGUSON an academic appointment soon after occurred, for a vacancy took place, in 1759, by the death of Dr JOHN STEWART, Professor of Natural Philosophy in the University of Edinburgh. The Town Council, who were the patrons of this Chair, after consultation with the ministers of the city, appointed FERGUSON on the 4th of July of the same year.

FERGUSON immediately began to prepare his lectures, so as to be ready to conduct his class when the University session was opened in October. Notwithstanding the shortness of the time at his disposal, he was so successful in his teaching, that "DAVID HUME said FERGUSON had more genius than any of them, as he had made himself so much master of a difficult science,—viz., natural philosophy, which he had never studied but when at college,—in three months, so as to be able to teach it."‡

In 1760, FERGUSON was instigated by Dr CARLYLE to publish a little volume, to which the following quaint title was given,—*The History of the Proceedings in the Case of Margaret, commonly called Peg, only lawful sister to John Bull, Esq.*§

The object of this publication, which went through two or three editions, was to turn into ridicule the opposers of the Scotch Militia Bill, which had been rejected in the preceding session of Parliament. The Act of Parliament by which the militia of England was constituted did not apply to Scotland, as, on account of the Rebellion of 1745, and the still existing jealousy of the Jacobites, Government had felt alarm at the proposal to extend to the sister kingdom a measure which should put arms into the hands of those who might turn them to revolutionary purposes. This jealous feeling towards Scotland was the cause of con-

* Hume's Life, by J. H. BURTON, vol. ii. p. 47.

† It may here be mentioned, that the Professorship of the Law of Nature and Nations—the patronage of which was vested in the Crown—was then a sinecure, and was abolished in 1832. It has, however, been again revived as the Chair of Public Law, by the Universities' Commissioners of 1858.

‡ Carlyle's Diary, p. 283.

§ Lond. 1760, 12mo.—It may be interesting to those who possess this curious little volume, which (from its having been published anonymously) has sometimes been attributed to SWIFT, to have the following key to the principal characters referred to in it:—*John Bull*, England; *Sister Peg*, Scotland; *Nurse*, Lord Hardwicke; *Jowler*, Mr Pitt; *Hubble Bubble*, Duke of Newcastle; *Boy George*, George Townshend, Esq.; *Bumbo*, Lord President Dundas; *M'Lurcher*, The Highlanders; *Sir Thomas*, The King; *Gilbert*, Sir Gilbert Elliot, Bart.; *Squire Geoffrey*, The Pretender; *Small Trash*, Charles Hope Weir; *Lick Pelf*, Earl of Hopetoun; *James*, James Oswald, Esq.; *Suckfist*, General Watson.

siderable agitation in Edinburgh, and called forth several pamphlets, none of which, except that of FERGUSON, are now deserving of attention.*

FERGUSON and the other members of the Select Society, which had been instituted in 1754 for the promotion of philosophical discussion, but which was now in a declining state, in 1762 revived it in a different form, as a means of agitating the militia question, and keeping alive the flame of patriotic feeling. To this new society was given, at the suggestion of FERGUSON, the name of the "Poker Club," which numbered among its members nearly all the *literati* of Edinburgh and its neighbourhood.†

This Club continued in existence till 1784, when from various causes it dwindled away, without achieving the object for which it was instituted,—viz., the extension of the Militia Bill to Scotland. It was not till 1793 that the Government agreed to place Scotland on the same footing as England in regard to an establishment so essential for the safety of the country.

As the salaries allowed at this time to the Professors in the University of Edinburgh were very small, it was not uncommon for them to receive into their families the sons of noblemen and gentlemen while they attended the University. From the reputation which FERGUSON had now obtained, he was, in 1763, entrusted with the education of the Honourable CHARLES and ROBERT GREVILLE, the sons of the Earl of Warwick, whose eldest son, Lord GREVILLE, had been educated under the care of Principal ROBERTSON. These young gentlemen remained with him for some years, and always retained a lively sense of the benefits they received under his care. The connection thus formed was of great service to FERGUSON, as it brought him more immediately to the notice of many persons of rank, and the fame he acquired shortly afterwards by his writings greatly extended his influence among his contemporaries.

Whilst these young noblemen were residing with him, FERGUSON employed one of his most promising students, who afterwards became a very distinguished man, to aid him in superintending their studies. This was JOHN, afterwards Sir JOHN M'PHERSON, Governor-General of India, who always acknowledged that it was to his intercourse and co-operation with FERGUSON that he owed most of his knowledge and success in life.

FERGUSON also took a warm interest in JAMES M'PHERSON, the translator of Ossian, who, in 1760, had anonymously published his "Fragments of Ancient Poetry, collected in the Highlands of Scotland."‡ A curious incident which occurred about this time, with reference to the Ossianic Poems, will be subsequently noticed.

* An account of the manner in which this singular work was written is given in Carlyle's Diary, page 407; and an interesting letter of HUME, in which he avowed himself as its author, is given in his Life, by J. H. BURTON, vol. ii. page 88.

† See Carlyle's Diary, page 419.

‡ Along with Patrick Lord ELIBANK, Principal ROBERTSON, Dr BLAIR, and J. HOME, FERGU-

He continued Professor of Natural Philosophy for about five years, and conducted his class in a manner which gave universal satisfaction. By adapting his lectures to the capacities of his students, he contrived to render his subject more attractive than it had been hitherto considered, and he also published for the use of his class a short analysis of his Course.

The study of Ethical and Political Philosophy, however, in which he had distinguished himself as a young man, had always a greater attraction for FERGUSON than the physical sciences, and on the transference of Professor BALFOUR, of Pilrig, to the Chair of the Law of Nature and Nations in 1764, FERGUSON was elected his successor as Professor of Pneumatics and Moral Philosophy. About ten years before this, when Mr CLEGHORN, the predecessor of Mr BALFOUR, was on his deathbed, he urged FERGUSON to apply for this office, for which he conceived him to be particularly qualified. "Mr CLEGHORN, after expressing his regret at not having influence with the patrons to secure such an arrangement, added, as FERGUSON sometimes related with much emotion, 'I can only say of you as Hamlet did of Fortinbras, *He has my dying voice.*'" *

On being appointed to this Chair, which had long been the object of his ambition, FERGUSON applied himself to its duties with the greatest activity, and his lectures were attended not only by the regular students, but by the most distinguished men of the country.

Within little more than a year after his appointment he published his *Essay on the History of Civil Society*,† which contributed to raise him still more in the estimation of the public. This celebrated work, a portion of which had been written several years previously, had been, in 1759, submitted in manuscript to the critical opinion of DAVID HUME, as a 'Treatise on Refinement.' HUME gave it his approval, and stated that with some amendments it would make an admirable book, "as it discovered an elegant and a singular genius."

The 'Essay' was again submitted in its finished state to HUME, who now recommended FERGUSON'S friends to prevail on him to suppress it, as likely to be injurious to his literary reputation. HUME had heard an opinion expressed, by the French philosophers HELVETIUS and SAURIN,‡ with which he at the time concurred, that the fame of MONTESQUIEU'S 'Esprit des Lois' would not be lasting. As FERGUSON'S Essay may be regarded to a certain extent as a commentary on MONTESQUIEU, HUME, perhaps, hastily adopted the same opinion with regard to the work of his friend. When he found that the general opinion was favourable to the work, he heartily joined in the congratulations which FERGUSON now received.

son made zealous efforts to induce M'PHERSON to promote his further researches for the discovery of ancient Gaelic Poetry, and he took part in a meeting convened by Dr BLAIR, in 1760, to provide funds for the purpose of enabling M'PHERSON to do so.—*Browne's Hist. of the Highlands*, vol. i. p. 43.

* Encyc. Brit. Suppt. vol. iv. art. FERGUSON.

† Edinburgh, 1766. 8vo.

‡ Hume's Life, by J. H. BURTON, vol. ii. p. 387.

Writing to the author, in February 1767, he says:—"Dear FERGUSON,—I happen'd yesterday to visit a person three hours after a copy of your Performance was open'd for the first time in London. It was by Lord MANSFIELD. I accept the omen of its future success. He was extremely pleas'd with it; said it was perfectly well wrote; assured me that he would not stop a moment till he had finished it, and recommended it strongly to the Perusal of the Archbishop of YORK who was present. Tho' I set out with reluctance, I do not repent my journey. Direct to me at Miss ELLIOT'S, in Brewer's Street. I have not seen SMITH; Judge of my hurry."*

In another letter to FERGUSON he says, that he had "met with nobody that had read it who did not praise it. Lord MANSFIELD is very loud to that purpose in his Sunday Societies. I heard Lord CHESTERFIELD and Lord LYTTLETON express the same sentiment; and what is above all, CADDELL, I am told, is already projecting a second edition of the same quarto size." †

Writing to Dr BLAIR, HUME further remarks,—"I hear good things said of FERGUSON'S book every day. Lord HOLDERNESS showed me a letter from the Archbishop of YORK, where his Grace says, that in many things it surpasses MONTESQUIEU. My friend, Mr DODWELL, says, that it is an admirable book, elegantly wrote, and with great purity of language. Pray tell to FERGUSON and to others all these things." ‡

Writing to Principal ROBERTSON from London, on the 19th March, HUME makes the following interesting statement:—"FERGUSON'S book goes on here with great success. A few days ago I saw Mrs MONTAGUE § who has just finished it with great pleasure. I asked her, Whether she was satisfied with the style? Whether it did not savour somewhat of the country? 'O yes,' she said, 'a good deal; it seems almost impossible that any one could write such a style except a Scotchman.'" ‖

Dr BEATTIE of Aberdeen, writing to the Poet GRAY, on 30th March, states,—"A Professor at Edinburgh has published an Essay on the History of Civil Society, but I have not seen it. It is a fault common to almost all our Scottish authors that they are too metaphysical. I wish they would learn to speak more to the heart and less to the understanding; but alas, this is a talent which heaven only can bestow; whereas the philosophical spirit (as we call it) is merely artificial, and level to the capacity of every man who has much patience, a little learning, and no taste."¶

* His hurry was so great that he apparently had not time to sign the letter. It is in the possession of D. LAING, Esq.

† Encyc. Brit. Suppl. vol. iv. art. FERGUSON.

‡ Life of Hume, by BURTON, vol. ii. p. 386.

§ The elegant author of an Essay on the genius of SHAKESPEARE.

‖ Stewart's Works, vol. x. p. 223.

¶ Gray's Works, vol. ii. p. 295.

GRAY, in reply to BEATTIE, thus refers to the Essay of FERGUSON:—"I have read over (but too hastily) Mr FERGUSON's book. There are uncommon strains of eloquence in it; and I was surprised to find not one single idiom of his country (I think) in the whole work. He has not the *fault you mention;* his application to the heart is frequent, and often successful. His love of MONTESQUIEU and TACITUS has led him into a manner of writing too short winded and too sententious, which those great men, had they lived in better times and under a better government, would have avoided."*

Besides the interesting letters relating to the publication of this valuable work, which are to be found in the lives of HUME and Lord KAMES, the following letter from the Baron D'HOLBACH to FERGUSON is very characteristic:—

"SIR,—I receiv'd with the deepest sense of gratitude the undeserv'd favour of your kind letter dated the 3d of March; tho', your valuable work is not yet come to my hands according to the orders you were so good to give your Bookseller in London, I shall expect the favour you intended with thankfulness, and even with patience; having had the good fortune of getting the perusal of a copy belonging to an acquaintance of mine. I found it answering completely to the high opinion I had conceived of your great abilities and ingenuity, by the testimonies given of you by Mr ANDREW STEWART, Colonel CLERK, and several other gentlemen from your country, with whom I have had the pleasure of conversing in this place. Tho' you don't seem to set a high value on theory, it must necessarily precede practice, and I think that given in your grand performance, by enlightening the human mind, may contribute to render their practice better; for I don't despair of the perfectibility of mankind: I believe they have been mere children in matters the most important for them. I am of opinion that the greatest part of our distresses arise from our ignorance, and give me leave, Sir, to tell you sincerely, that I am persuaded that your valuable work is, and will be, very able to dispel the foggs that hang over our understandings. We are always indebted to great men for useful inventions, that are the fruits of their invention and theory. What they have found out with a great deal of trouble, becomes by and by popular; and by degrees truth, when become general, influences the general practice, even in spite of those who think it their interest to keep mankind in the dark. As to the virtues that preserve nations, or at least put off long their decline, I believe they must be the effects of learning; when morality shall be clear'd, or rescued from the hands of those who have made it their study to render it obscure. I think every individual will be more virtuous, and even the powerful movers of men will find it their own interest in governing according to the rules of reason. I have the honour to be, with the highest consideration, Sir, yours, &c.,

"*Paris, 15th of June,* 1767.† "D'HOLBACH."

* Gray's Works, vol. ii. p. 295.

† MSS. University of Edinburgh.

The praise with which this Essay was received was well deserved, as it was the result of a great amount of research into the history of ancient and modern times, and of a remarkable knowledge of the springs of human action. The author considers the condition of man, under various forms of government, at different periods, and traces him through the several steps from his first rude efforts at civilization and arts, to a high state of politeness and refinement. The gradual efforts of the human mind, rising from the simple perceptions of sense to the heights of moral and political knowledge, are delineated in elegant and classical language. Dr REID had about two years previously published his 'Enquiry into the Human Mind;' a work which was the first systematic attempt to carry out in the study of human nature the same plan of inductive investigation which had conducted NEWTON to the properties of light and to the law of gravitation. FERGUSON was the first to applaud REID's success, and his Essay on Civil Society is to be regarded as one of the earliest applications of the same method of research to the development of society and to national policy.* FERGUSON was of opinion that mankind should be studied in groups, and that all speculation as to their progress should relate to entire societies, and not to single individuals. In this point of view, he discusses the subjects of self-preservation, war and dissention, intellectual powers, moral sentiments, happiness, and national felicity. In the treatment of these important subjects, FERGUSON particularly endeavours to inculcate that the happiness of man consists in the exercise of his faculties as a member of society, and with the view of promoting public utility; that the power of states depends principally on the national character and public spirit, which are counteracted and sometimes annihilated amongst modern nations by selfishness and by the spirit of commerce. Adopting the views of MONTESQUIEU, he ascribes to climate and situation a great influence on the literature, commerce, and policy of nations; and justly observes that man has always attained to the principal honours of his species under the temperate zone. He further considers the laws which ought to regulate political establishments, and is of opinion that these, while they may vary according to the diversities of character and circumstances, should not interfere with that firm and resolute spirit, with which the liberal mind is always prepared to resist indignities, and that the power of restraint should be exercised in an inverse proportion to the general knowledge and virtue of a people.

The enthusiasm of his own nature may be traced in his opposition to despotic governments and to political slavery. He viewed with solicitude the tendency to despotism which characterised some of the military governments of the continent, and he expresses his fear of a renewal of those revolutions so frequently described, which out of the ruins of several nations form those colossal powers always fatal to liberty and to the wellbeing of man.

* Stewart's Works, x. p. 261.

On the whole, this Essay must be regarded rather as an exposition of general principles, than an application of these principles to particular instances. It is defective in so far as the subject of religion, which in every age has had so considerable an influence on society, has been omitted. But, notwithstanding its defects, it must be admitted that the disquisitions which it embraces have as much interest at the present time as they had one hundred years ago, because they trace the same affinities between man and man, generation and generation, in the delineation of common passions, affections, and desires. To say that human society is modified by our present circumstances, and affected by the progress of modern civilisation, is only to render it still more amenable to those laws of moral and intellectual symmetry which regulate the destinies of our species, and which FERGUSON has with much ingenuity attempted to evolve.

The fame which FERGUSON had now acquired, and the connection he had formed with many persons of influence, led to suggestions of higher preferment. The following letter from his friend Colonel (afterwards General) CLERK, brother of Sir JAMES CLERK of Pennicuik, shows that at this time he was regarded as a suitable person to fill some political office. The Colonel had pressed him to dedicate his Essay to Lord SHELBURNE, but this was declined, and the book appeared without any dedication :—

"To ADAM FERGUSON, Esq.—R. CLERK." "*London, October* 10*th*,1766.

"I have not wrote you for some time. I suppose that your book is printing. Lord SHELBURNE told me one day that he supposed Governor JOHNSON would not perhaps return to West Florida, as he is coming home, and sayd, that he saw no reason why he should not offer the government of it to you. I answered that I should write to you of his kindness for you long before it should be an object of deliberation, but that I thought you would be happier in your present situation, and more independent, for the other was uncertain, though, in the common way of thinking in the world, it was a great favour. Besides, I thought that you was of more service to mankind where you was. He laughed at me. We shall have time to consider of this. However, it shows Lord SHELBURNE's kindness for you, and good opinion of you. You asked my opinion upon a subject which I shall give you when at leisure.—Yours affectionately."

In 1766, FERGUSON revisited Logierait, and delighted the villagers by his recollections of themselves and their kindred, while they, in their turn, were no less proud of the distinction attained by the son of their former pastor. This was also the year of his marriage to Miss KATHERINE BURNET of Aberdeenshire, the amiable niece of Professor JOSEPH BLACK. This union was one of unmingled happiness to both, till it was broken by the death of Mrs FERGUSON in 1795.

Among the many allusions to FERGUSON, contained in the Diary of his friend Dr CARLYLE, we learn that he and FERGUSON had, about ten years before this,

paid, their addresses to the same lady, but without success. CARLYLE has not favoured us with the name of the lady, who must have possessed many attractions; but he remarks, that "after having rejected rich and poor, young and old, to thenumber of half a score, she gave her hand, at forty-five, to the worst tempered and most foolish of all her lovers, who had a bare competency, and which, added to her fortune, hardly made them independent. They led a miserable life, and parted, soon after which he died, and she then lived respectably to an advanced age.

In December 1766, FERGUSON received the honorary degree of LL.D. from the Senatus Academicus of the University of Edinburgh.

He also published, in the same year, a short syllabus of his lectures, entitled, *Analysis of Pneumatics and Moral Philosophy. For the use of the Students in the College of Edinburgh.* This work was afterwards enlarged, and published as the *Institutes of Moral Philosophy*,—a book which was found so useful, that it was translated into French, German, and Russian, and was made a text-book in several foreign universities. It exhibited a clear outline of his Course.

About the close of the year 1773, FERGUSON was solicited to undertake the charge of the education of CHARLES, Earl of Chesterfield (nephew of the celebrated Earl), by his guardians Lord STANHOPE,* Mr HEWITT, and Sir GEORGE SAVILLE. The offer of this appointment was made by Lord STANHOPE in the most complimentary terms on the recommendations of Dr ADAM SMITH,† who endeavoured with great earnestness to induce FERGUSON to accept of it. The young Earl was then in his nineteenth year; and it was proposed that he should travel on the Continent for several years, under the charge of FERGUSON, who by his care was expected to make up for the neglect of the Earl's previous instructors.

At the present time it may seem strange that such a proposal should have been seriously entertained by any one holding a Professorship in the University:

* Editor of Dr ROBERT SIMSON's posthumous works.

† Writing to SMITH with reference to this appointment, FERGUSON alludes to BEATTIE's celebrated Essay on Truth, and the corpulence of HUME, in the following letter. BEATTIE's "Essay on the Nature and Immutability of Truth in opposition to Sophistry and Scepticism," was so popular a work, that in four years five large editions of it were sold off. It was first published in 1771; and the letter of FERGUSON refers to the 3d edition, which appeared in 1773:—

"*Edin. Sept. 2d*, 1773.

"MY DEAR SIR,—I am told that Dr BEATY, or his party, give out that he has not only refuted but killed D. HUME. I should be very glad of the first, but sorry for the other; and I have the pleasure to inform you that he is in perfect good health; if he had been otherwise I should have certainly mentioned it in some of my letters. He had a cough, and lost flesh, soon after you went from home, which we did not know what to think of, but it turned out a mere cold, and it went off without leaving any ill effects; he has still some less flesh than usual, which nobody regrets, but in point of health and spirits I never saw him better. You seemed to doubt whether I should not write to Lord STANHOPE. I had inclination enough, but was not so decided as to send my letter to himself without putting it in your power to withhold it if proper, and therefore I stayed for a frank; what is disagreeable is, laying him under the obligation to make a ceremonious answer, and, if he be gone, subjecting him to Continental postage, so you will judge. I have not seen J. FERGUSON, but he must acquiesce.—I am, dear Sir, most affectionately yours, ADAM FERGUSON."

but the emoluments from his Chair were at that time so small, and the terms offered by the Earl,—an allowance of L.400 a-year during the Earl's minority, and an annuity of L.200 for life—were so tempting, that FERGUSON, not without some hesitation, undertook the responsibility. His reasons are fully given in the following letter to his friend ADAM SMITH:—

"*Edinburgh, January 23d*, 1774.

"MY DEAR FRIEND,—It has given me great pleasure that you have avoided doing anything that might tend to urge Lord STANHOPE farther than he has already gone in the proposal respecting Lord CHESTERFIELD. If I had known the part he took in that business, I should certainly at first have either frankly accepted of the offer made me, or declined it in a way that could not imply an intention to raise the terms. This is certainly the only alternative that is now left me. I have revolved the subject all night and this morning, and the possibility of my becoming a burden on Lord STANHOPE'S family weighs much, but the odds on Lord CHESTERFIELD'S life is so great as very much to reduce that consideration. My place here, a few years ago, was worth about L.300 a-year, but this and the preceding year it has fallen considerably short; and while the present alarm of the scarcity of money, and the expense of education at Edinburgh, continues, it may not rise again to its former value. To this I must add, that in case of debility or old age, I shall probably be reduced to my salary, which is no more than L.100 a-year. For these reasons I think that I can fully justify myself to my family in accepting of L.200 a-year certain, with the privilege of choosing my place and my occupations; and if my Lord CHESTERFIELD'S guardians should be of opinion that he ought, when he comes of age, not only to relieve my Lord STANHOPE of his engagement, but likewise, in case I shall have acquitted myself faithfully and properly, to make some such addition to my annuity as I mentioned, I shall then likewise think that I can justify my conduct to the world, who rate men commonly as they do horses, by the price that is put upon them. But of this I would not have the least hint to my Lord CHESTERFIELD at present. I have so far proceeded without consulting anybody, and have formed an opinion subject to correction. I mean to read your letters, and this I am writing to one or two of my friends. If they approve, it shall go to you; and if you agree with me, be so good as intimate my resolution to the guardians of my Lord CHESTERFIELD; or, if you have any objections of moment, delay it till I shall have heard from you. My own present feeling is, that I should be to blame if I omitted putting myself and family under the protection of persons so worthy and so respectable, when I have an opportunity of doing it without any real hazard to my interest But I shall not enter on this subject, my heart, indeed, being too full, especially with respect to Lord STANHOPE. I am, &c., ADAM FERGUSON."*

Having, through ADAM SMITH, arranged satisfactorily the terms of his engage-

* MSS. University of Edinburgh.

ment with Lord CHESTERFIELD, FERGUSON prepared for his travels on the Continent. He accordingly, in February 1774, wrote to the Town-Council, as Patrons of the University, requesting permission to name persons to teach his Classes during the remainder of the session, viz. Dr JAMES LIND, for the Natural Philosophy Class,* and Professor BRUCE for that of Moral Philosophy. The Council, however, refused to consent to this arrangement, and ordered that FERGUSON should teach in person during the remainder of the session.

Notwithstanding the refusal of the Town Council, FERGUSON joined his pupil in London at the close of that winter session of the University, in the belief that the Provost and the greater part of the Council would be disposed to sanction his absence for the subsequent session.

When that session, however, commenced in the following October, the Council appointed Professor BRUCE to conduct the class of Moral Philosopy, and took steps to punish the contumacy of his colleague. Accordingly, in April 1775, they passed the following act:—"The Council, considering that upon the 16th of February 1774, they had refused an application of Mr ADAM FERGUSON, Professor of Pneumatics and Moral Philosophy in this city's University, where he requested that he might be allowed to substitute proper persons in what remained of his business in the College that winter; and also considering, that notwithstanding thereof he has deserted his office, and come under engagements incompatible with his discharging the duties thereof; and the act of the 23d of May 1764, electing Mr ADAM FERGUSON into the said office being read, the Council did, and hereby do, rescind the said act of Council, with all that has followed thereupon, and declared the said office of Professor of Pneumatics and Moral Philosophy in the University of this city vacant."†

Whatever may now be thought of the propriety of this step of FERGUSON, still it had not been without precedent in the history of the University. His friends in the Senatus Academicus gave him their support, and he took measures to vindicate his conduct, and to stay the somewhat arbitrary proceedings of the Town Council. The following notes for his defence, drawn up by his friend Dr BLAIR, are interesting, as showing his warm sympathy with his colleague:—

"Mr FERGUSON, on his going away, engaged one of his colleagues, Mr BRUCE lately elected Professor of Logic, to supply his place in teaching this winter.

"He wrote a letter to the Town Council, begging leave of absence for one season, and proposing Mr BRUCE to be allowed by the Council to teach in his place. This letter, indeed, was not delivered; because the member of Council to whom it was addressed, upon its being mentioned to him, advised, as more for Mr FERGUSON's interest, that it should not be presented.

* On the death of his relation Mr RUSSELL, FERGUSON had undertaken the additional duty of teaching the Natural Philosophy class during sessions 1773, and 1774.

† DALZEL's Hist. of the University of Edin., vol. ii. p. 445.

"But by a minute of the Town Council in the beginning of winter, Mr BRUCE was appointed to teach in Mr FERGUSON'S place. As this gave the sanction of the Council to the substitute whom Mr FERGUSON proposed, it was considered by him and all his friends, as equivalent to giving him leave of absence for this session, and he had not the smallest apprehension of any intention to deprive him of his office, without at least giving him warning of his danger.

"On Wednesday last, 5th April, just upon the close of the session, the Town Council, upon a motion made by the Provost concerning the impropriety of professors in the College strolling through the country as governours, found the office of Professor of Moral Philosophy vacant, and were desired to have their thoughts on a proper person for filling it up; and this without any summons given to Mr FERGUSON to attend, or any intimation whatever made to him or any of his friends.

"The Professor of Moral Philosophy, by his Commission from the Town Council, holds his office *ad vitam aut culpam* must not the *culpa*, therefore, be first properly found, and the Professor summoned to see what he can say in his defence, before he can legally be deprived of his office?

"The words of King James' Charter respecting the power which he gives the Magistrates over the Professors are,—'*cum potestate imponendi et removendi ipsos sicuti expediverit.*" Do these words authorise every arbitrary and wanton exercise of power over the Professors? Or does the clause *sicuti expediverit* restrain it to what is profitable, and expedient, and fit?

"Do not the words in the charter which immediately precede these '*avisamento tamen ministrorum eorum*,' connect with the words before quoted, and was not the *avisamentum* necessary to have been taken on this occasion?

"Mr FERGUSON has not only for many years, ever since he was elected Professor, regularly discharged all the duties of his office, but in the session immediately preceding this, when the Chair of Natural Philosophy became vacant in the beginning of the session, taught both the classes of Natural and Moral Philosophy.

"Sir JOHN PRINGLE, who was a predecessor of Mr FERGUSON'S in the same Chair, went abroad when in that office as physician to the army, and for a year (or for years, uncertain which) taught his class by a substitute without quarrel, until he thought proper to demit.

"The Professor of Mathematics has been absent for two years, and taught his class by his son without quarrel.

"Dr DRUMMOND was elected two years ago by the Town Council, Professor of the Theory of Medicine, and has never appeared to discharge any of the duties of his office, which for two sessions have been discharged by substitutes without quarrel, and no step taken for finding the office vacant.

"When Mr FERGUSON. after being absent only for five months, is suddenly

deprived of his office, without any requisition given him to attend, without any communication previously made to the Principal, or any members of the University, but the intention of depriving him kept profoundly secret till the moment of its execution, does not this plainly indicate that the Town Council did not seek to bring back Mr FERGUSON to the discharge of his office, but had formed a design to turn him out with a view to bestow his office on another? and can such violent and unjust proceedings towards an eminent Professor and a respectable University be warranted by law?" *

These grounds of objection to the harsh measure of the Town Council were embodied by FERGUSON, in an application to the Court of Session for a bill of suspension of the sentence of deprivation,† which had the desired effect of causing the Council to rescind their act, and restore the Professor to the peaceable enjoyment of his office.

The tour which FERGUSON made with his pupil Lord CHESTERFIELD through France and other parts of the Continent, although it brought about this disagreeable quarrel with the Town Council, proved highly advantageous to his improvement. In a letter to his friend ADAM SMITH, he thus describes the pleasure which his appointment afforded him.

"*Geneva, June* 1*st*, 1774.

"MY DEAR SMITH.—You see I have taken full benefit of the time you allowed me to form my opinion of this situation, and have the pleasure to inform you it is in most material circumstances very agreeable. I was received with great politeness, and continue to be treated with sufficient marks of regard. I have found not only vivacity and parts as I was made to expect, but likewise good dispositions and attachments, servants all of an old standing, and become friends without any improper influence or disorder that I have yet observed. I was made to expect great jealousy of control, and set out with a resolution to employ no other than what a sense of my great regard might give me. It is likely that a person of a different character was expected, and the disappointment, I believe, has had a good effect. My journey hither furnished no adventures worth relating. My Lord STANHOPE's being at Paris gave me access, for the few days I stayed, to some very respectable and agreeable company, in which I was questioned concerning you, particularly by the Duchess D'ENVILLE, who complained of your French, as she did of mine, but said that before you left Paris she had the happiness to learn your language. I likewise met with your friend, Count SARSFIELD, to whom I had great obligations, and if you write I beg that you will thank him, &c. &c. ADAM FERGUSON."‡

FERGUSON, and his pupil Lord CHESTERFIELD, after residing for some time at Geneva, returned to London in the Spring of 1775, and the following interesting

* MSS. University, Edinburgh. † Ably drawn up by ILAY CAMPBELL, afterwards Lord President.
‡ Ibid.

letter addressed to Dr CARLYLE, gives an account of their proceedings while on the Continent:—

"*Blackheath, April 29th*, 1775.

"MY DEAR CARLYLE,—In answer to the two or three letters which you have written to me, I can give you five or six which I had written in my own mind to you, before I received any of yours. The first was from Geneva, where, having had the advantage of lodging in CALVIN'S own house, and having access to some of his most secret manuscripts, I thought myself, without vanity, qualified to give you some light into the more intricate recesses of our Church. My second was from Ferney, the seat of that renowned and pious apostle, VOLTAIRE, who saluted me with a compliment on a gentleman of my family who had civilized the Russians.* I owned this relation, and at this and every successive visit encouraged every attempt at conversation—even jokes against Moses, Adam and Eve, and the rest of the Prophets—till I began to be considered as a person who, tho' true to my own faith, had no ill humour to the freedom of fancy in others. As my own compliment had come all the way from Russia, I wished to know how some of my friends would fare, but I found the old man in a state of perfect indifference to all authors except two sorts—one, those who write Panegyrics, another who write Invectives on himself. There is a third kind, whose names he has been used to repeat, fifty or sixty years, without knowing anything of them—such as LOCKE, BOYLE, NEWTON, &c. I forgot his competitors for fame, of whom he is always either silent, or speaks slightingly. The fact is, that he reads little or none, his mind exists by reminiscence, and by doing over and over what it has been used to do. Dictates tales, dissertations, and tragedies; even the latter with all his elegance, tho' not with his former force. His conversation is among the pleasantest I ever met with; he lets you forget the superiority which the public opinion gives him, which is indeed greater than what we conceive in this Island. But he is like to make me forget all the rest of my letters. The third was from the face of a snowy mountain in Savoye, higher than all the mountains of Scotland piled upon one another, and containing more eternal ice in its recesses than is to be found in all Scotland in the hardest winter. The bottom of this ice is continually melting in the valleys, like the bottom of a roll of butter placed on end in a frying pan. It is perpetually creeping down from the mountain, where fresh snows continually fall in snotters. Masses come down from the mountains sometimes, and shake all the rocks with a force that nothing but an earthquake can imitate, and drive the air out of the narrow valleys with the force of a hurricane, that roots up trees on the opposite hills. I wrote you this letter in the full belief that you are a great natural philosopher, and disposed

* FERGUSON'S 'Institutes of Moral Philosophy,' having been translated into Russian, was used as a Text-Book in the Russian universities.

to believe every word I say. My fourth letter was written from the innermost parts of Swisserland, on a Sunday afternoon, when I saw the militia exercise. They have uniform clothes and accoutrements all at their own expense, which is not a great hardship, for it is their only public burden. They appear to me to be a very effective military establishment, and as they were the only body of men I ever saw under arms on the true principle for which arms should be carried, I felt much secret emotion, and could have shed tears. But to conclude, my fifth and last letter was from the neighbourhood of this place, where everything, from a pair of snuffers to the Venus of Medicis, and the great Diana of the Ephesians, is better provided than anywhere else; where every one is busily enjoying, and no one thinks whence it came nor how it is to be kept. I thought to have finished all my letters here; but as a frank will carry another sheet, I shall take room, at least, to sign my name. As I have already written you five letters, and this new sheet may pass for another, you will please to observe that you are, at least, four letters in my debt. I am much obliged to you for your goodness to my wife and my bairns. If I live to return to them, we shall not part so easily again. You may believe I was much surprised at the attempt of the Town Council to shut the door against me; but am obliged to them for opening it again. I may be a great loser; but the end for which I am persecuted cannot be gained while I have it in my option to return. I have been much obliged to the general voice that was raised in my favour, as well as to the ardent zeal of particular friends. ILAY CAMPBELL has given me proofs of friendship which I can never forget. PULTENEY has behaved to me in everything, as he would have done at the beginning of the Poker Club. I have always been an advocate for mankind, and am a more determined one than ever; the fools and knaves are no more than necessary to give others something to do. I saw J. HOME in town yesterday morning, he goes on as usual. MAC* is listening to the reports of his History. I do not live among readers, and am really ignorant of the general verdict. I have been living here above three weeks. A charming villa, in a magnificent scene, *sed quis me sistat gelidis in montibus Pentland;* and this I do not say on account of the hot weather, tho' it has been for three days the greatest I ever saw in this country.

"Remember my blessing to Mrs CARLYLE and your young ones, of whose thriving state I am happy to hear. Tell EDGAR, when you see him, that I have lately a letter from CLERK, and shall write to him—meaning EDGAR—soon. I am, dear CARLYLE, yours most affectionately, ADAM FERGUSON."†

The engagement which FERGUSON had with Lord CHESTERFIELD terminated rather abruptly shortly after this; and on returning to Edinburgh, he continued his literary pursuits with renewed activity.

* JAMES M'PHERSON (Ossian).

† MSS. University, Edinburgh.

The following interesting letter, addressed to ADAM SMITH at this time, has reference to the publication of the 'Inquiry into the Wealth of Nations:'—

"*Edinburgh, April* 18*th*, 1776.

"MY DEAR SIR,—I have been for some time so busy reading you, and recommending and quoting you, to my students, that I have not had leisure to trouble you with letters. I suppose, however, that of all the opinions on which you have any curiosity, mine is among the least doubtful. You may believe, that on further acquaintance with your work my esteem is not a little increased. You are surely to reign alone on these subjects, to form the opinions, and I hope to govern at least the coming generations. I see no addition your work can receive except such little matters as may occur to yourself in subsequent editions. You are not to expect the run of a novel, nor even of a true history; but you may venture to assure your booksellers of a steady and continual sale, as long as people wish for information on these subjects. You have provoked, it is true, the church, the universities,* and the merchants, against all of whom I am willing to take your part; but you have likewise provoked the militia, and there I must be against you. The gentlemen and peasants of this country do not need the authority of philosophers to make them supine and negligent of every resource they might have in themselves, in the case of certain extremities, of which the pressure, God knows, may be at no great distance. But of this more at Philippi. You have heard from BLACK of our worthy friend D. HUME. If anything in such a case could be agreeable, the easy and pleasant state of his mind and spirits would be really so. I believe he will be prevailed on at last to get in motion, and to try the effect of Bath, or anything else Sir JNO. PRINGLE may recommend. I have said more on this subject to Mr GIBBON, who, if you be found at London, will communicate to you. If not, I hope we shall soon meet here. And am, &c.

"ADAM FERGUSON."†

For several years FERGUSON had meditated the publication of a History of the Roman Republic; and he now began with greater perseverance to collect his materials for the projected work. He was also stimulated to bring his labours on this subject to completion, as Gibbon had, in 1776, begun the publication of his 'History of the Decline and Fall of the Roman Empire;' and the following correspondence is valuable, as showing the friendly relations which existed between these eminent men:—

"*Edinburgh, March* 19*th*, 1776.

"DEAR SIR,—I received, about eight days ago, after I had been reading your History, the copy which you have been so good as send me, and for which I now trouble you with my thanks. But even if I had not been thus called upon to offer you my respects, I could not have refrained from congratulating you on the

* See 'Wealth of Nations,' book v. chap. i. part 3, art. 2.

† The original letter is in the possession of the Rev. Mr CUNNINGHAM, Prestonpans.

merit, and undoubted success, of this valuable performance. The persons of this place whose judgment you will value most, agree in opinion, that you have made a great addition to the classical literature of England, and given us what Thucydides proposed leaving with his own countrymen, *a possession in perpetuity.* Men of a certain modesty and merit always exceed the expectations of their friends; and it is with very great pleasure I tell you, that although you must have observed in me every mark of consideration and regard, that this is, nevertheless, the case, I receive your instruction, and study your model, with great deference, and join with every one else in applauding the extent of your plan, in hands so well able to execute it. Some of your readers, I find, were impatient to get at the fifteenth chapter, and began at that place. I have not heard much of their criticism, but am told that many doubt of your orthodoxy. I wish to be always of the charitable side, while I own you have proved that the clearest stream may become foul when it comes to run over the muddy bottom of human nature. I have not stayed to make any particular remarks. If any should occur on the second reading, I shall not fail to lay in my claim to a more needed and more useful admonition from you, in case I ever produce anything that merits your attention. And am, with the greatest respect, dear Sir, your most obliged, and most humble servant, ADAM FERGUSON."*

GIBBON'S reply to this letter was as follows:—

"*Bentick Street, April the* 1*st*, 1776.

"DEAR SIR,—I shall not pretend to deny that your approbation, and that of your literary friends at Edinburgh, has given me very great pleasure. I am not proud enough to be above vanity; and I have always looked up with the most sincere respect towards the northern part of our island, whither taste and philosophy seemed to have retired from the smoke and hurry of this immense capital. Your good opinion, in particular, I should wish to cultivate; and am pleased to understand from some passages in your letter that you are engaged in a work, which I am convinced will stand in the same proportion to my imperfect essay, as the Roman Republic may be considered to have done, if compared with the lower ages of the declining empire.

"What an excellent work is that with which our common friend Mr ADAM SMITH has enriched the public!—an extensive science in a single book, and the most profound ideas expressed in the most perspicuous language. He proposes visiting you very soon; and I find that he means to exert his most strenuous endeavours to persuade Mr HUME to return with him to town. I am sorry to hear that the health and spirits of that truly great man are in a less favourable state than his friends could wish; and I am sure that you will join your efforts in convincing him of the benefits of exercise, dissipation, and change of air.

* Gibbon's Miscellaneous Works. By Lord Sheffield, vol. ii. p. 499.

"If I were not afraid of being too troublesome, I would desire you to inform me by a line of the particulars of his present condition, as well as of his intentions. I am, dear Sir, your most faithful and obedient servant, E. GIBBON."*

To this letter of GIBBON, FERGUSON returned the following answer:—

"*Edin.*, 18*th April* 1776.

"DEAR SIR,—I should make some apology for not writing you sooner an answer to your obliging letter; but if you should honour me frequently with such requests, you will find that, with very good intentions, I am a very dilatory and irregular correspondent. I am sorry to tell you that our respectable friend, Mr HUME, is still declining in his health; he is greatly emaciated, and loses strength. He talks familiarly of his near prospect of dying. His mother, it seems, died under the same symptoms; and it appears so little necessary or proper to flatter him, that no one attempts it. I never observed his understanding more clear, or his humour more pleasant or lively. He has a great aversion to leaving the tranquillity of his own house, to go in search of health among inns and hostlers. And his friends here gave way to him for some time; but now think it necessary that he should make an effort to try what change of place and air, or anything else Sir JOHN PRINGLE may advise, can do for him. I left him this morning in the mind to comply in this article, and I hope that he will be prevailed on to set out in a few days. He is just now sixty-four.†

* Dalzel's Hist. of the University of Edinburgh, vol. i. p. 22.

† It was principally at the desire of FERGUSON that DAVID HUME, a few days after the date of this letter, was induced to undertake a journey to London, to try the effect of change of air in mitigating the severity of his disease. FERGUSON had also written to their mutual friend ADAM SMITH, giving him an account of HUME's critical state at this time; and thus describes his condition —"DAVID, I am afraid, loses ground. He is cheerful, and in good spirits as usual; but I confess that my hopes from the effects of the turn of the season towards spring have very much abated." In consequence of this letter, SMITH and JOHN HOME set out from London to visit HUME at Edinburgh, and accidentally met him at Morpeth on his way south. HOME returned to London with HUME, and preserved a diary of the journey, which has been printed in his life, by MACKENZIE. In this diary is the following interesting entry:—

"*Newcastle, Wednesday*, 24*th April.*

"Mr HUME not quite so well in the morning,—says that he had set out merely to please his friends; that he would go on to please them; that FERGUSON and ANDREW STUART (about whom we had been talking) were answerable for shortening his life one week a-piece: for, says he, you will allow Xenophon to be good authority; and he lays it down, that suppose a man is dying, nobody has a right to kill him. He set out in this vein, and continued all the stage in his cheerful and talking humour. It was a fine day, and we went on to Durham—from that to Darlington, where we passed the night."

The illness of HUME, feelingly alluded to in the above letters of GIBBON and FERGUSON, was the cause of his death on the 25th of August in the same year. The following interesting letter (belonging to Mr DAVID LAING), dated at Edinburgh, on the 9th of July before his decease, is very characteristic of the cheerfulness which he displayed up to his last moments. It is addressed to "JOHN HUME at Kilduff, near Haddington:"—

"MY DEAR JOHN,—I offered to give you a letter along with you, informing you how I should be on Tuesday thereafter, viz., weaker and more infirm than when you saw me. This, indeed, would have sav'd postage; and I can do no more at present than confirm the same truth, only that the matter seems now to proceed with an accelerated motion. I had yesterday a grand jury of physicians who sat upon me, the Doctors CULLEN, BLACK, and HOME. They all declare the opinion of

"I am very glad that the pleasure you give us recoils a little on yourself, through our feeble testimony. I have, as you suppose, been employed, at any intervals of leisure or rest I have had for some years, in taking notes or collecting materials for a history of the destruction that broke down the Roman Republic, and ended in the establishment of Augustus and his immediate successors. The compliment you are pleased to pay, I cannot accept of, even to my subject. Your subject now appears with advantages it was not supposed to have had, and I suspect that the magnificence of the mouldering ruin will appear more striking than the same building, when the view is perplexed with scaffolding, workmen, and disorderly lodgers, and the ear is stunned with the noise of destruction, and repairs, and the alarms of fire. The night which you begin to describe is solemn, and there are gleams of light superior to what is to be found in any other time. I comfort myself, that as my trade is the study of human nature, I could not fix on a more interesting source of it than the end of the Roman Republic. Whether my compilations should ever deserve the attention of any one beside myself, must remain to be determined after they are farther advanced. I take the liberty to trouble you with the enclosed for Mr SMITH,* whose uncertain stay in London makes me at a loss how to direct for him. You have both such reason to be pleased with the world just now, that I hope you are pleased with each other. I am, with the greatest respect, dear Sir, your most obedient and humble servant, ADAM FERGUSON."†

The progress of his labours, in collecting materials for the History of Rome, was, however, interrupted by circumstances which turned his attention for a time to other inquiries.

The Revolution in America had now drawn more general attention to the affairs which were passing on that great continent. It is unnecessary here to relate the different steps which led to the institution of the American Congress in 1773. It is sufficient to remark that the Congress had, in 1776, assumed the functions of sovereignty, and required all persons to abjure the British Government, and swear allegiance to the Congress itself.‡

the English physicians absurd and erroneous. They own a small tumour in my liver; but so small and trivial, that it never could do me any material injury; and they say that I might have liv'd twenty years with it, and never have felt any inconvenience from it; each of them has had patients who have had tumours in that part ten times larger without almost complaining for years together. They have thoroughly persuaded me to be of their opinion; and, according to their united sentiments, my distemper is now a hæmorrhage as before, which is an illness that I had as lief dye of as any other. The first part of the text being now discuss'd, we proceed to the second, viz., the cure, which I leave to another opportunity. I send you a letter which my nephew opened by mistake: but finding, after he had read a few lines, that it was not meant for him, he proceeded no further. Yours sincerely, DAVID HUME."

In token of the long friendship which had existed between HUME and FERGUSON, HUME bequeathed him a legacy of L.200.

* Dr ADAM SMITH.

† Gibbon's Misc. Works. By Lord Sheffield, vol. ii. p. 501.

‡ FERGUSON was in the habit of discussing from time to time in his correspondence with General CLERK, Mr JOHNSTONE (afterwards Sir WILLIAM PULTENEY), and other friends, the various

The endeavours of the Americans to throw off the yoke of the British Government, and to assert their independence, were warmly defended by Dr RICHARD PRICE, a dissenting clergyman in London, well known as the ingenious author of a 'Review of the Principal Question in Morals,' and of some works relating to the theory of annuities, and the finances of the country. PRICE had, in 1775, published his 'Observations on the Nature of Civil Liberty, the Principles of Government, and the Justice and Policy of the War with America.'

This work took up the ground, that from the nature of civil liberty one country could have no power over the property or legislation of another which was not incorporated with it by a just and adequate representation. It drew, in contrast to this country, the most flattering picture of America, where, as its author observes, "we see a number of rising states, in the vigour of youth, inspired by the noblest of all sentiments, the passion for being free, and animated by piety—*Here* we see an old state, great indeed, but inflated and irreligious, enervated by luxury, encumbered with debts, and hanging by a thread. Can any one look without pain to the issue? May we not expect calamities that shall recover to reflection (perhaps to devotion) our libertines and atheists?"* It concluded by prophesying ruin to England, through the addition of many millions to the national debt, unless some plan of reconciliation were speedily to be carried out.

The publication of these views, which had the greater weight from their author's reputation as a sound financial writer, created an immense sensation both in England and America. In the course of a few months 60,000 copies of this book were disposed of; and while PRICE was lauded by the friends of American freedom, he was subjected to abuse and misrepresentation by those who supported measures of repression.

Along with other writers of note, FERGUSON sympathised in his views with Government, and he communicated his objections to the pamphlet of PRICE in a letter to Mr GREY COOPER, one of the Secretaries of the Treasury.

Mr COOPER was so much pleased with the observations of FERGUSON, that he sent the following letter in acknowledgment:—

"*Parliament Street, March* 23, 1776.

"SIR,—It was my duty to have thanked you sooner for your letter, and the very masterly and judicious paper which accompanied it, and which I have read

political changes which were taking place at this period. The following extract from a letter addressed by General CLERK to him when he was at Geneva, in 1775, with Lord CHESTERFIELD, is interesting in connection with recent events in America. The General says: "When I saw you at Paris, you said that the American Colonies would end in military governments. You astonished me, and though I contradicted you, I had not patience to discuss it at that time, as it required the clearing up of so many points of which you and I had different opinions. However, I never doubted of its being a very disagreeable affair for us, and I think now that it has the appearance of being as bad as ever I imagined it."—*MSS. University of Edinburgh.*

* 4th ed. p. 98.

with great attention and pleasure. Dr PRICE's pamphlet has been circulated with the same zeal that the Methodists circulate their manuals and practices of piety. Like base coin struck in times of disorder and confusion, it has had a value and a currency in the world which no other times could have given it. In that respect he deserves and demands what neither the weight of his arguments or the accuracy of his knowledge entitle him to expect—an answer from a good and able writer. I have ordered the observations to be printed by Mr STRAHAN, without its being known who is the author of them. I am happy of having this opportunity of corresponding with Professor FERGUSON; and if *idem sentire de republica* be the basis of friendship, I can very fairly pretend to yours; for I entirely concur with you in your noble sentiment, that the great object is to lay the demon of discord on both sides of the ocean; and I am, dear Sir, with great regard and esteem, your very faithful, humble servant, GREY COOPER."*

The reply of FERGUSON was accordingly published anonymously as *Remarks on a Pamphlet lately published by Dr Price, intitled Observations on Civil Liberty*, &c.; and was acknowledged to be written with less invective and with more moderation than the publications previously issued on that side of the American question. FERGUSON contended that, although the Colonies were by their charters and original compacts bound to submit to Parliamentary taxation, their altered circumstances now required a change of policy; and suggested that, as Commissioners were to be appointed to settle all differences, negotiation should speedily take place. He was led, however, into various positions of a questionable nature, that weakened the effect which his conciliatory views would otherwise have had upon the public mind.

The British Government, which had at first treated the disputes in America with contempt, now began to take measures to vindicate their authority, and sent reinforcements to their army in that country. At the same time they appointed General HOWE and his brother, Lord HOWE, Commissioners, to settle all disputes in an amicable manner, as the feeling indicated in FERGUSON's pamphlet began to gain ground, that measures of conciliation should be attempted.

The Americans, however, flushed with several advantages gained over the British troops and by the promise of assistance from France, were determined that no proposal for reconciliation should be entertained except upon the footing of a treaty between two independent powers.

In 1778, GEORGE III., who throughout the whole of the American disputes had inflexibly opposed pacific measures, began, when too late, to yield to a more liberal policy. In that year two bills for effecting a reconciliation with America were introduced into Parliament by Lord NORTH. Commissioners were again to be sent over to treat with the Congress; and as it had been objected

* MSS. University of Edinburgh.

that the powers of the former commissioners had been unduly restricted, the new commissioners were expressly authorised to discuss and settle every point in dispute between Great Britain and her colonies.

The commissioners were the Earl of CARLISLE; Mr EDEN, one of the Commissioners of Trade, and Under-Secretary to Lord SUFFOLK; and GEORGE JOHNSTONE, originally a captain in the navy, and at one time Governor of West Florida. With these three commissioners were conjoined Lord HOWE, and his brother General Sir WILLIAM HOWE, the members of the commission formerly appointed.

The three newly appointed commissioners met at Portsmouth in April 1778, and proceeded to open their instructions, after which they embarked at Spithead on board the Trident, and arrived at Philadelphia, on the 5th of June 1778. The appointment of a Secretary was one of their first acts on reaching America. They had expected that Mr HENRY STRACHEY, the Secretary to the former commissioners, would continue his services to the new commission, but they found that as he had already returned to England a new appointment was necessary. By virtue of their powers they elected FERGUSON as their Secretary on the 6th of June, having special confidence in his ability for discharging the difficult and delicate matters intrusted to them.

The commissioners, on proceeding to business, found many unforeseen circumstances of discouragement in an undertaking which had never been very hopeful. In consequence of the expected war with France, orders had been sent from England in March for the British troops to evacuate Philadelphia and retire to New York, and these orders, of which no previous intimation had been made to the commissioners, were in process of execution when they landed. The treaty between the Colonies and France, concluded by FRANKLIN on the 6th of February, had also arrived in America, and was the occasion of great rejoicing to the American people.

Nothing daunted by these untoward symptoms, the commissioners proceeded to open negotiations with General WASHINGTON and the Congress. They intimated to the former that it was their intention to send FERGUSON with despatches to Congress, and requested that he might receive the necessary passport for that purpose. They then drew up a letter to that body, in which they stated their powers, and expressed their desire to concur in every just arrangement for the cessation of hostilities and the restoration of free intercourse between Britain and the Colonies. This letter was ordered to be delivered to Congress by FERGUSON in person.

On reaching the outposts of the American army with this letter, FERGUSON was met by the officer commanding the piquets, who informed him that he could not be allowed to proceed to headquarters without a passport, and that the application for this document previously made could not be granted until the pleasure of Congress was known. In order, however, that the object of the com-

missioners might not suffer from unnecessary delay, it was determined to send the letter by the ordinary conveyance of the military posts, and it was accordingly delivered on the same day to the American piquets by Lord CATHCART.

The commissioners, while awaiting with considerable anxiety the reply of Congress to their conciliatory letter, in which they proposed concessions of the most liberal nature, gave an account of their proceedings to Lord GEORGE GERMAIN, Secretary of State for the American Department. In this letter they plainly informed Government of the difficult position in which they were placed by the unfortunate order for the evacuation of Philadelphia. They also admitted that, in consequence of the state in which they found the country, they had offered terms to the Americans of a more liberal nature than their instructions allowed.

These papers of the commissioners caused some dissatisfaction to the ministry, and were not at the time made public.

The following letter, addressed by Sir WILLIAM PULTENEY (brother of GEORGE JOHNSTONE) to FERGUSON, is interesting, as showing the state of feeling in England with reference to these proceedings of the commissioners:—

"*London, 4th August* 1778.

DEAR FERGUSON,—I was much obliged to you for your letter of the 19th June, which arrived a fortnight ago, and was delivered by Mr MACKENZIE. I enter entirely into your sentiments, and those of my brother, concerning the unfortunate order of the 24th March. I have done all I can in consequence of the despatches I have received, and I have hopes that I have not laboured in vain. I have wrote a long letter to my brother, which will give you all the information that seems to me material. Firmness, wisdom, and exertion were never more wanted for any country than now. I approve much of the letter to Government, and the letter to the Congress, and I believe they will meet with general approbation, though the ministers do not, I guess, relish the first, and neither have been given to the public.—I am, dear FERGUSON, most affectionately yours,

"WILLIAM PULTENEY.

"I think it right to suggest to your private ear an observation or two. Though I am not surprised at the heat with which the commissioners took up the concealment of the order and the order itself, yet I have my doubts whether it was prudent to let it transpire in America that they disapproved of the measure, or that they were ignorant of it till they arrived. I can see many advantages which might have resulted from their appearing satisfied, but none from the contrary. It is true, the misery of the departed inhabitants and their complaints must have made it next to impossible for the commissioners not to vindicate themselves from having had any hand in the measure; but I think it right to make this observation with a view to the future.

"I also think it would have been as well if the opinions of the commissioners had been communicated by letter to fewer persons here, because I think it was a piece of knowledge which ought to have been withheld from the American Deputies at Paris, and the Court of France. By communicating only to a few proper persons, every good end of this communication might, I think, have been attained without the disadvantages. I make this observation with a view to the future.

"I have some reason to think that Dr FRANKLIN has acted a double part. From some facts I have heard, I suspect, that notwithstanding his solemn promise to me that no use should be made of what passed between us, he did from the first make use of it to urge the French Court to a further immediate treaty, to be put over and to be ratified before the commissioners should arrive, from a fear that the Americans would certainly accept our terms. The date of the last treaty will throw light upon this, when compared with the dates of my conversations with him. He was told of my arrival in Paris, and my errand on Thursday the 11th March. I saw him first on Saturday the 13th, and again on Sunday the 14th. The declaration of the French ambassador here was made on Friday the 12th. I saw him again on Sunday the 29th, and Monday the 30th, and for the last time on Saturday the 5th April.

"I am informed by ANDREW STEWART, that DA. HUME told him the following remarkable fact:—HUME went to visit Mr OSWALD of Dunnikier, then, I believe, a Lord of Trade, soon after Dr FRANKLIN came to England, which was in 1758; and as he entered the room Dr FRANKLIN was coming out. HUME took notice that FRANKLIN, who was just gone, was a very ingenious man. OSWALD said he had been with him on business relating to the Colonies, and added these remarkable words,—'He is certainly a man of genius; but if I am not much mistaken in characters, that man has more of faction in his mind than is sufficient to embroil any country in the world.'" *

The commissioners, after despatching the letters above referred to, and feeling discouraged by the effects of the order for the evacuation of Philadelphia, re-embarked on board the vessel which had carried them to America, and set sail for New York.

While in that city they received a communication from Congress, intimating that the only ground on which they could enter on a treaty would be an acknowledgment of the independence of the States, and the withdrawal of the British force from America.

The commissioners then issued a proclamation, calling upon all persons in America to aid them in bringing the unhappy quarrel to a speedy termination.

After some correspondence with the Congress relative to the performance of

* MSS. University, Edinburgh.

the stipulations contained in the convention of Saratoga, the commissioners found at length that the decision of the American disputes was to be left to the sword. They accordingly set sail from America about the end of November, and reached Plymouth on the 19th December 1778. The time for which they had been appointed expired on the 1st of June 1779, when they formally demitted office.

They had the honour to receive a formal intimation of the royal approbation of their services, through Lord GEORGE GERMAIN, who also expressed his regret that his correspondence with them, from which he had received so much information, had come to a conclusion.

On his return to Edinburgh in 1779, FERGUSON resumed the charge of his class, which had been conducted during his absence by Mr DUGALD STEWART, and continued the preparation of his 'Roman History.' But before that work made its appearance, a serious illness befell its author. Towards the end of 1780, FERGUSON had an attack of paralysis, probably occasioned by his free manner of living. His recovery from this illness is still quoted by medical authors as one of the most remarkable on record.

Under the treatment of his distinguished relative Dr BLACK,* the symptoms gradually became more favourable, and FERGUSON was able, after some months, to undertake a journey to Bath. But he did not receive so much benefit from the use of the waters there as from the Pythagorean course of diet which he adopted, and which brought about a complete restoration. During the long period of thirty-six years that elapsed between his paralytic attack and his death, FERGUSON enjoyed remarkably good health. The occasional ailments he had seem to have been in no way connected with the disorder from which he made so wonderful a recovery.

Of his many sympathising friends, no one was more sincere than Sir JOHN M'PHERSON, now about to proceed to India as a member of the Supreme Council. The following letter expresses his feelings on this occasion:—

"*Kensington Gore*, 13*th January* 1781.

"MY DEAR FRIEND,—Though your illness has not filled me with despondency, the first reports I had of it took away the happiness I should naturally have had in announcing to you my India appointment. The truth is, I was so little disposed to mention that event to any of my friends in Scotland—while I under-

* The resemblance between this case and the attack which ultimately proved fatal to M. DE SAUSSURE in 1799 rendered that eminent French philosopher anxious to learn the mode of treatment employed by Dr BLACK, under which FERGUSON had recovered. DE SAUSSURE's physician, Dr ODIER, accordingly requested Dr MARCET, then a student at the University of Edinburgh, to obtain from Dr BLACK the desired information. Dr MARCET, accompanied by Professor DUGALD STEWART, waited on Dr BLACK, who, after a long and interesting conversation, delivered to him, in writing, for transmission to DE SAUSSURE, an account of the case and its treatment, which has been printed in the 'Medico-Chirurgical Transactions' (vol. vii. p. 230), and is the more interesting, as it is, perhaps, the only existing memorial of the medical practice of that distinguished chemist.

stood you were in a situation not to communicate it to them first—that I never wrote to any person here of it; and the only correspondence I have had with your capital of late, is an answer which I thought myself obliged to send to the Duke of GORDON. I have likewise written to the Duchess this night.

"Dr CARLYLE'S letter of to-day has set my mind more at ease. You have naturally a good constitution; and I place every hope in your Highland stamina, your philosophy, and knowledge of nature.

"My friend CARLYLE has written me, with an interest in your welfare, and all that belongs to you, that adds, if possible, to my attachment to him. There is a circumstance which, with all his love for you and me, he is not fully known to—it is that I met you when I lost my father, and that your children and I are of but one family.—Farewell. May the power of affection be a power to give health and happiness! If you do not recover your health before I leave this country, I leave it with half my spring of satisfaction and soul.—Yours ever,

JOHN M'PHERSON." *

The intimate friendship between FERGUSON and Sir JOHN M'PHERSON has already been mentioned. It began in 1763, when the Honourable CHARLES and ROBERT GREVILLE, sons of the Earl of WARWICK, were attending the University. Sir JOHN was son of the minister of Sleat, in Skye, and, when a student, had been private tutor to these young noblemen while they lived under FERGUSON'S care.

A circumstance occurred at that time (1765) which singularly enough gave rise to a controversy in 1781 between FERGUSON and the celebrated Dr PERCY, Dean of Carlisle, afterwards Bishop of Dromore. The occasion of this controversy was the keen discussion regarding the authenticity of the poems of Ossian, in which the most eminent literary men were at this time engaged.

As is well known, JAMES M'PHERSON, the translator of "Ossian," first published his "Fragments of Ancient Poetry" in 1760. The work was anonymous; but as it professed to give a specimen of a great amount of ancient Celtic poetry still existing in the mainland and isles of Scotland, it was received with the utmost enthusiasm. M'PHERSON made a tour to obtain further materials, after which he gave to the world 'Fingal,' an ancient epic poem in six books; shortly afterwards followed by 'Temora,' in eight books, with other poems of 'Ossian.'

These productions caused an immense sensation, and were translated into several European languages, while M'PHERSON was hailed as the preserver of these relics of ancient culture. A few years later, however, a suspicion began to be entertained that these poems were not authentic, and their genuineness became the subject of as warm a controversy as ever was waged in the annals of literature.

* MSS. University of Edinburgh.

From JAMES M'PHERSON's friendship with FERGUSON, BLAIR, and Principal ROBERTSON, and from the high approval which these eminent men bestowed on his labours, they were subjected along with M'PHERSON to various attacks and misrepresentations. In particular, FERGUSON and BLAIR were, in 1781, charged by Dr PERCY with having perpetrated upon him a practical joke relative to the poems of Ossian, when he visited Dr BLAIR at Edinburgh in 1765.

The immediate cause of this matter being revived so long after the time when it happened was the publication, in 1781, of 'An Enquiry into the Authenticity of the Poems ascribed to Ossian,' by WILLIAM SHAW, the author of various Gaelic works.

From the several letters written on the occasion, we learn that in October 1765 Dr PERCY, when travelling in Scotland, had been for a few days the guest of Dr BLAIR, at the time an enthusiastic admirer of the Ossianic poems. BLAIR, according to PERCY's statement, carried him to FERGUSON's house, that he might have an opportunity of hearing some of the original Gaelic of the poem of Fingal recited to him by a native of the Highlands. After the recitation took place, Dr PERCY was called upon by BLAIR to mention in print this circumstance, as a proof of the genuineness of the Gaelic poetry of Scotland. The Doctor, who was then preparing for the press the second edition of his famous 'Reliques of English Poetry,' accordingly inserted the following paragraph:—"Concerning the bards of Gaul . . . no remains of their poetry are now extant; but as for those of Britain and Ireland, they have been more fortunate. . . . For an account of the Irish bards, the curious reader may consult O'CONNOR's 'Dissertations on the History of Ireland,' Dublin, 1776; SPENCER's 'View of the State of Ireland,' &c. &c. But no pieces of their poetry have been translated, unless their claim may be allowed to those beautiful pieces of Erse poetry which were lately given to the world in an English dress by Mr M'PHERSON; several fragments of which the editor of this book has heard sung in the original language, and translated *vivâ voce* by a native of the Highlands, who had at the time no opportunity of consulting Mr M'PHERSON's book."*

In 1781, when the controversy regarding the genuineness of Ossian was at its height, this statement of Dr PERCY was adduced in favour of M'PHERSON, by the Rev. JOHN SMITH, minister of Kilbrandon, in his 'Gallic Antiquities,' a work which contained, among other matters, a 'Dissertation on the authenticity of the Poems of Ossian.' In this dissertation the author stated, that "amidst the general wreck to which our traditions and poems have fallen for some time back, many pieces of Ossian are still remaining, and are found to correspond with the translation. A Highlander may perhaps be suspected of partiality in making this assertion; but several gentlemen of candour from other countries have made

* "Reliques," 2d ed.; vol. i. p. 45.

the experiment, by causing such as had never had any access to see the translation, to give the meaning of those pieces which they repeated; and they declare that, on comparing the Gaelic and the English, they were entirely satisfied with the justness of the translation. Mr PERCY, in his preface to 'Reliques of old English Poetry,' tells, that he himself had often done this, and found the interpretation which he had got *extempore* correspond with the English translation, with which they had no access to be acquainted. Either these persons were inspired, or Ossian's Poems are authentic."*

In the bitter attack on M'PHERSON by SHAW, that writer refers to this statement of SMITH as follows;—"Mr SMITH mentions Dr PERCY's '*Reliques of Ancient Poetry*,' in which he says the Doctor confesseth, that he himself heard pieces of it recited; and being compared with the *translation*, exactly corresponded. Dr PERCY does not understand a syllable of the Erse, and therefore could be no judge. The truth is, Dr BLAIR and Professor FERGUSON, when Dr PERCY was at Edinburgh, took care to introduce a young student from the Highlands, who repeated some verses, of which Professor FERGUSON said, such and such sentences in Fingal were the *translation*. Mr SMITH, if he looks into the second and third editions of the 'Reliques,' will find the observations there no longer; and that Dr PERCY, on reflection, had just reason to suspect that this young student had previously been taught the part he recited, and the lines might as readily be any common song as the original of Fingal; for they knew it was impossible for an Englishman to detect it."†

This treatise gave FERGUSON some annoyance, and on the 21st of July 1781, he gave a formal denial to SHAW's statement, in the public prints.

In his advertisement, he quoted the passage from SHAW, and added, "to prevent any inferences which might be drawn from my silence, I think it material to declare that the above passage, so far as it relates to me, is altogether false; and that I never was present at the repetition of verses to Dr PERCY by a young student from the Highlands."

When the pamphlet of SHAW and the advertisement of FERGUSON were brought under the notice of Dr PERCY, he also wished to vindicate himself. Being, however, at a distance from his papers, he could only trust to his recollection at the time, and wrote the following letter to Dr BLAIR, enclosing the draft of an advertisement which he proposed to insert in the newspapers.

"*Alnwick Castle*, 17*th August* 1781.

"DEAR SIR,—I have at length gained a few moments of leisure, and will now endeavour to give you a full and true account of what may have occasioned the indecent liberty which has been taken with our names in the pamphlet you mention.

"In autumn 1765 I spent a week with you most agreeably at Edinburgh,

* Gallic Antiquities, p. 96. † Shaw's Inquiry, p. 25

when, among other kind instances of your friendship, you introduced me to many worthy and ingenious men, and among the rest to Mr Professor FERGUSON. I believe you mentioned to him, that I had entertained doubts of the authenticity of Ossian's Poems, to remove which, he sent for a student that was a native of the Highlands, who told me he had heard lines of the original sung by the servants and country people there; and being asked if he could repeat any lines himself, he recited some passages in Earse, which being then translated to me, contained part of the description of Fingal's Chariot (a part of the poem of which I had entertained the greatest doubt). You then desired me, in a future edition of my 'Reliques of Ancient Poetry,' &c., to testify what I had heard. To this I could not reasonably object, and accordingly, in my second edition, 1767, I related in a note what had occurred. Some years after, I became acquainted with a gentleman, who is also intimately so with Mr M'PHERSON; but whose name I will now never mention, because I will not expose him to the inconvenience of being dragged before the public, as I have unfortunately been myself. This gentleman, in the most solemn manner, assured me (as one perfectly well informed) that the Poems of Ossian were almost all the productions of Mr M'PHERSON's own genius; that what was really original hardly exceeded in quantity our ballad of Chevy Chase. When I urged to him the transaction, at which I myself had been present, he assured me I had been imposed on, and advised me to suppress the note in my next edition, which accordingly I did in my third impression in 1775, silently and quietly, never intending to enter at all into the controversy concerning the genuineness of Ossian's Poems, of which I was so incompetent a judge from my utter ignorance of the Earse language.

"From the positive repeated testimony above mentioned, together with some other observations, which I occasionally made myself, I own I began to believe them to be *modern*, but no *less brilliant*, proofs of Scottish genius, equally tending to do honour to the country that gave birth to their author. But as I never intended to publish one word on the subject, I fondly hoped I might have gone out of the world without having my name ever mentioned in the controversy.

"This, however, was unluckily not to be my fate, for Mr SHAW having called on me just before he set out for the Highlands, when he assured me he would inquire with the utmost impartiality into the genuineness of the poetry attributed to Ossian. To him I unreservedly expressed my sentiments on that subject, without concealing anything I knew or believed concerning it, not intending to influence his opinion (which would have been absurd in one who knew so little of the matter), but to spur his diligence to remove my objections.

"I accordingly related the transaction at which I had been present, and the positive assurances I had since received from Mr M'PHERSON's friend, that I must have been deceived. I also urged the suspicious circumstance of the wolf being

never mentioned, or alluded to in these volumes. But this was not my own observation, it had before occurred to others;* only that I have occasionally in my researches found abundant proof that the wolf existed in England long after the Norman Conquest, and much later in Scotland; and in Anglo-Saxon poetry, I find such reference to the wolf, as would naturally happen in a country where it abounded, and was the only animal of terror.

"Little did I imagine that this writer would quote my name at length, and assign whole sentences to me, without ever asking my consent, or allowing me to revise what I might have inaccurately let fall in conversation. Yet this he has done, and till I saw his pamphlet in print, I never knew or suspected that I was to make such a figure in it.

"Thus the matter stands: I never in my life had the most distant suspicion that you were privy to the imposition, if it was one; and as for Mr FERGUSON, he may also have been free from any share of the deception, which may have originated only from the reciter himself; but the lines were certainly recited in his presence, as I perfectly well remember, although he may have entirely forgot the occurrence.

"Thus far I had written, before I saw the advertisement published by Mr FERGUSON. As he hath *committed* himself to me, I am now compelled to give my testimony to the public. I perfectly well remember the transaction, though Mr FERGUSON may have forgot it; we may both be sincere, though my recollection may, in this case, have been better than his.

"As I have been unwillingly forced into this controversy, I shall desire to get out of it as soon as I can; and if not again attacked, it is not my intention to push the matter any farther. Upon the whole, I hope you will think the advertisement which follows is written with temper and decency, and such as may have a tendency to compose the dispute, so far as I am engaged in it.

"As for yourself, my good friend, I hope it will make no breach in our friendship. I know your generous and enlarged heart can extend its regard to persons who may differ from you in points of the most sacred importance; even (as our Liturgy expresses it) to all Jews, Turks, Infidels, and Heretics. And though, on this question, I may have the misfortune to be one of the *latter*, yet I hope you will still allow me to subscribe myself, your ever affectionate friend,

"THOMAS PERCY."

"*P.S.*—Pray write to me without delay, under cover, to his Grace the Duke of NORTHUMBERLAND, at Alnwick Castle, in Northumberland. Mrs PERCY joins with me in respects to you and Mrs BLAIR.

"*P.S.*—I have some notion that the student who was produced to me by Mr FERGUSON was (the Indian) Mr MACPHERSON, then (I believe) his pupil. Perhaps

* See Johnson's Life by BOSWELL, vol. ii. page 303.

this circumstance may serve and awaken the recollection of you both. Pray, inform me how Mr or Dr FERGUSON is usually styled. Is he called Mr or Dr?*

Dr PERCY, after some time had elapsed, began to think that he might possibly be mistaken on some points, and wrote the following letter to Dr BLAIR, after consulting the memoranda which he had made, when on his visit to Scotland:—

"*Alnwick Castle, September* 10, 1781.

"DEAR SIR,—You will excuse my having remained so long silent since I was favoured with your last (inclosing the polite letter from Mr Professor FERGUSON), when I inform you, that I delayed my answer till I could send into Northamptonshire, to have my papers there searched for minutes, which I remembered to have made, of some of the particulars that occurred to me during my short visit to Edinburgh in 1765; for, as I have the misfortune to differ about a matter of fact from a gentleman of so respectable a character as Mr Professor FERGUSON, I thought it would not be treating him with due regard, to neglect any means of information that could contribute to settle the point between us. After all, I think he would have recollected the recital made to me by the student, as well as he has done some of the other circumstances, at least he would not have been so positive on this head as he appears to be in his letter, if you had reminded him that the student produced to me was his own pupil, Mr MACPHERSON, who, I believe, then boarded with him in his house.

"I have, however, recovered a pocket-book, in which I had written down minutes at the time, expressing how and where I spent every day during my short stay at Edinburgh in 1765, where I was only five days; for I arrived there from Stirling on Tuesday, October 8th, and departed thence for England early on Monday morning, October 13th.

"Now by these minutes I find, that on Wednesday, October 9th, Mr FERGUSON dined with me at your house; and on the Sunday following, October 13th, after evening service (wherein I well remember to have heard a most eloquent sermon from you), you caused me to drink tea with Mr FERGUSON.

"At his house it was, during that visit, that Mr FERGUSON, I believe, gave me the written specimens of Earse poetry, which he mentions; but very certain it is, that then and there the student was produced to me, who recited *vivâ voce* the passages in Earse, as I have related in my former letter. To which I can now add this farther circumstance, that it being Sunday, he could not decently sing the tune, which I had a great curiosity to hear; and as I was obliged to leave Edinburgh early the next morning, and was not likely to see him again, he in the evening, as we were going away, took me aside, and in a low voice, hummed a few notes to me, as a specimen of the old Highland tune.

"This having been the case I can have made no jumble, as Mr FERGUSON is pleased to suppose, nor could I possibly confound this with any other occurence, for

* MSS. University of Edinburgh.

I not only never heard the sound of the Highland language from any other persons but Mr FERGUSON and his pupil, during my stay in Edinburgh; but I do not find that I was ever there in company with any other natives of the Highlands but themselves; and for this I can appeal to my memorandum-book, which mentions persons (and there were many very worthy and ingenious ones) to whom you then introduced me; and also how and where I spent the whole five days among them.

"I might even proceed further and aver, that I never heard the recital of Earse poetry, either before or since, in my whole life; but that I now recollect, I once heard a short song or two from an old Highland soldier, who, travelling home to Scotland, begged at my door, but who I could not find knew anything of the subjects of Ossian's Epic poetry. On his account I shall suppress the circumstance of my never having heard the sound of Earse poetry, except at that single recital of the student, and, in my intended advertisement, which I shall also, in other respects, shorten as much as possible, for I heartily wish to rid my hands of this foolish business; and unless Mr FERGUSON is more desirous of committing his name in print than I am, our controversy shall soon be at an end; for I shall only attribute to him a want of recollection, which surely might happen to the best memory at so great a distance of time.

"In truth, I cannot but think Mr FERGUSON'S name too respectable to deserve to be tacked to slight appeals and rejoinders in the common newspapers; and I must acknowledge I have some reverence for my own; and, therefore, when I have once supported my own veracity in as few words as possible, I hope the matter will drop, and neither of us be ever mentioned more on this subject. But if he still persists in denying publicly the existence of a recital, which at your desire I once mentioned in print (though, upon since reflecting how little I knew of the matter, and for other reasons assigned in my former letter, I have since suppressed it), I must then be compelled, much against my will, to produce at large necessary proofs in support of my own affirmation, which yet, I trust, I shall with temper and decency, and still continue to approve myself—Dear Sir, your affectionate friend and very faithful servant,

"THOS. PERCY."

"*P.S.*—My Lord ALGERNON PERCY, who has been here since I wrote to you last, but who is since gone away, could not distinctly remember, as I had at first understood him, that Mr FERGUSON was present when Mr MACPHERSON repeated the Earse poetry to me; but he remembered that fact better than could have been expected, after six years' interval, considering too, he was but a boy when it happened.

"The Duke desires his compliments, and pray deliver mine in the kindest manner to Mrs BLAIR. I am now removing to Carlisle, where I hope to receive your next favour." *

* MSS., University of Edinburgh.

This letter was sent to FERGUSON, who wrote the following answer to Dr BLAIR :—

"*Edinburgh*, 15*th September*, 1781.

"DEAR SIR,—I return Dr PERCY's letter of the 10th inst., on this disagreeable subject, of the recital of Erse poetry. I am sorry he has had so much trouble; but cannot blame myself, as I am satisfied the trouble did not originate with me. I have in what is past, and shall continue in what may follow, to confine myself strictly to what is necessary in my own defence. I found it alledged in print, that Dr PERCY had a cheat put upon him when at Edinburgh, to which I was accessory. In such cases it is often argued that until such or such assertions be contradicted they must be supposed true; and I did not choose that my character should rest upon that footing. I was free to deny any concurrence in the cheat, and even free to deny my having ever been present at any such scene as that in which the cheat was said to be practised. With respect to the last point, indeed, it may be thought that I could speak only negatively, and deny my having any memory of the transaction; and so it is no doubt of all past transactions. But there are circumstances which entitle a person to be more or less positive. In this case the cheat that is said to have been put upon Dr PERCY could not be practised in my presence without my concurrence; and this every feeling of my mind warrants me in denying in the most positive terms. As I never questioned the fidelity of Mr JAMES M'PHERSON in his publications, I was none of those who busied themselves in finding evidence of it. It has happened to me, indeed, to mention a very few particulars of Erse poetry that were known to myself; and from my knowledge of which I had taken a very early impression of what mere genius, without the aid of literature or foreign models, may do where the human mind is free and the passions have scope in recital as well as in action. I imagined that evidence of its power might have been found in every country if collected before language and manners had so far changed as to obliterate or efface its productions. There being any remnants of it in the Highlands of Scotland, I imputed to the manners and language having changed less than they have done elsewhere in equal periods of time. Whether or no this be honourable for the people I will not at present try to determine. It appeared to me matter of some curiosity in the history of mankind, but very little as matter of vanity to one corner of this island, much less of jealousy to any other corner of it. The scraps I showed to Dr PERCY had a reference to this idea, not the fidelity of Mr M'PHERSON's publications. And I was surprised to find myself, contrary to the general tenor of my feelings, stated as a fabricator of evidence on that subject. I thought myself free to deny in very positive terms my having ever been present at the repetition of verses to Dr PERCY by a student from the Highlands; because I never knew a student who pretended to repeat any part or specimen of Ossian's heroic poetry. And the mention of Mr JOHN M'PHERSON's name does not at all

alter the state of my recollection, for my memory of him is, among other particulars in which he is well known to me, that he never appeared to be in possession of any part of Ossian's poetry. I well remember that he was in some degree a singer, though I do not recollect any particular song but one, which, with a very few words of any meaning, consisted chiefly of a chorus or burden, not more significant than lullabolaro or derry down. If he repeated this or any other song that Dr PERCY might hear the sound of the language, it is no wonder that I should forget that circumstance, especially as I have totally forgot Dr PERCY's visit with you at my house. But I hope that Dr PERCY, now he has seen his minutes, will be sensible that a person may mistake what he thinks he remembers, as well as another may forget what has really passed. What he wrote from his memory in a former letter was, that I had sent for a student to your house. What he writes now is, that he came to the student at my house. Some other very easy mistake in the circumstances, if recollected, might acquit me entirely of any share in the imposition that was put upon Dr PERCY. I confess that I was astonished at the ease with which this charge was stated against me in the pamphlet which has given rise to this correspondence. If I had the honour of being sufficiently known to Dr PERCY, I should certainly request that he would compare probabilities, and consider which is most likely, that I should be accessory to a cheat, or that he should mistake some material circumstance of a story sixteen years old. Although I may not be entitled to employ this plea with Dr PERCY, I certainly must be allowed to submit it, in case I am under a necessity of more publications, to persons to whom I am better known. There is certainly hitherto no reason to apprehend from me, as Dr PERCY mentions, any improper desire of committing my name in print. I appeared, from necessity, to prevent inferences which might be drawn from my silence against me. I do not pretend to set up my affirmation against that of any other person; but as often as occasion is given to the same inference, I must appear again to the same purpose. Dr PERCY is pleased to say in the letter which I return to you, that if I persist in denying publicly the existence of a recital, &c., he must then be compelled, much against his will, to produce at large necessary proofs in support of his own affirmation. Dr PERCY will be pleased to observe, that I do not pretend to know what recitals he may have had made to him. I only deny that I ever was present at any imposition put upon Dr PERCY by any pretended recital of Erse, and that I ever was present at any such recital. I am persuaded that there are no proofs to the contrary, of which Dr PERCY will not perceive the weakness the more he considers them. At any rate, he must be sensible that if any such proofs are supposed, I cannot possibly consent to have them secreted. When they appear, I hope that I too shall proceed with temper and decency, although I have a little more at stake than Dr PERCY, and have my integrity to defend against a most unexpected attack, which it seems is to be carried on against me in support of his

accuracy in conversation.—I am, with much obligation for your good offices in this business, dear sir, your most affectionate humble servant,

"ADAM FERGUSON."*

This singular dispute about a matter of fact is only interesting from the eminence of the persons engaged in it; at the same time, the question of the genuineness of the Ossianic Poems given to the world by M'PHERSON, is still an interesting subject of inquiry.

The advertisement of Dr PERCY, followed by a further statement of FERGUSON, duly appeared in the public prints, after which the subject dropped. †

There can be no doubt that BLAIR, when writing his elegant dissertation on the Ossianic poetry, was an enthusiastic believer in the genuineness of these poems, and that the recitation of Gaelic poetry had taken place at his instigation. On the occasion of this correspondence, he seems to have shown a forgetfulness, or possibly a fear of admitting any statements tending to compromise his opinions, which caused some annoyance both to Dr PERCY and FERGUSON.

From the letters above given, we learn that FERGUSON was a supporter of the genuineness of the Ossianic Poems, as he states that he "never questioned the fidelity of Mr JAMES M'PHERSON in his publications." ‡ A correspondence with M'PHERSON relative to the publication of the original Gaelic of Ossian will be subsequently referred to.

In 1782, FERGUSON entered warmly into the scheme proposed by Principal ROBERTSON to institute in Edinburgh a society, similar to the foreign Academies, for the cultivation of every branch of science and literature. The immediate cause of this proposal was the application of the Society of Antiquaries to be incorporated by Royal Charter. §

The Senatus Academicus drew up a memorial to Government, proposing that, "instead of granting a charter to the Scots Antiquaries as a separate society, a society shall be established by a charter upon a more extensive plan, which may be denominated 'The Royal Society of Scotland,' and shall have for its object all the various departments of science, erudition, and belles lettres. That a certain number of persons, respectable for their rank, their standing, or their knowledge, shall be named by the Royal Charter, with powers to choose the original members of the Society, and to frame regulations for conducting their inquiries and proceedings, and for the future election of members." ‖

* MSS. University of Edinburgh.

† See Shaw's 'Inquiry' for FERGUSON's vindication, Appendix, p. 82.

‡ See his letter to Mr M'KENZIE, Secretary of the Highland Society, in that Society's Report on the Poems of Ossian, Appendix, p. 62.

§ Smellie's Account of the Antiquarian Society of Edinburgh, p. 12.

‖ It was due to the persevering efforts of Principal ROBERTSON that the Royal Society was instituted. After memorialising Government, about the end of the year 1782, to the effect above stated, the Records of the University bear that the Principal, on the 10th of February 1783, informed the Senatus Academicus, "that the Lord Advocate and Mr HUNTER BLAIR had desired him to acquaint

The members of the Philosophical Society, which had long existed in Edinburgh, were at this time also anxious to be incorporated by Royal Charter. They, however, adopted the views of the Senatus Academicus, and entered heartily into the scheme for the establishment of a society, on the model of those at St Petersburg and Berlin, for the purpose of cultivating every branch of science, erudition, and taste.

The Royal Society was accordingly incorporated in June 1783. The following is a list of the noblemen and gentlemen named in its Charter:—HENRY, Duke of Buccleuch; Lord President DUNDAS; JAMES MONTGOMERY, Lord Chief-Baron of Exchequer; Lord Justice-Clerk MILLER; JOHN GRIEVE, Lord Provost; Sir ALEXANDER DICK; Sir GEORGE CLERK; Principal ROBERTSON; Professors CULLEN, MONRO, BLAIR, WALKER, FERGUSON, DALZEL, ROBISON, MACONOCHIE; ILAY CAMPBELL, Solicitor-General; J. HUNTER BLAIR, and ADAM SMITH, Esqrs.; and J. MACLAURIN, W. NAIRNE, and ROBERT CULLEN, Advocates.

FERGUSON took a warm interest in the progress of the infant society, and was elected one of the Councillors. His only contribution, however, to its literary labours was a sketch of the Life of his relative, Dr JOSEPH BLACK, published in 1801.

them, that as they had the prospect of being in Edinburgh during the recess of Parliament, they had not returned any answer to the letters which the Principal had written to them, in obedience to the appointment of the meeting held on the second day of December last, but that they had laid the Memorial transmitted to them before His Majesty's ministers, and had good reason to think that what was requested in the aforesaid Memorial would be granted. That in order to obtain this, it would be necessary that a petition from the Principal and Professors of the University, in respectful and general terms, should be addressed to His Majesty, which the Lord Advocate undertook to present."

"The Principal produced a scroll of such a petition, the tenor whereof follows:—'Unto the King's most excellent Majesty, the Petition of the Principal and Professors of the University of Edinburgh, humbly sheweth—That literary societies having been found by experience to contribute greatly towards promoting useful science and good taste in every country where they have been established, many persons eminent in rank, or in learning, have long expressed an earnest desire that a literary society, formed on the plan suited to the state of this part of the United Kingdom, might be instituted in Edinburgh, being fully persuaded that its labours and researches will be of considerable advantage to the nation.

"'We, therefore, deeply sensible of your Majesty's paternal attention to the welfare of your people in every instance, and confiding in the gracious disposition of a Sovereign who has distinguished his reign by the splendour of his efforts to extend the knowledge of nature, and the liberality of his institutions for encouraging the arts of elegance, are humble suitors to your Majesty, that you may be graciously pleased to establish, by Charter, a literary society, to be denominated, The Royal Society of Edinburgh, for the advancement of learning and useful knowledge, empowering the Members of it to have, as the objects of their investigation and discussion, not only the Sciences of Mathematics, Natural Philosophy, Chemistry, Medicine, and Natural History, but those relating to Antiquities, Philology, and Literature.

"'We humbly request that your Majesty will take our petition into your gracious consideration, and be pleased, as Founder and Patron, to give a beginning and form to this Royal Society, in that mode, and under those Regulations, which to your Royal wisdom shall seem most proper.'

"Which being maturely considered by the Senatus Academicus, was approved of, and the Principal empowered to sign it in their name, and to transmit it to the Lord Advocate and Mr HUNTER BLAIR, with thanks for their obliging attention to the former application of the Senatus Academicus, and to request that they will still continue to attend to this business, until it be brought to the desired issue."

In 1783 FERGUSON gave to the world his principal work, entitled *The History of the Progress and Termination of the Roman Republic.* This title strictly embraces the period between the end of the early Roman monarchy, and the elevation of Julius Cæsar as the first Emperor of Rome. But, in order to bring the narrative nearer to the point at which GIBBON begins his History of the Decline and Fall of the Roman Empire, FERGUSON continues his work down to the death of Tiberius, the time when the succession to the throne began to be considered as hereditary.

FERGUSON has thus written the History of Rome from A.U.C. 240 to A.U.C. 790, a period of 550 years, and has given a lucid and compendious account of the leading events of that history.

In preparing his work, he of course availed himself of the classical authors, and, amongst modern writers, he made use of the researches of GUAZESSI and VESTRINI, the Annals of PIGHIUS, and the celebrated Essay of MONTESQUIEU, on the Grandeur and Decline of the Roman People.

His aim was rather to give in a connected and elegant form a narrative of the great facts of Roman history, than to indulge in discussions of the many matters of controversy which so extensive a subject necessarily involves. He does not enter upon the story of the origin of Rome, or even of the rise of the Republican form of government, but leans to the view previously held by DE BEAUFORT, and more fully developed by Sir GEORGE CORNEWALL LEWIS, that the early history of Rome was so involved in fable that no profit could result from such inquiries.

In this way FERGUSON'S History, ably and elegantly as it was written, does not afford the rich fund of information to be obtained from the more recent works of NIEBUHR and MOMMSEN, who, with infinite skill, have elucidated the early history of Rome by a critical examination of the remains of the classical authors; and who, by the comparison of their fragmentary details, by the examination of institutions existing in later and more historical times, and by the study of analogous phenomena among other nations, have endeavoured to place that history on a more trustworthy basis.

FERGUSON was led to undertake this work from a conviction that the history of the Roman people, during the period of their greatness, was a practical illustration of those ethical and political doctrines which were the object of his peculiar study; and he has remarked, that to know the history of Rome well was to know mankind, and to have seen our species under the fairest aspect of great ability, integrity, and courage. He regarded the great Roman statesmen and warriors during the Republican period, as exhibiting the utmost range of the human powers; while he reckoned the steps, by which the republican form of government was exchanged for despotism, as well deserving the careful attention of the student of political history and human nature.

As was before remarked, the military experience which he had seen in his youth

was of material service to him in writing his vivid account of the wars in which the Roman people were so constantly engaged; and his knowledge of human nature enabled him faithfully to portray the characters of the principal Roman leaders, and to test them by the laws of a high morality.

The many editions of this work which have been published show the estimation in which it has been regarded by the literary world.* The errors and omissions of the first edition were subsequently carefully corrected; and FERGUSON himself, ten years after it first appeared, visited Italy to inspect the scenes of the more important events which his work describes.

In 1785, FERGUSON, now in his sixty-second year, finding the anxiety attendant upon his professorial labours pressing upon his health and spirits, resigned the Professorship of Moral Philosophy. That he might retain his salary, he was, according to the custom of the Town Council, appointed to the Chair of Mathematics in conjunction with a junior professor, Mr PLAYFAIR, while Professor DUGALD STEWART was transferred from the Chair of Mathematics to that of Moral Philosophy.

STEWART had been the pupil of FERGUSON, and owed to his instructions the development of that taste for metaphysical speculation, by which in his lectures and writings he shed so much lustre on the University.

As Professor of Moral Philosophy, FERGUSON amply sustained the reputation of the institution with which he was so long connected. He was manly and impressive as a lecturer, but at the same time persuasive and elegant. In one particular his mode of teaching was peculiar, and not easily imitated. As he had delineated the general plan of his course in his 'Institutes of Moral Philosophy,' he had for many years no written lectures, but trusted to his mastery of the subject for the expression of his ideas on the spur of the moment. When his health gave way in 1781, however, he found it necessary to write out his course, which, during the leisure of his retirement, he corrected for the press and published in 1792.

Amongst the many proofs he received of the value of his professorial instructions, none were more agreeable to him than the attentions shown by Sir JOHN M'PHERSON, who had now attained the high position of Governor-General of India. The following letter, which enclosed a munificent gift, is no less creditable to his kindness of heart than to the merits of the veteran Professor:—

"12*th January* 1786.

"MY DEAR FRIEND,—When I was but a Company's writer in the Carnatic, I remember I sent you a small bill, which you told me you accepted with pleasure, as it came from me, and you bought French cloth with it, being then on a visit

* It was translated into French by De Meunier and Gibelin in 1784, also into German and Italian.

to Paris. I have been near a year Governor-General of India, and four years a Supreme Counsellor, and I have sent you nothing but a little madeira, yet you are the friend next to my heart, and your interests are dearer to me than my own, as they involve the concerns of a numerous family depending on the state of your health. If I have been thus inattentive to your situation, you are yourself the cause, for to you am I indebted for those rules of conduct in my public trusts, which have bound my generosity to your or to my own private interests within narrow limits. You have been occasionally informed of the line pursued by me since I left Europe, the situation in which I found affairs, my labours to retrieve them, and the disbursement of my own income in various attentions to those who were recommended to me, and whom I could not oblige at the public expense. If the line I have pursued was not necessary from its satisfaction to my own mind, the example of it was a *sine qua non* to enable me, when affairs devolved upon me, to reduce the expenses of this colony about a million sterling per annum, and to silence the cries of thousands who might otherwise have just grounds for charging me with partiality and selfishness.

"I have followed your maxims in the practice of affairs,—upon perhaps the greatest theatre of affairs, if the greatness of affairs is founded in the numbers of men, and the extent of their interests—the concerns to be extricated or forfeited —the wealth that might have been acquired, and the consequences that might ensue to individuals, tribes, and nations.

"The events that hinged upon my ideas and conduct four years ago were more important than those which I can now influence, though I stand at the head and in absolute charge of all our affairs in India.

"It is, my friend, one-and-twenty years since I began under you the rudiments of these affairs; and as there is no period of my life that I look to with such a conscious sensation of joy and pride, as that which I passed with you and our noble pupils, so to you is due the account which I can in truth give, and which I am bound to notice to you: It must be interesting to you, and it is for the benefit of our native school, and perhaps of society in general, that I should enable you to know the result, that you might hereafter be the more confirmed in your system.

"I have amply experienced the truth of three of your favourite positions:—

"1*st*. That the pursuits of an active mind are its greatest happiness, when they are directed to good objects, which unite our own happiness with that of our friends and the general advantage of society. Hence the first success in the Carnatic; the subsequent efforts in London; the return to India; the visit to Europe in '77; the intercourse with men in business; the friendships of the ministers; Lord N.'s * selection of me for my trust in 1778.

"2*d*. I have likewise experienced, that he who has not been in contact with

* Lord North.

his fellow creatures knows but half of the human heart. But such are the necessary taxes of occupation, of business, and perhaps of life.

"3*d*. That all that rests with us individually, is to act our own parts to the best of our ability, and to endeavour to do good for its own sake, independent of events, disappointments, or sufferings.

"Under these impressions I have acted and I now act; and if the India Company, the ministers, and the Legislature extend their views to the necessity of affairs, and to the future prosperity of Britain and India, as they stand united; and if they will adopt the plans I have laid before them, I am steady in believing that the greatest benefits to Britain from Thule to the Land's End, and to Asia, from Cape Comorin to Tartary, may flow from the practical operation of the commercial and political systems I have opened for the adoption of the empire. The outlines are clear and strong, as well as the ground of the operations themselves. Look on the map and see the field of empire marked by the Thibet Hills from Tartary to Chitagong, by the Ganges from its source to its embrace of the ocean, and by opposite chains of hills and of wild tribes from Balasore to the Jumna.

"This empire asks nothing from Britain but protection and some staples; and it sends to Europe every year about twenty fine Indiamen, loaded with the industry and the productions of its extraordinary soil. Each ship is worth L.100,000, one with another. The improvements made in navigation, and the knowledge of climates, and the care of health, enable Britain to carry on this trade, if she adopted a liberal plan for it, on a footing to employ a fleet in going and returning, including China and the coasts of the great Peninsula, about seventy ships—now equal in size to 50-gun ships, why not to 64 and 74? Commerce would then create a navy for Britain, at least such as would command the Indian seas; and as in King William's days, the first great operations of our state began by converting our debts into funds or property by regular payments of the interest; so we may here employ the present interest of our debts to be a medium for remitting the whole to Britain in an additional investment of goods. Upon this system, which necessity forced us to begin here in 1782, by providing what was called a subscription investment, and drawing bills upon the proceeds of the goods, India was saved from the jaws of war and the chains of a little monopolist policy, which forced all remittances to Britain through the channels of foreign trade, and which paid the tribute of custom to Lisbon and Copenhagen, at a rate that has turned the exchange from Copenhagen against England to about 18 per cent.

But my system does more; it pours in upon Britain more streams of friendship and of aid, which every officer, civil and military, in these colonies wishes to send partially to his relations, and which, in the general remittance and receipt, give the British heart on this and your side of the ocean its most delightful

exercises, and which gladden every village and place, from the cottages of the Isle of Skye to the palaces of London.

"I think a still greater scene opens by this commercial intercourse, if our rivals in Europe wished but for a proper share in it. It would embrace much of the repose of the universe, the happy communications of all the inhabitants of the globe from the sources of the Mississippi to those of the Ganges, and from west to east, till the east and the west are united.

"I have at this moment at Calcutta ambassadors from Tidore, in the eastern seas, from Thibet, from all the states of India, and from Timur Shaw, who is crossing the Attock; and as Manilla is opening her trade, I hope to hear direct from Lima before I leave India, and to make the Incas of Peru acquainted with the Brahmin Rajahs on the banks of the Ganges.

"Curious are, besides, the treasures in literature and the oblivious history of nations that are dawning upon us from the researches of Sir WILLIAM JONES and others, in Shanscrit, Arabic, and Persian. Even Anacreon and Euclid's best and happiest labours may have been long asleep in the translations of this country. And what seems to complete our prospect of elegant and useful information, is that the present Governor of Chinsura, who was for seven years in Japan, has brought in the wonders of that country. Their Encyclopædia is in his hands, and in some of the arts of life and of government, those islanders of Asia, those Anglo-Asiaticks, have left all other nations far behind.

"While devoting all my moments that are my own to such general considerations, I have perused, and am perusing again, your story of the Roman State and their rule of India—Thanks! thanks! my dear friend, but one ambition remains—it is to converse with you at your town over these affairs. Has life in reserve for us this happiness? or is our expectation of it enough? May I be able to meet you there worthy in every respect of your esteem as of your affection,—and is it possible to go through the remaining acts of my service here with progressive dignity and success. Hitherto all is as you could wish. But all may not be at the farm as you wish.* I know the feu-duty embarrasses you, and the *dignitas* without the *otium* may be there. Receive, then, the inclosed bill upon my masters, the India Company. Let the amount of it be sunk to discharge the annual feu-duty of the farm during your life, Mrs FERGUSON'S, and the lives of all your children and their descendants. It will be a future business to buy off the feu-duty altogether; at present I can send you no more. And should fate have deprived me of the future happiness of knowing that you can be conscious of this little attention, those nearest and dearest to you I must consider as what remains to me of you. To them I address this letter; also, in such event, JOHN FLETCHER, HOME, M'PHERSON. FERGUSON will keep a room for me, or any remembrance of

* FERGUSON, for several years after his marriage, had cultivated the farm of Bankhead, near Currie, at a considerable sacrifice of his private means.

the farm-house. Tell him (for I will not admit the idea that you have left us) that he is my son. His father was more than the father of your ever affectionate, "JOHN MACPHERSON.

"Mind me to Drs BLAIR, HOME, ROBERTSON, CARLYLE, BLACK, &c. &c."

About the time of the resignation of his University duties, FERGUSON resided in what was then a southern suburb of Edinburgh, named "the Sciennes." This suburb, which now forms part of the city, was then considered so far distant that his friends used to call his house "Kamtschatka;" and there, in the beginning of 1787, an interesting occurrence took place, which shows the pleasure he always took in the recognition of youthful genius.

BURNS had come to Edinburgh at the close of the previous year, to superintend the printing of the second edition of his poems. His arrival in the capital had produced a sensation, and he received great attention from many of the literati of the time. FERGUSON'S colleagues, Professors DALZEL and STEWART, have recorded the feelings of interest which the arrival of BURNS excited in the literary society of Edinburgh. Being desirous to converse with so remarkable a man, FERGUSON invited a small party to meet him at his house, amongst whom were Drs HUTTON and BLACK, Mr DUGALD STEWART, and the famous aeronaut LUNARDI. Trifling as this incident may seem, it afforded to Sir WALTER SCOTT, then a boy and companion of FERGUSON'S sons, the only opportunity he ever had of meeting with BURNS. On that occasion also he displayed that wonderful acquaintance with poetry for which he afterwards was so remarkable.

In a letter to LOCKHART, SCOTT thus describes this interesting meeting:—"I saw him one day at the late venerable Professor FERGUSON'S, where there were several gentlemen of literary reputation, among whom I remember the celebrated Mr DUGALD STEWART. Of course, we youngsters sate silent, looked and listened. The only thing I remember, which was remarkable in BURNS' manner, was an effect produced upon him by a print of BUNBURY'S representing a soldier lying dead upon the snow, his dog sitting in misery on the one side, on the other his widow, with a child in her arms. These lines were written beneath:—

"Cold on Canadian hills, or Minden's Plain,
Perhaps that parent wept her soldier slain;
Bent o'er her babe, her eye dissolved in dew,
The big drops mingling with the milk he drew,
Gave the sad presage of his future years,
The child of misery baptised in tears."

"BURNS seemed much affected by the print or rather the ideas which it suggested to his mind. He actually shed tears. He asked whose the lines were, and it chanced that nobody but myself remembered that they occur in a half-forgotten poem of LANGHORNE'S, called by the unpromising title of "The Justice of the Peace;" I whispered my information to a friend present, who mentioned it to

BURNS, who rewarded me with a look and a word, which, though of mere civility, I then received and still recollect with very great pleasure." *

In 1792, FERGUSON published his lectures, under the title of *Principles of Moral and Political Science, being chiefly a Retrospect of Lectures delivered in the College of Edinburgh.* This work was the first extensive contribution to mental philosophy which emanated from the University of Edinburgh. HUTCHESON in the University of Glasgow, REID and BEATTIE in that of Aberdeen, had previously laid the foundation of the Scottish school; and FERGUSON has the merit of having introduced its doctrines into a new sphere, where they were destined to attain a further development. He also had the advantage of bringing to his speculations a greater amount of historical knowledge, and a much more extended acquaintance with human character.

He divides his subject into two parts, the *first* of which states historically the most general appearances in the nature and state of man; embracing his description and place in the scale of being, mind or the characteristics of intelligence,

* Life of SCOTT, vol. i. p. 136.

Some interesting reminiscences of FERGUSON's son, Sir ADAM, who was the life-long friend of SCOTT, printed in Chambers's Journal, No. 60, 1855, supply one or two particulars which SCOTT's modesty suppressed. "The large black eyes of BURNS, which literally glowed when he spoke with feeling or interest, overflowed as he read the above lines, and he turned with an agitated voice to the company, asking if any one knew who wrote them. The philosophers sat mute; and after an interval, young WALTER said half aloud and very carelessly, 'The're written by one LANGHORNE.' BURNS caught the response, and seeming, both surprised and amused that a boy should know what all those eminent men were ignorant of, he said to SCOTT, 'You'll be a man yet, sir.' Rather oddly, we have found on an inspection of the print, that the name 'LANGHORNE' is inscribed below the lines, though in so small a character, that where the picture hung on a wall, it might well have escaped the notice both of BURNS and SCOTT."

In the same interesting article, an amusing anecdote is recorded of Principal ROBERTSON, when dining one day at FERGUSON's house:—

"FERGUSON, while devotedly attached to Dr ROBERTSON, and a great admirer of his works, found reason to complain of the manner in which he conducted himself in private society, particularly at dinner parties. It was the worthy Principal's custom, as soon as the cloth had been removed, to settle himself in his chair, and throwing out a subject, commence lecturing upon it to the destruction of conversation, and the no small weariness of the company. By way of giving him a check, Dr FERGUSON took his friend Dr CARLYLE of Inveresk into counsel; and it was speedily arranged between them that, immediately after dinner, Dr CARLYLE should anticipate the ordinary lecture of Dr ROBERTSON, by commencing a long tirade, in an enthusiastic manner, on the virtues of an article then in the course of being puffed in the newspaper advertisements, namely, patent mustard! FERGUSON, in the meantime, had a private conversation with the Principal, in which he took occasion to remark, that he had lately begun to fear there was something wrong with CARLYLE's mind; he was getting so addicted to speak loudly in praise of trivial things,—for example, he was unable for the present to converse about anything but patent mustard! ROBERTSON expressed his concern for the case, but hoped it was only a passing whim. The dinner party accordingly assembled at Dr FERGUSON'S, and ROBERTSON was about to commence as usual with one of his long-winded formal palavers, when all at once Dr CARLYLE broke in,—'This was,' he said, 'an age most notable for its inventions and discoveries. Human ingenuity was exerted on the noblest and the meanest things, and often with the most admirable effects on the meanest. There was, for instance, an article of a humble kind which had lately been wonderfully improved by a particular mode of preparation, and he for his part was inclined to say, that *patent mustard* was the thing above all others which gave a distinguishing glory to this age. In the first place,'—it is needless, however, to pursue his discourse further; suffice it, that Dr ROBERTSON sat paralysed, and could not afterwards, during the whole night, muster power or spirits to utter more than an occasional sentence."

and the principles of his progressive nature. Having laid this foundation of his course in history, FERGUSON proceeds in the *second* part of his work, to examine the specific good incident to human nature, and to treat of moral law or the distinction of good and evil, and its systematic applications, which are explained under the heads of Ethics, Jurisprudence, and Politics.

In the treatment of the metaphysical part of his course, FERGUSON declares himself an enemy to the scepticism of HUME, and opposed to the doctrine of ideas as maintained by HOBBES, LOCKE, and BERKELEY, but he coincides in his views with the metaphysical doctrines of ARISTOTLE as revived by REID. He is also a valuable exponent of the inductive method of observation as applied to the mind, so well laid down by REID, and consistently recommends the employment of this mode of procedure in all investigations. His metaphysical discussions are also valuable, as showing clearly the characteristics of mental as distinguished from material action, and establishing those primary truths on which all useful philosophical speculation is founded.

In his moral system, FERGUSON was a philosopher of the Stoic school. He avoided, however, the exaggerations and paradoxes into which many of its disciples have fallen, and endeavoured, by selecting what seemed reasonable and just from that and other theories of morals, to enunciate a more perfect system.

In opposition to HUTCHESON, who confounds the Will with Desire, FERGUSON first of all establishes Free-will as the subject and foundation of Moral Science. To the laws which regulate the Will—viz., *the Law of Self-preservation—the Law of Society—and the Law of Estimation or of Progression*, FERGUSON refers all moral facts, and all systems of morality. By this theory also he attempts to refute or reconcile the different theories previously promulgated.

In supporting his system, FERGUSON was opposed to that of CLARKE, who regarded the Intellectual principle as the arbiter of right and wrong, and who thus made virtue a matter of mere calculation. He was opposed to HUME, who places the foundation of morals in Utility, and shows, that if utility and virtue often unite to urge us in the same direction, they are often also at variance and mutually contradictory; and that whilst virtue is that which is most definitely useful, and most certain to promote our happiness, it nevertheless is not confounded in our minds with any idea of private interest. He was also opposed to SMITH's theory of Sympathy as the principle of morality; and proves, that to sympathise with a person, and to approve of his conduct, are two very different things. He thus also disposes of HUTCHESON's celebrated theory of the *moral sense*:—"If," says he, "moral sense be no more than a figurative expression, by which to distinguish the discernment of right and wrong, admitting this to be an ultimate fact in the constitution of our nature, it may appear nugatory to dispute about words, or to require any other form of expression than is fit to point out the fact in question."

FERGUSON endeavours to reconcile all these systems of morals, by comprehending them in his own classification. With HOBBES and HUME he admits the power of self-interest or utility, and makes it enter into morals as the law of Self-preservation. HUTCHESON'S theory of universal benevolence, and SMITH'S idea of sympathy, he combines under the law of Society. But as these laws of Self-preservation and Society are the means rather than the end of human destiny, they are subordinate to a supreme end, and this supreme end is Perfection.

It was in this *Idea of Perfection*, then, that FERGUSON placed the principle of moral approbation, and considered it as the law which every intelligent being forms to himself, by which to judge of every sentiment of esteem or contempt, and every expression of commendation or censure.

The philosophic speculations of FERGUSON have been carefully criticised by COUSIN, who thus expresses himself with reference to this theory of *Perfection*:— "We find in his method the wisdom and circumspection of the Scottish school, with something more masculine and decisive in the results. The principle of *perfection* is a new one, at once more rational and comprehensive than benevolence and sympathy, and which, in our view, places FERGUSON as a moralist above all his predecessors."*

In treating, in the latter part of his course, on the fundamental law of morality, and its applications and sanctions, FERGUSON observes, that some of these sanctions may be enforced, whereas others may be left to operate on the free will of the agent. Obligations and sanctions which may be enforced form the subject of Jurisprudence; those which cannot be enforced, are the applications of morality to the Duties of men.

In treating of Jurisprudence, FERGUSON explains the laws relating to peace and war, and follows GROTIUS in acknowledging the law of self-defence to be the only just foundation for employing force or stratagem in the case of independent or unconnected individuals.

The Duties of men FERGUSON divides into two classes,—those which may be considered as prohibitory, forbidding the commission of wrongs, and those which regard conscience, and can only be recommended by way of persuasion. The duties which involve in regard to others the right of constraint or prohibition are the foundations of natural law; and in this way FERGUSON enters upon the last portion of his subject, which is Politics.

FERGUSON treats of Politics under the heads of Population, Manners, and Wealth, and Civil Liberty. In this department, he follows MONTESQUIEU and HUME, and eloquently pleads the cause of well-regulated liberty and free government. His views in 1792 were, however, somewhat modified from those

* Philosophie Écossaise, 3d ed., p. 512

he had enunciated in 1769, when he published his "Institutes of Moral Philosophy." In that work, when treating of Political Institutions, he had thus expressed himself,—"Institutions that preserve equality, that engage the minds of the citizens in public duties, that teach them to estimate rank by the measure of personal qualities, tend to preserve and to cultivate virtue." The progress of the French Revolution, however, had, by the time when he published his lectures, cooled the general enthusiasm for liberty; and FERGUSON, who seems in 1769 to have held extreme views, at length admitted the necessity for inequality in rank, and the expediency of hereditary distinctions and of an aristocracy. *

In 1793 FERGUSON was elected an honorary member of the Academy of Sciences of Berlin. He was also a member of the Academy at Florence, of the Etruscan Society of Antiquaries at Cortona, and of the Arcadia at Rome.

From his knowledge of the Gaelic language, and from his early friendship with JAMES M'PHERSON, he was at this time consulted as to the proposed publication of the original Gaelic of the Poems of OSSIAN.

M'PHERSON had from the first professed his willingness to satisfy the public as to the authenticity of OSSIAN'S Poems, by printing the originals which had come into his possession. At the same time, when urged by the Committee of the Highland Society, he always pleaded want of leisure as his excuse for withholding them from the world. In 1793, however, a few years before his death, he prepared to comply with the generally expressed desire, but a difficulty arose as to the selection of the character and spelling to be adopted in printing the Gaelic language. It appears from the following exceedingly interesting letter, addressed to FERGUSON, that he had resolved to adopt the letters of the Greek alphabet, as more adequately representing the niceties of Gaelic pronunciation. With the view of making a trial of this method, he printed a specimen sheet, containing a passage from the Gaelic translation of the Bible in Greek characters, which he submitted to the criticism of his friends:—

"*London, May 21st*, 1793.

"MY DEAR SIR,—I wrote you a few lines some time ago, wherein, if I recollect aright, I promised to send you soon after an answer to your letter of the 8th of April, on the subject of the proposed printing the original of the Poems of OSSIAN in the Greek character. Having been, at the time of receiving your letter, immersed in a hurry of business, from which I have not, as yet, wholly extricated myself, I desired a gentleman, who has for many years, in conjunction with myself, thought *critically*, of the Gaelic language, to throw our opinion upon paper, at his convenience, more for your satisfaction than from either a wish or expectation of making converts of others. This he has done accordingly,

* Institutes, 2d ed., page 293. The Lectures on Moral Philosophy were translated into French, and attracted much attention abroad.

as you will find under another cover, which goes by to-morrow's post. As my friend has left little that is material for me to add, I shall not trouble you with a long letter.

"Our friend, Dr BLAIR, I perceive, labours under much want of information on the subject; for there is not one of the points on which he states his objections founded in fact, and, that being the case, his arguments and reasons require no answer. I cannot conceive what interest, except it was a silly degree of vanity, to give themselves a consequence on account of their knowledge in the Gaelic, those persons who gave the information had in deceiving our friend.

"Mr DAVIDSON writes rationally, but he seems not to know that there is scarce any manuscript to be followed, except, indeed, a very few mutilated ones in a kind of Saxon characters, which is as utterly unknown to the Highlanders as either the Greek or Hebrew letters. With respect to the cheap copy he mentions, if there should arise a wish for having a small edition, there is scarce any common printer but can metamorphose the Greek character into something like it in the Roman. With respect to the splendid edition now intended, it was never my intention to put it up to sale, so that its grandeur will not keep it out of the hands of those who would enjoy it most. I believe it will appear, from the accompanying observations, that there are not many of those amateurs between Glotta and Tarvisium.

"Mr DAVIDSON should be informed, that neither the Irish nor the Scotch Highlanders had ever any alphabet of their own. When they wrote, or attempted to write, they made use of the Saxon characters, which are much more confined than even the Roman, from which they are derived.

"As I have heard that Mr DAVIDSON is an excellent Greek scholar, he may be induced perhaps to try the effect of the specimen now sent on the Highland porter or chairman, in the manner recommended in the accompanying observations. Our friend Mr HOME, and even Dr BLAIR, who are both good Grecians, will be able, I trust, to read the original of OSSIAN, as it is to be printed in Greek, in a manner that will be intelligible to such Highlanders as *understand* their native tongue. But these, I apprehend, are much more circumscribed in number than is generally supposed.

"The result of the whole is, that I have resolved to follow the example of the old Druids, in writing the Celtic language in Greek characters. I shall not, therefore, with Dr BLAIR, agree, 'That it is the opinion of some of the *learned* in *Earse* that must determine the point, and that to them it *must be submitted*.' Where those *learned men* are I have never been able to learn. With respect to the clergy, I would rather take their ghostly advice on matters of religion than accept of their opinion about the manner of printing profane poetry. I consequently request, that instead of submitting the decision to them you will be pleased to return to me the specimen, already in your hands, at your convenience. And

after having weighed the observations at your full leisure, and at your own time, you will please to put them also under a cover to me. You will easily perceive, that this letter is meant only for your own eye; for few men wish to *know* that they have been so long deceived, on a point which the smallest attention might at once ascertain.—With my best respects to all friends, I am, with great esteem, yours most faithfully, JAMES MACPHERSON."*

The observations on the method to be adopted in printing the Gaelic language in Greek characters, drawn up at M'PHERSON's desire, and sent to FERGUSON along with the above letter, were to the effect, that the existing Gaelic orthography does not give the pronunciation of that language with truth and certainty, for the same letters represent different sounds, and the same sounds are expressed by different letters, and this in a promiscuous manner, according to the fancy of the writer. That in Gaelic a large number of letters are absolutely quiescent, which were probably introduced to represent in a clumsy manner a coarse pronunciation used chiefly in Ireland. That from these irregularities in the use of the letters which are really needful, and from the absurd accumulation of those which are useless, confusion has arisen, that renders the writing of the language arbitrary and the reading of it a matter of conjecture. With the view of making an experiment, a Scripture story—the finding of Moses by Pharaoh's daughter—was copied from the Gaelic translation of the Bible, and on the opposite page the same words were written in Greek characters. This specimen of the proposed system was circulated by M'PHERSON among his friends, who were requested to make the experiment of reading the specimens to some illiterate Highlander, with the view of ascertaining which of the two would be best understood. In answer to the objection, that the use of the Greek alphabet would be a great inconvenience and innovation, it was urged, that the Highland gentry do not generally read the Gaelic; that it would be but the labour of a few hours to master the Greek letters, and their use would smooth the way for those who wished to read and write the Gaelic language. It was further expected, that the familiar use of the Greek letters would naturally lead to the study of the Greek language itself, then much neglected in Scotland: and that it would be "no degradation of its characters to express the compositions of a poet, which the taste and learning of Europe have long since ranked among the admirable works of antiquity."

To the letter of M'PHERSON, and these observations, FERGUSON replied in the following terms:—

"*Edinburgh*, 30*th May* 1793.

MY DEAR SIR,—I am glad you are decided on the form in which Ossian is to be recorded. You may expect to hear different opinions on the subject; but if

* MSS. University of Edinburgh.

any one thinks he can do better in a more portable form, or in Roman character, this he can easily accomplish from your standard copy; and I shall cease to reason on the subject. Being but a bastard Gaelic man, my ear is a very uncertain rule for pronunciation or orthography. I will, however, mention what occurs under correction of your better judgment. Will it not be proper to prefix an alphabet, with notice of the power of each letter? If so, I think the two sigmas should be distinguished, the one *s* the other *sh*. I think the alpha is sufficiently full and broad in the sound without any additional vowel, as (*o*), for instance; and I think the upsilon should have the power of the English (*u*) uniformly given to it. The modern Greeks always pronounce it so. The (ει), falsely numbered with the diphthongs, should always stand for the Italian (*u*) or English double (*oo*), as in moon or boon, &c. To illustrate these remarks, I have ventured to mark the changes they would make in the specimen. Αχαϝ, I see, you spell with a kappa, to my ear it is rather a (γ), gamma; however you know much better. Query, also whether the nasal sound, when the article αν precedes a word beginning with gamma or kappa, may not be marked with the double gamma, as in the tale of Pharaoh's daughter (αγγ κοφαν); so much for remarks which you will not make any use of, as you see cause. I have conformed to your former injunction exactly in consulting no more persons. There are few persons of any education in the Highlands, whether clergy or laity, that do not know the Greek alphabet; and perhaps will have easier access to your Ossian in that alphabet than they would in the barbarous orthography which few, and I among the rest, never learned to read. I know that this would make many a learned man stare. For there is no persuading people south of Tay, that all the works of the bards are not to be found in booksellers' shops in Lochaber or Morven the capital of the country at least. I tryed your experiment on J. HOME, and he made it much more intelligible from the Greek orthography than from the Roman. I showed him in confidence your flagellation of the Edinburgh critics, and he is much diverted. I admire the fair hand and current writing of Greek in your amanuensis.—And am, dear sir, your most obedient and most humble servant,

"ADAM FERGUSON."*

M'PHERSON does not seem to have received much encouragement towards using the Greek characters for his projected edition. He died three years after this, and the Poems of Ossian, which were printed in Gaelic after his death, appeared in the ordinary Roman characters.

About the end of the year 1793, FERGUSON, although now in his seventieth year, finding his health much improved, formed the resolution of visiting Italy, that he might be the better able to prepare a new edition of his Roman History. He accordingly set out for the Continent on his way to Rome, and visited the chief cities of Germany, in all of which he was received with much distinction.

* This letter has been kindly furnished by Sir DAVID BREWSTER.

The following letters, addressed to Sir JOHN M'PHERSON, were written from Frankfort, Munich, and Venice, and are interesting from their allusions to passing events at this memorable period:—

"*Frankfort, 25th Sept.* 1792.

"MY DEAR FRIEND,—I wrote a line from Ostend, to give notice of my safe arrival on the Continent. I have since made out so far of my journey to this place, where I halt a day or two; but do not find that I can venture to go in search of the MARQUIS LUCCHESINI, and therefore enclose your letter to him, and consign it to the post, with my regret for not being able to do more. Military matters are well here, a division of French prisoners has just past, a second is expected at night, and a third to-morrow, amounting in all to about three thousand men taken in battle lately by the Duke of BRUNSWICK, but I cannot learn where. You pelted me with letters from the Continent, to which I was not enabled to make any answer. I should be sorry to return you the compliment exactly. My pelting will be very moderate, and your answers, I hope, will come, though I don't at present know where to direct them nearer than Rome, to the care of Mr JENKINS, banker, and there, in the name of God, let them come as many, and as soon as possible; that is to say, much sooner than *gleich* and *geschwind*, which I have generally found to be as slow as possible. All I have to say for the present is, that travelling even here is certainly a very healthy business, for I thrive wonderfully upon it. I have some inducements to go by Munich, and to take the inland route by Nurenburg, &c., as I know less of it than I do of the other, and the road, I am told, is good. I sometimes torment myself with thinking what is to become of the world; but as I have no commission to govern it, the wisest course is to mind my route, and so I shall do in the best humour I can muster.—Believe me to be yours most affectionately,

"ADAM FERGUSON."*

"*Munich, 5th Oct.* 1793.

"MY DEAR FRIEND,—Here I am at Munich, in a most prosperous course of travelling, waxing in strength and patience. I sent you a line from Frankfort, intimating my intention of sending your letter to the MARQUIS LUCCHESINI, with my regret for not being able to hunt for military quarters in person. I did so in the best French I could muster. The elector of Bavaria said at his levee yesterday that the king of Prussia has declared his intention to winter at Berlin, and to leave his army under the Duke of BRUNSWICK. There is, I find, a hankering inclination to censure his Majesty, on a supposition that more might have been done in the campaign; but I am of the opinion, which I guess is also yours, that to hem in the French, and give them as few opportunities as possible to take

* MSS. University, Edinburgh.

what we call crop to themselves, is the very perfection of conduct. There is a report here that the Emperor is about setting out for Brussels, and that even part of his equipage is in readiness. I surprised JAMES STUART by meeting him here, and find we shall be much together at Rome, &c. &c. It is now about forty years since I have known him to be one of the pleasantest, naivé, and best hearted creatures in the world. I am introduced to Mr WALPOLE here, and was at a *vrai diner d'Embassadeur*, all English, at his house yesterday; but I shall make no stay, being very impatient to get within the precincts of the Old Republic, and no less impatient to be at some place where I can hope to hear from you, and learn something of what is doing in the world; for in this way of life we are hood-winked, and know no more than can be seen when the glasses of the sulky are down.—I am, my dear friend, yours most affectionately,

"ADAM FERGUSON."*

"*Venice*, 19*th Oct.* 1793.

"MY DEAR FRIEND,—I write merely to let you know what is become of me, and the sum total is that I am well, and have come on as prosperously as a speculative master and a dumb servant could do without any other aid. I wrote a line also from Frankfort or Munich, with an account of what I did with your letter to COUNT LUCCHESINI. I see from newspapers since, that if I had stayed but a few days more at Frankfort I should have seen him there; but the secrets of kings who can know? and I should have thought myself in a scrape amidst the scarcity of horses, caused by his Majesty's motions. In the way I took by Nurenburg and Munich I avoided that distress, came prosperously through the Tyroll, and at Verona began to reap the fruits of my labours. If you remember, the Cimbri or Teutones are said to have performed wonders against CATULUS the Roman general in that neighbourhood; and though it be not of much consequence whether that tale be exaggerated or no, yet I was anxious to judge of its credibility on the spot, and got on horseback from Verona for that purpose, and reconnoitred the banks of the Adige for some little way. So far I had come post; but there I fell in with a Florentine veturino, who had brought some travellers from Florence. I rode his horses at Verona, and agreed with him to drive my Titbo thing to Padua. We agreed so well on the road, that I have lodged and boarded myself with him all the way to Rome. He seems to be a good-humoured, careful creature; and I am happy to escape the blustering postilions of the ecclesiastical state. I told him I should be at Florence soon, though at present I go by Loretto; and if any distress befal me, my point of rallyment will be Florence, being under the special protection of Count MANFREDINI, so that Antonio Lopini, this veturino, and I are already a sort of compatriots. I languish for news from England. I call for newspapers everywhere, but nothing has yet overtaken me

* MSS. University, Edinburgh.

more than I knew, and in part witnessed in passing through Flanders. I sometimes flatter myself that you will not have waited for accounts of my arrival at Rome, but will have written under care of P. MOIR, and JENKINS the banker; if you have not, *pour l'amour de Dieu* delay it no longer. I could not pass this place, though it is much too modern to be any object to me; I wonder at it; but am not much delighted. *Si je n'avois que soixante et dix ans*, as VOLTAIRE used to say, I would read its history with great avidity; but that is for the world to come. I went to the opera last night, and was truly entertained with the audience.—I am, my dear friend, yours most affectionately,

"ADAM FERGUSON."*

From the disturbed state of the Continent at the time, owing to the effects of the French Revolution, FERGUSON's stay at Rome was shorter than he anticipated, but he returned to Edinburgh much pleased with his tour.

After his return he continued to reside at his villa at "the Sciennes," where he enjoyed the society of his literary friends. Principal ROBERTSON dwelt in the neighbourhood, at the Grange House, and Mr COCKBURN, father of Lord COCKBURN, had his abode in the immediate vicinity, at Hope Park, midway between the houses of the Principal and his late colleague. Lord COCKBURN informs us, in the "Memorials of his Time," that he, when a boy, was frequently at the houses both of ROBERTSON and FERGUSON. He thus gives us his recollection of FERGUSON'S appearance:—"Our neighbour on the east was old ADAM FERGUSON, the historian of Rome, and STEWART'S predecessor in our Moral Chair—a singular apparition. In his younger years, he was a handsome and resolute man. . . . . Time and illness, however, had been dealing with him, and, when I first knew him, he was a spectacle well worth beholding. His hair was silky and white; his eyes animated and light-blue; his cheeks sprinkled with broken red, like autumnal apples, but fresh and healthy; his lips thin, and the under one curled. A severe paralytic attack had reduced his animal vitality, though it left no external appearance, and he required considerable artificial heat. His raiment, therefore, consisted of half-boots, lined with fur; cloth breeches; a long cloth waistcoat, with capacious pockets; a single-breasted coat; a cloth greatcoat, also lined with fur; and a felt hat, commonly tied by a ribbon below the chin. His boots were black, but with this exception, the whole coverings, including the hat, were of a quaker-grey colour, or of a whitish-brown; and he generally wore the furred greatcoat even within doors. When he walked forth, he used a tall staff, which he commonly held at arm's-length, out towards the right side; and his two coats, each buttoned by only the upper button, flowed open below, and exposed the whole of his curious and venerable figure. His gait and air were noble; his gesture slow; his look full of dignity and composed fire. He looked

* MSS. University, Edinburgh.

like a philosopher from Lapland. His palsy ought to have killed him in his fiftieth year, but rigid care enabled him to live, uncrippled either in body or mind, nearly fifty years more. Wine and animal food besought his appetite in vain, but huge messes of milk and vegetables disappeared before him, always in the never-failing cloth and fur. I never heard of his dining out, except at his relation, JOSEPH BLACK'S, where his son, Sir ADAM (the friend of SCOTT), used to say it was delightful to see the two philosophers rioting over a boiled turnip. Domestically he was kind, but anxious and peppery. His temperature was regulated by Fahrenheit; and often, when sitting quite comfortably, he would start up, and put his wife and daughters into commotion, because his eye had fallen on the instrument, and discovered that he was a degree too hot or too cold. He always locked the door of his study when he left it, and took the key in his pocket; and no housemaid got in till the accumulation of dust and rubbish made it impossible to put the evil day off any longer, and then woe on the family. He shook hands with us boys one day in summer, 1793, on setting off, in a strange sort of carriage, and with no companion except his servant James, to visit Italy for a new edition of his History. He was then about seventy-two, and had to pass through a good deal of war, but returned in about a year younger than ever." *

In 1795, FERGUSON received a severe blow to his domestic happiness by the death of his wife, who had been his faithful partner for nearly thirty years. Being now well advanced in years, and taking but little pleasure in society, he began to look about for a spot where he could spend the rest of his days in peaceful seclusion. It happened at this time that the old castle of Neidpath, overhanging the Tweed, near Peebles, was left untenanted by the Duke of QUEENSBERRY, and FERGUSON, charmed by the beauty of its situation, interested himself to procure a lease of the old castle, and a few acres of ground, from the Duke. The following letter fully shows the eagerness with which this new arrangement was entered into by the veteran philosopher:—

"*Edinburgh*, 20*th May* 1795.

"MY DEAR FRIEND,—Tho' the time is now approaching at which I have for some time past flattered myself with the hopes of seeing you here, I take my chance of overtaking you at Brompton with a few lines. The scheme of a country life, which you proposed to dispute, still remains with me, and I have been looking out for some place at which to settle. Among others, I have seen the castle of Nydpath, on the Tweed, belonging to the Duke of QUEENSBERRY. It has been lately dismantled, or stript of its furniture, and so far destined to become the habitation of bats and owls, or what is little better, such a tenant as I am. The servant who showed the place told me that his Grace has been asked to let it, but declined, which makes my prospect somewhat desperate. I have, neverthe-

* Memorials of his Time, p. 48.

less, made proposals in form to the man of business here. And beg the favour that, if you should see the Duke of QUEENSBERRY, you will try to incline his Grace not to forbid any transaction with me. And I undertake to satisfy you that the scheme I propose is the best for my family as well as myself. I am, your most affectionate humble servant, ADAM FERGUSON."*

It was in the month of May that FERGUSON had removed his household gods to Neidpath, and during the summer the old Castle was found to be a most desirable residence. "The woods, the hills, and the river, are Elysian, and the atmosphere all composed of vital air." These were his expressions in September; but when the cold blasts of winter approached, the Castle was anything but an enviable residence in its existing condition. The following letter to Sir JOHN M'PHERSON gives a glimpse of his situation in the winter season there:—

"*Nydpath, 9th January* 1796.

"MY DEAR FRIEND,—I have just now received your affectionate letter, with the inclosed commission of business for ADAM the Writer to the Signet, and write merely to get out my breath on this plaguy situation into which I have got, without the accommodation of either town or country. . . . I now see the mistake of having thrust myself into this situation before it was cleared for me one way or another; but I reasoned that I must either occupy the Castle before winter to keep it in repair, or lay aside thoughts of it altogether, to the last of which I was extremely reluctant. I am sensible what I should do now is to wait the chapter of accidents, but patience, the great virtue for succeeding in anything, has been but very scantily dealt to me. Old as I am, I had rather be doing anything, than wait doing nothing, of which this very letter is a sufficient proof; for it certainly will do you no good, nor me any other than employing some minutes of this woful time. I have to wait for some instruction to his man of business from the Duke of Q. So much for one Duke; if ever I have to do with another, I will give them leave to duck me in the first horse-pond. I am, most affectionately yours, ADAM FERGUSON."*

It has been remarked, that no Stoic philosopher more completely subjected his passions and his feelings to his reason than did FERGUSON; but the discomforts attendant upon his residence at Neidpath were a sore trial for his philosophy. Writing about them, in February, he says, "if any body think me a philosopher, he is grievously mistaken. I have done nothing but *peste* and scold inwardly for three or four weeks, not to say months."

The arrangements necessary to get quit of the lease of the Castle were, however, easily made; and he took up his residence at Hallyards, a farm in the

* MSS. University of Edinburgh.

immediate neighbourhood, where he lived for the next fourteen years,—a longer period than he had ever spent in any of his previous places of abode. During this period he still enjoyed good health, and interested himself in farming with all the ardour of a young agriculturist.

It was to an incident which occurred while FERGUSON lived at Hallyards, that we owe one of Sir W. SCOTT's most characteristic novels. Among the hills near the house lived an eccentric and misshapen dwarf, called DAVID RITCHIE, and SCOTT, then a young advocate, when he came in 1797 to pay a visit to the FERGUSONS, was taken to see DAVID as one of the lions of the district. The strong impression which the interview with the hermit, who was supposed to be possessed of magical powers, made on SCOTT, was never effaced; and the tale of the 'Black Dwarf,' published twenty years afterwards, owed its origin to this remarkable occurrence.*

The following letter addressed to Sir JOHN M'PHERSON, contains FERGUSON's views as to the epitaph which he wished to be inscribed on his tomb, and also an allusion to the energy which was the distinguishing feature of the character of the late Sir JOHN SINCLAIR.

"*Hallyards*, 3*d July* 1798.

"MY DEAR FRIEND,—My silence is not negligence nor forgetfulness. If I had ten thousand of the best letters that ever were written, you should have them all; but what can I write from this post, at which my prime consolation is, that I have nothing to do but to wait quietly till my time comes. The French, I trust, although they may teaze, cannot subdue this armed nation; and all speculation on the subject is at an end. I have in my view a most delightful kirkyard, retired and green, on the bank of a running water, and facing a verdant hill, which in your part of the world would pass for a tremendous mountain; but to me it gives the idea of silence and solitude away from the noise of folly; and so I fancy myself laid there, with a stone to tell the rustic moralist what he will not understand, because I sometimes project it should be in Greek, as follows:—ὡς ἐγω τον κοσμον ἐθαυμασα, και συ θεασαμενος χαιρε; but then, again, I wish to explain it, and so it should be, 'I have seen the works of God, it is now your turn, do you behold them and rejoice.' I would speak my verse for agriculture in Greek also,—Ανθρωπου ἐργος γεωργια,—and you may judge of my willingness to write when I put all this on paper to you. I have not stirred from home for many months past till lately, when Admiral and Mrs NUGENT being at Edinburgh led me thither to gratify my sense of their kindness to my little seaman. And I am still the more convinced, that NUGENT is the most amiable, faultless creature upon earth. In that excursion I met our friend, Sir JOHN SINCLAIR, in the street; this put it in his head to write to me since my return hither, an account of works he is projecting to promote what he calls statistical philosophy. I hinted that his project is too vast;

* CHAMBERS's Hist. of Peeblesshire, p. 402.

but he tells me that his mind is made up to draw it on a great scale, and on as perfect a plan as possible; and that he never started at any difficulty that could be surmounted, ever since he collected, as far back as the year 1780, one thousand and two hundred men, and in one day's time made a road of six miles long over a mountain, till then thought impassable. The fact is, that he has got an instinct to be doing, which other people ought to know how to employ without turning him out of place. Although I have so many excuses for writing so seldom, I am not willing to allow you any; so I pray, when you are writing however, let there be a scrap for me, even if you should not be able to tell me what is become of BUONAPARTÉ. By the by, is that a genuine Prussian paper in answer to the French demands, which we have in the newspapers? It is menacing, and I do not see how the great nation can give way to it, without appearing to be cowed. They certainly meant to gall us, and to secure the co-operation of Prussia against us, by transferring Hanover, &c. &c.—Yours most affectionately,

"ADAM FERGUSON."*

The following letter, also addressed to Sir JOHN M'PHERSON, concerning the purchase of an estate in Peeblesshire for a ward of Sir JOHN'S, contains an allusion to ALLAN RAMSAY and the Edinburgh writers:—

"*Hallyards*, 1*st August*, 1798.

"MY DEAR FRIEND,—To begin where your letter ends *amicus amicissimus indefatigabilis*. After having splashed you before with bad Greek, you are well off that there is nothing more now than bad Latin. I do recollect hearing of S. J. E.'s desire to have some land property near his native spot, but at present know nothing more, nor do I know of any fit place at Peebles for your ward, but shall inquire. There is no property in this country you know without a doer, as ALLAN RAMSAY used to call the writers when he was angry with them, which he was, indeed, for the greatest part of every hour of his life. If there be any subject on which to make us your doer, we shall not neglect to do what is proper; and for the sequel, if there be any sequel, it must come as God will have it in the whims and inclinations of those concerned. As to the world, I am glad you think BUONAPARTÉ is gone upon a mere trading or plundering voyage. In that way he cannot be long without having the seas disputed with him, and I patiently wait for the consequence, without supposing that every encounter of ours must be *veni, vidi, vici*, for even the great JULIUS was a puppy at a time, and more so than has yet appeared of BUONAPARTÉ. A combination of Europe, including Russia, if not properly directed would do us no good. You may possibly remember my bull, that the proper way to make war on the great nation is to make peace with them. In this they are too wise to be caught, I mean their directors; but I think we may make a war as like peace as possible, especially if Europe com-

* MSS. University of Edinburgh.

bine in it, by keeping them at bay,—leaving them no outlet from home, nor goading them with any trifling attacks to keep their attention and animosities directed abroad. There was an expedition to Ostend, and there is one again now, the newspapers say, from Margate, mere proofs that we have not yet learned the character of our enemy or the nature of our contest, but of that no more. I am no oppositionist, and this moment think the nation in a most prosperous state—that is to say, we have men, arms, and spirit, and if we should come to have less wealth we must consume the less, either by having fewer mouths or putting less in them. I was in Edinburgh for a day or two when your last letter came here to Hallyards, otherwise, having now three or four such favours to acknowledge, should have done it sooner. The 'Roman History' advances but slowly. The printers have much other work when our law courts are sitting; then much of the business proceeds by a kind of paper war from the press. Five octavo volumes are projected, but little more than one is yet printed. I shall be obliged to your German author for his prolongation scheme, though having annuities and salaries from other people, 'tis like they think I have prolonged enough. I went to Edinburgh to see our friend G. JOHNSTONE, and was highly gratified.—I am, my dear friend, most affectionately yours,

"ADAM FERGUSON."*

In the following extract from a letter, written in 1799, FERGUSON thus expresses his opinion of the views then published by Sir JAMES MACINTOSH.

At that time MACINTOSH had been recently called to the English bar, and with the view of bringing himself into notice, he delivered a course of lectures on the Law of Nature and Nations.

The introductory lecture was published as 'A Discourse on the Law of Nature and Nations,' and it, with the other lectures of his course, received the highest praise from men of every shade of political opinion.

FERGUSON states, "I hear very favourable accounts of Mr MACINTOSH'S performances at Lincoln's Inn. As I judge only from his pamphlet, his tone, though perhaps more harmonious, is in unison with mine. He had his reasons, probably, for not mentioning me, and I am not solicitous about them. He will probably procure to Moral Philosophy that popularity in England which I wished for, but have been unable to obtain. His taking his ground in the law is not so apt to alarm the Universities and the Church as if he had called his object Moral Philosophy, which those authorities sometimes mention among the corruptions of the times." *

The literary labours of FERGUSON were not yet over. In 1801 was published his already mentioned contribution to the Transactions of the Royal Society, under the title of *Minutes of the Life and Character of Joseph Black, M.D.* In it there is

* MSS. University of Edinburgh.

given a concise but interesting account of BLACK's discoveries of carbonic acid and latent heat, which entitle him to be regarded as the father of modern chemistry.

It was his relationship to the family of JOSEPH BLACK,* which was probably the indirect means of forming FERGUSON's own philosophical views. The father of Dr BLACK had been a wine merchant at Bourdeaux, and when residing there enjoyed the intimate friendship of the great MONTESQUIEU, who was the president of the parliament or court of justice of that province. The letters and scraps of correspondence which passed between MONTESQUIEU and Mr BLACK, the descendants of the latter preserved as though they had been titles of honour belonging to their race. In his own Philosophy, FERGUSON has in many places followed the views of MONTESQUIEU, and his 'Essay on Civil Society' may be regarded as an eloquent introduction to the immortal work, 'The Spirit of Laws.'

The infirmities accompanying advanced life now made FERGUSON desirous of again residing in a town, where he might have more opportunities of conversing with intelligent friends.

As St Andrews was the place where he had been educated, his early predilection for that ancient city returned, and in 1808 he retired thither to spend the remainder of his life.

He there enjoyed the society of the Professors of the University, and that of the patriotic GEORGE DEMPSTER of Dunnichen, whose endeavours to extend the manufactures of Scotland are well known.

Among FERGUSON's letters, perhaps not the least curious is the following, addressed to Mr CARLYLE BELL, in which he expresses his opinion of the 'Diary' of Dr CARLYLE of Inveresk. CARLYLE died in 1805, and it was proposed in 1810 by his executors to edit this work, which, however, has only recently appeared under the editorial superintendence of Mr J. HILL BURTON. Had it been published in 1810, in place of 1860, the Diary would not have excited the same interest:—

"*St Andrews*, 21*st July* 1810.

"MY DEAR SIR,—I have received your letter acquainting me that trustees whom you do not name, are now deliberating on the publication of my worthy friend and your late uncle's manuscripts. Of this you must be sensible that I cannot give any opinion. The small part I saw, or with my impaired sight could decipher, did not appear to me intended for publication; but rather the amusement of leisure in the exercise of a talent in which our friend excelled; the easy and satisfactory detail of familiar occurrences affording a pleasure which his correspondents experienced in every letter he wrote to them. I was so pleased in reading the part you showed me or I could attempt to read, but it related to things and persons most of us so obscure as not to be entitled to public notice,

* The mother of FERGUSON was aunt of the mothers of JOSEPH BLACK and JAMES RUSSELL Professor of Natural Philosophy. FERGUSON was also married to Dr BLACK's niece.

that I should not be willing to exceed what I believed to be the writer's original intention by publication; and I thought myself the more at liberty to give this opinion that I found my own name repeated with that partial favour which I always experienced from my friend. It was our lot through great part of our time to be neighbours so near as to be frequently together, and the opportunities, I believe, were never willingly omitted by either. We were *socii criminis* in the countenance we gave to the first representation of our friend J. HOME's tragedy of Douglas, a charge for which I was never called to account. But Dr CARLYLE was more known, and had more enemies, who, by prosecuting him for this offence, declared him innocent of anything more likely to serve their spite. We were also accessory to the formation of a Poker Club, and survived most of its members, and thus had occasions of regret which are but ill repaired in the solitary comforts of sequestered old age. You cannot doubt my desire to promote the respect which is due to the memory of Dr CARLYLE, but how I know not, beyond the testimony, if it were called for, that I never knew a more steady friend or more agreeable companion, and in this I should have so many concurring witnesses as to make my words of little account. I shall be anxious to know how you proceed, and I beg I may hear from you.—I am, with best respects to Mrs BELL, yours most affectionately,

ADAM FERGUSON."

During his residence at St Andrews FERGUSON's mind was almost as vigorous as in his younger days, and his bodily functions, with the exception of his sight, were scarcely impaired by age.* Even in 1815, the year before his death, his health was better than it had been for some years, and his spirits were elevated by the successful termination of the war with France, in which he had always been much interested.

About the beginning of February 1816, however, he was attacked by a febrile complaint, to which he had been occasionally subject. This illness, after continuing for four days, proved fatal on the 22d of that month, when he was in the ninety-third year of his age.

FERGUSON had a family of seven children, four sons and three daughters. His

* In 1812 FERGUSON was requested by Mr HENRY MACKENZIE—'The Man of Feeling'—to furnish him with some memoranda relative to his early acquaintance with JOHN HOME, author of 'Douglas.' MACKENZIE's last literary effort was a Memoir of HOME, which he read before the Royal Society in 1822, and which was afterwards published in a separate form. To that volume an Appendix is added, containing a remarkable letter, written when FERGUSON was in his ninetieth year. Besides referring to his early connection with JOHN HOME, it contains further information as to his views with reference to the Ossian controversy.

There was also published, after his death, a short biographical sketch or memoir of his friend, Lieut.-Col. PATRICK FERGUSON (second son of JAMES FERGUSON, of Pitfour, one of the Lords of Justiciary in Scotland), which he had written some short time previously. It was intended as an article for the Encyclopædia Britannica, but it was considered by the editor too long for that work; and as FERGUSON declined to abridge it, it was not inserted. A few copies were printed in 1817 from the original sketch, for private distribution.

eldest son, Sir ADAM,—one of the most genial and kind-hearted of men—was an early friend of Sir WALTER SCOTT, and is frequently alluded to, under the soubriquet of Linton, in Sir WALTER'S Life by LOCKHART. His second son, JOSEPH, entered the army, and died in India in 1800 as Captain in Lord SEAFORTH'S regiment. His other sons were, JAMES, Colonel H.E.I.C.S., and JOHN, Rear-Admiral, Royal Navy, who survived their father.

His daughters, ISABELLA, MARY, and MARGARET are frequently noticed in LOCKHART'S Life of SCOTT, as having, when residing at Huntly Burn, formed part of the delightful circle which SCOTT gathered around him at Abbotsford.

In these Memorials of ADAM FERGUSON, which we now conclude, we renew our converse with many persons of whom Scotland has every reason to be proud, and amongst whom FERGUSON deservedly holds a high place. Whether viewed as a historian, a moralist, or a man, FERGUSON was eminently distinguished by a vigour and a simplicity of character which well entitle him, as the last survivor of a galaxy of great contemporaries, to be designated Ultimus Romanorum!

On the monument erected to his memory by his family, within the grounds of the old Cathedral of St Andrews, is the following elegant inscription, from the pen of Sir WALTER SCOTT:—

HERE REST
THE MORTAL REMAINS OF ADAM FERGUSON, LL.D.,
PROFESSOR OF MORAL PHILOSOPHY IN THE UNIVERSITY OF EDINBURGH.
HE WAS BORN AT LOGIERAIT, IN THE COUNTY OF PERTH, ON THE 20TH OF JUNE 1723,
AND DIED IN THIS CITY OF ST ANDREWS, ON THE 22D DAY OF FEBRUARY 1816.
UNSEDUCED BY THE TEMPTATIONS OF PLEASURE, POWER, OR AMBITION,
HE EMPLOYED THE INTERVAL BETWIXT HIS CRADLE AND THE GRAVE WITH
UNOSTENTATIOUS AND STEADY PERSEVERANCE IN ACQUIRING
AND DIFFUSING KNOWLEDGE,
AND IN THE PRACTICE OF PUBLIC AND OF DOMESTIC VIRTUE.
TO HIS VENERATED MEMORY
THIS MONUMENT IS ERECTED BY HIS CHILDREN,
THAT THEY MAY RECORD HIS PIETY TO GOD AND BENEVOLENCE TO MAN,
AND COMMEMORATE THE ELOQUENCE AND ENERGY WITH WHICH HE INCULCATED
THE PRECEPTS OF MORALITY,
AND PREPARED THE YOUTHFUL MIND FOR VIRTUOUS ACTIONS.
BUT A MORE IMPERISHABLE MEMORIAL OF HIS GENIUS EXISTS IN HIS
PHILOSOPHICAL AND HISTORICAL WORKS,
WHERE CLASSIC ELEGANCE, STRENGTH OF REASONING, AND CLEARNESS OF DETAIL
SECURED THE APPLAUSE OF THE AGE IN WHICH HE LIVED,
AND WILL LONG CONTINUE TO DESERVE THE GRATITUDE, AND COMMAND
THE ADMIRATION OF POSTERITY.

XLIII.—*On the Reputed Metrological System of the Great Pyramid.* By Professor C. Piazzi Smyth, Astronomer-Royal for Scotland. (Plates XXIII.—XXVII.)

(Read 21st March 1864.)

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In the year 1859, a book of remarkable power and originality was published by Mr John Taylor, of London, entitled "The Great Pyramid, why was it built?"

On first looking into it, I was unfortunately inclined to fear that its results were unlikely to be very sound; though merely because they seemed to bear, in a hitherto nearly barren, or difficult, and certainly most mysterious field, such a remarkably large crop of rich and promising-looking fruit. But considering afterwards, that that was not the proper frame of mind to be indulged in connection with, and certainly not in place of, strict scientific investigation into the merits of the case,—I read the book carefully, and then searched for the data required to test it, both in the original authors appealed to, and in some others.

Without attempting to follow Mr Taylor in all the learnedly copious breadth with which he treats the subject, and which would be quite beyond my powers,—I have endeavoured to submit to a still more searching examination than he himself has done, such part of his inquiry as I felt myself in some measure professionally conversant with the general principles of,—and have now to submit the results to the indulgence of the Royal Society of Edinburgh.

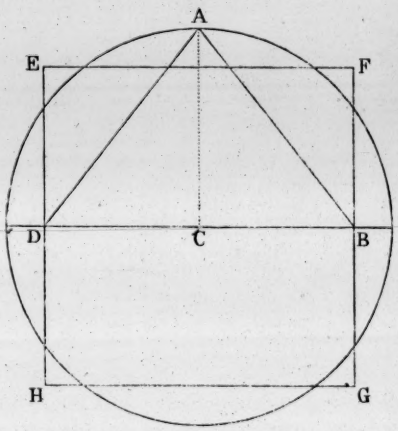
(1.) *Circumferential Analogy of the Pyramid.*

The first proposition or statement which Mr Taylor puts forward, admitting of scientific examination is, that—"the vertical height of the Pyramid (which rises from a square bed, see Plate XXVII. fig. 7), is, to twice the length of one side of its base, as the diameter to the circumference of a circle:" or, as in fig. 1 on the next page;

Where, AC	:	DB × 2	: :	Diam.	:	Circumference
	:	DEFB	: :	1	:	3·14159, &c.

Now this statement alludes of course to the primitive condition of the Pyramid, as it came out of the hands of its builders nearly 4000 years ago; and to identify, without any doubt, which Pyramid is referred to, we may mention that it is the largest of the group of stone Pyramids near the modern village of Jizeh, Jeezeh, or Gheezeh, and some 15 miles north of the ancient city of Memphis; standing therefore on the western or opposite side of the river Nile, to that on which the modern city of Cairo is presently found. Altogether it is the largest, most solidly, and compactly built, and most exquisitely finished in its interior, as well as one of the earliest of all the Egyptian Pyramids; has generally been known amongst all nations for ages as "the Great Pyramid;" and has been in later years tried by more theories than any other, yet without yielding hitherto any fully satisfactory account of its objects or intentions.

Fig. 1.



For nearly 3000 years after its erection, the external surface of the Great Pyramid remained untouched; and would have at that time allowed Mr TAYLOR's proposition to be instantly tested with severity; for the Pyramid was then as beautiful a realisation of the mathematical solid so called, as well could be imagined; but after that interval, the Arabian Caliphs in Cairo began systematically to carry away the whole of the external marble casing; leaving at last only the rude steps of the inner component masonry, which made the new exterior assume sensibly another figure. These stone steps too, serving unfortunately after that, to render the climbing of the Pyramid easy to any one and every one,—European, as well as other travellers have been for ages past seized with a strange madness to clamber up to the top of the structure, and then begin throwing down some of the uppermost stones, for the ignorant pleasure of seeing them smashing and thundering down the steep sides of the monument; whence it comes to pass, says the learned "Description de l'Egypte" of the French nation, that "the *area* which now exists, in place of the original sharp point, at the top of the Pyramid, is daily growing larger, and the height becoming smaller."

Hence the *original* height of the Pyramid cannot now be determined by direct measurement; and though, abstractly, we might compute the proportions required in Mr TAYLOR'S proposition,—either, from the measured axial height and the length of one of the sides of the base,—or, from the *angle* made by one of the sides with the base, without reference to linear measures at all—yet we must in practice confine ourselves to the last method alone; that is, to procuring

accurately the angle at the Pyramid, represented by the angle ABC, in our fig. 1.

Now, early travellers, it seems, though attempting many other measures, seldom ventured upon angles; or when they did, mentioned this particular angle of the Pyramid, as being almost anywhere between 30° and 60°; and even Dr PERRY in 1743, who was dissatisfied with all previous measures, and found fault with the regularity of the Pyramid also, stating that the angles of every side were different, and ran thus 40°, 37°·5, 35°, and 42°·5,—was wofully far from the truth with every one of them.

Hence there is no angle observation on record worth any attention, until that of the French savans in the celebrated Napoleonic expedition of 1799; and they, confining their attention chiefly to the Northern face of the Pyramid, or that upon which the one and only entrance into the Pyramid is found, and looking as well as they could along the broken line of the steps of stones as left by the ravages of the Caliphs, made the angle,—

51° 19′ 4″

This was in truth an exceedingly close approach, compared to anything that had been previously accomplished; and it was confirmed by Mr HAMILTON, who visited the Pyramid immediately after they had left, or in 1801, and recorded the angle as

51° 23′ 46″

And though M. CAVIGLIA said the angle, in 1817, was so much as 58°, it is plain that he made no attempt at refinement in his measures.

In this state, therefore, the matter remained, until Colonel HOWARD VYSE'S all-important discovery in 1839, of two of the original casing stones, well preserved under the hill of rubbish below the entrance into the Pyramid; and he found them still firmly *in situ*; that is, cemented securely to the grand and admirably smoothed as well as levelled plateau of rock on which the whole pyramid stands.

Colonel VYSE'S three volumes should be carefully read, and some fifty other authors, to establish the extraordinary importance of this discovery.

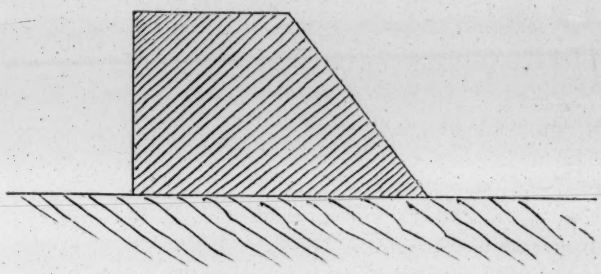
The stones were very large and of the most exquisitely truthful workmanship: one of them appearing thus in cross section (see fig. 2, on next page).

The angle of the sloping side, being carefully measured by Mr BRETTEL, C.E., for Colonel VYSE, came out 51° 50′; but on computing the angle from the linear sides as given above, in Mr PERRING'S measures, it appears to be = 51° 52′ 30·3″.

There is little doubt of the truth really lying between the two results, for neither of them is to be implicitly trusted; not the angle observed, because it is exceedingly improbable that the clinometer employed was equal to the unusually high precision required in this case; and not the angle computed from the sides, because those sides were not only measured with a rudeness, of merely to "the

nearest inch;" but because an error of more than an inch may be suspected in measuring the lower side, or that lying flat on the rock.

Fig. 2.



	Feet.	Inches.
Its lower side being .	8	3
„ upper	4	3
„ vertical	4	11
„ inclined	6	3

This suspicion arises, from the circumstance of the lower left-hand corner in the section, which should be a right angle, coming out in the computation more than one degree and a half different therefrom; an amount of error in their easiest angle, which the builders were not likely to commit, when they had arrived within 1 or 2 minutes only, of what was required of them in their most difficult angle. We have therefore as follows, from that unique opportunity of the casing stones, which has now passed away for ever; as the stones are since reported to have been irretrievably injured by the hammers of specimen seekers, and the mischief-working of curiosity-mongers.

By angle direct	=	51° 50′ 0″
By vertical and inclined sides	=	51 52 30·3
Mean	=	51 51 15·2

By angle direct	=	51° 50′ 0″
By vertical and inclined sides	=	51 52 30·3
By three measured sides	=	51 50 46·5
Mean	=	51 51 5·6

By angle direct	=	51° 50′ 0″
By vertical and inclined sides	=	51 52 30·3
By three sides	=	51 50 46·5
By two sides, and base shortened by one inch?	=	51 52 12·2
Mean	=	51 51 22·2

By all three means	=	51°	51′	15″·2
	=	51	51	5·6
	=	51	51	22·2
First Mean	=	51	51	14·3

Hence we may arrange the authors, angles, and the circumferential proportion (or π) which they attribute to the Pyramid, agreeably with Mr Taylor's proposition, in this manner—

					π
Caviglia	58°	0′	0″	=	2·49950
French Academy	51	19	4	=	3·20257
Mr Hamilton	51	23	46	=	3·19360
Vyse and Brettel	51	50	0	=	3·14393
Vyse and Perring	51	50	46·5	=	3·14268
Vyse and Perring ½	51	52	30·3	=	3·13922
Vyse and Perring corrected	51	52	12·2	=	3·13978
Final Mean, V., P., & B.	51	51	14·3	=	3·14159

Now the last result for π, is absolutely correct so far as it goes; and though there may be some doubt about the best way of taking the mean of all the observations, where the observers had such very large probable errors as their measures of the stones exhibit,*—yet there can be none, as to the Pyramid result coming out better and better, in proportion as it is more closely examined into.

The Pyramid has in fact proved itself to have been built closer to the truth, than the best and greatest of modern savans from all existing civilised countries have been able to measure it to: and if we are obliged to draw rein in pushing on the severity of our inquiry to still further places of decimals, the reason is not that the Pyramid fails in accuracy, but that modern observations are not sufficiently good to bear more than has been already put upon them.

This first result, therefore, of applying an independent test to one of Mr Taylor's propositions, has ended entirely in his favour. And if the finding has depended on the angle, A B C in fig. 1, alone, it may be as well to state here, that there is nothing in what remains to us of the original linear proportions of the Pyramid that tends in any way to invalidate the conclusion.

There have been, indeed, several ingenious suggestions as to various geometrical proportions existing between the several parts of the Pyramid, including its height, and base-breadth; but it is hardly worth while to allude to them further, as they are not accurate, and have not been advanced very confidently by any

* Though the Vyse and Perring observations were so rough, yet they seem eminently honest and fair: and it must add additional weight to their testimony in this case, that neither of these gentlemen seem to have had any idea at the time, of what refined results their observations might eventually be made to bring out, or what indeed the Pyramid itself contains in this direction; for in his 2d volume, Colonel Howard Vyse, enumerating his laborious assistant Mr Perring's conclusions about the Pyramid, says, that its height is to its base-side, as about 5 to 8; which gives no closer approximation to the value of π, then 3·200.

one as constituting the *intention* of the Pyramid; with the exception, however, of one very old statement, treated with much favour by Sir JOHN HERSCHEL in the Athenæum, April 1860; and asserting, that the area of each of the four inclined sides of the Pyramid equals the square inscribed on its height. Now to do this, the angle at the foot of the Pyramid, requires to be, as he states, 51° 49′ 46″; and that is an angle which Sir JOHN HERSCHEL declares to be "practically indistinguishable" from the angle required by the circular analogy of Mr TAYLOR, or 51° 51′ 14·3″; and, as HERODOTUS says that the Egyptian priests told him, that the area of the side to the square of the height was the intention of the Pyramid, —Sir JOHN accepts such explanation, and repudiates Mr TAYLOR's idea.

Answer may be made however, 1*st*, That the two angles are by no means practically indistinguishable; and that the measures already given, point clearly to the larger angle; 2*d*, That the Egyptian priests and people, even if inclined to instruct HERODOTUS, were ignorant of the full contents and objects of the Great Pyramid; and 3*d*, That the area analogy can rank merely as an isolated feat, while the circular one is the most essential foundation in all that higher metrology, for which it may presently appear the Pyramid was actually erected.

(2.) *Standard of Length.*

Having in the last section determined only the *proportion* existing between the height and base-breadth of the Great Pyramid, let us now endeavour to ascertain their values in *linear* measure. The following are actual observations, overlooking some comparatively small differences of French and English feet, reported to have been taken by different travellers at the dates specified:—

Name.	Date A.D.	Present Height in feet.	Northern Side of Base in feet.
Jean Palerme,	1581	600	660
J. Greaves,	1638	499	...
D. Monconys,	1647	520	682
M. Thevenot,	1655	520	682
Mr Melton,	1661	520	682
M. Vausleb,	1664	662	720
M. Lebrun,	1674	676	704
De Careri,	1693	520	682
Egmont,	1709	500	693
Dr Shaw,	1721	500	670
Dr Perry,	1743	687	789
M. Niebuhr,	1761	440	710
Mr Davison,	1763	461	746
M. Denon,	1799	448	728
French Savans,	1799	...	763·62
M. Caviglia,	1817	...	756
Howard Vyse,	1839	...	764

The list is interesting, as showing how little mere agreement amongst different reporters, as in 1647, 1655, 1661, and 1693, can be taken as proving that the truth has been arrived at. On the whole, the height has been exaggerated

by bad observers more than the base-breadth; the latter indeed, measured unfortunately on the northern side of the Pyramid only, having been apparently shortened as compared with the latest measures: but that is mainly due to the latter being carried on at the real base, or a broader part of the Pyramid than the older observers ever arrived at, and being also increased by application of the computed thickness of the casing stones to either side.

The French measure in 1799, and Col. Howard Vyse's in 1839, are indeed the only returns that can be accepted, because the only ones that really touched the original marks of the workmen; and these they obtained by sinking down through the sand-hills that have silted up all the lower part of the Pyramid for many ages, and then uncovering on the foundational rock-area of the whole structure the peculiar socket-marks of the old N.E. and N.W. corner stones, clearly and deeply cut into firm material. The honour of this very important discovery belongs to the French, and is detailed at length in their great work.

The mean of these two most useful French and English measures, gives 763·81 for the breadth of the side of the Pyramid base; and the angle we deduced before, gives with such base, the original vertical height equal to 486·2566 feet.

Now these are such extremely awkward fractions to have to deal with, that we may take them as conclusive against any measure like English feet having been employed in laying out the Pyramid. And if we seek on this principle for simpler numbers to give the proportion for the circular analogy, we find

116·5 and 366.

They are not exact, as no numbers can be when the proportion itself is incommensurable; but they are exceedingly close, the change for fuller accuracy being thus,

116·5014 and 366.
or 116·5 and 365·9956.

The number 366 in this case represents twice the base-breadth of the Pyramid, or what was employed against the height of the Pyramid (116·5) in representing the analogy of the circumference to the diameter of a circle. And as measures of space and time are often considered in company, it is worth while to remind, that 366 is also the nearest *even* number of days in a year; and more especially, that it is the number of whole turns made by the earth on its axis in the same time, as measured by the sidereal day. A length, therefore, of which 366 would measure the double base-breadth of the Pyramid, has some arithmetical, chronological, and dynamical arguments in its favour; in connection with the last of which, it further transpires, that such length has the very peculiar metrological import of being the $\frac{1}{10 \text{ millionth}}$ of the earth's axis of rotation; or of the only fully individual and correct reference for lasting national measures which the earth contains, now that its diameter at the Equator has been shown by recent geodesists to vary with the longitude.

To test such a point as this, we require, not only the nominal measures taken

at the Pyramid, but comparisons of the scales or rods employed there with those used in modern geodetic operations. These unfortunately we have not directly, and in all the cases which I have had an opportunity of examining, there had been a shortening of the scale in a hot, desert country. On a wooden scale, the dry weather might account for this; but on a steel rod where it certainly existed, it seemed only explainable by a slow contraction of the metal, gradually recovering itself from the operations of extension which it had undergone by hammering and rolling at the forge.

The French savans ought to have guarded against these sources of error, and their observation is evidently by far the more carefully taken of the two previously cited; and gives, reduced to inches, for the $\frac{1}{366}$th part of the double base-breadth, multiplied by 10 millions,

500,733,000;
Or, in using the $\frac{1}{365.25}$, 500,740,000.

Howard Vyse's measure, similarly treated, gives

500,984,000,
and 500,990,000.

But correcting his measure, by the proportions which his numbers for "the coffer" bear to those of Prof. Greaves, and correcting Prof. Greaves' measures by the French determination of the length of the foot which he engraved in the Pyramid on granite, immediately after measuring the coffer, and which foot is stated by M. Jomard, in the "Description de l'Egypte," to be 0·30460 of a metre (of which a whole length is equal to 39·37079 English inches), the above quantity should be reduced to

499,915,000,
and 499,921,000;

and the mean becomes, giving twice the weight to the French measure, as seems at least its due, from the superior care taken with it,

500,528,000,
and 500,535,000;
Mean = 500,532,000.

Now this quantity is quite contained within the variations of modern observations of the earth from each other, even in the best and latest geodetic comparisons; for they are given in the shape of different determinations of the length of the earth's polar axis in inches, by De Schubert, as varying between

500,560,000,
and 500,378,000;

but with much more inclination toward the former than the latter; and he is not materially differed from by the results of Bessel, Airy, Herschel, and Pratt; the latter, by his last refinements of taking into account the local disturbances at each astronomical station, making the polar axis equal to 500,523,000 inches nearly; while the others are all rather under 500,500,000.

We can hardly, therefore, do otherwise than conclude, if the supposition of a "lucky accident" be afterwards shown improbable, that the standard of length (=50·05 English inches nearly) in use at the Pyramid was, and was intended to be, in linear value $=\frac{1}{10 \text{ millionth}}$ of the earth's axis of rotation; and it is even further identified with the Pyramid in this manner:—

Mr Taylor has pointed out that there are reasons for concluding that inches were in use at the building of the Pyramid, and that these inches were slightly larger than ours, so that 500,000,000 of them measured the earth's axis of rotation precisely.

Now in the Pyramid, a body mathematically with 5 sides and 5 angles, everything may be expected to go by fives; and, accordingly, the inch, the *unit* of linear measure, is the one 5-hundred-millionth of the earth's axis of rotation.

The grand linear standard,—for scientific formative purposes, or those wherein the earth has to be alluded to as a whole,—and which we propose to call the "metron," viz. the $\frac{1}{500}$th of the double base, and the one ten-millionth part of the same axis, contains 5 times 10 units.

The smaller standard,—for convenient use, and for distance-measuring, wherein the half only of the earth requires to be referred to, because distances should be reckoned from the centre, and not either the nearer or further, surface of a sphere,—is $\frac{1}{500}$th of one side of the base; contains 5 times 5 units, and is the one ten-millionth of the earth's *radius* of rotation. While if a foot do not contain any even number of fives, it likewise is not any integral fraction of the earth's axis of rotation, is not a scientific standard, and, though tolerated in the base, has not been allowed to enter into any of the interior arrangements of the Pyramid.

(3.) *Figure of the Earth.*

If anything further could add weight to the belief of the polar diameter of the earth, so clearly hit by the actual measures of the double base-breadth, having been *intentionally* referred to for the linear unit of the Pyramid, it might well be the finding that certain other diameters are indicated in the building, and a knowledge of the full figure of the earth, with a purpose, thereby manifested.

Without having speculated originally about any such idea, something of the sort appeared spontaneously to me, when engaged in testing two "size-analogies," which Mr Taylor published for the second time in 1863, in his "Battle of the Standards." They had been examined by Sir J. Herschel in 1859 or 1860, and declared by him, in the Athenæum of 1860 for April, to be the only cases he then knew of, where a relation had been made out between the size of the Pyramid and that of the Earth; but he intimated that they were only rudely approximate. By aid, however, of the conclusions deduced from the base of the Pyramid in our last section, we can employ improved values, because the original ones, for both height and base, and therewith repeat the calculation under more favourable circumstances.

For the height, then, we take 116·5 metrons, or 5825 primal inches; for the base-breadth, 183 metrons, or 9150 primal inches; or 762·5 feet, *i.e.*, primal or Pyramid feet; and for the polar axis of the Earth, 500,000,000 of the same inches.

Then, the first of Mr Taylor's two analogies is, in the form chosen by Sir John Herschel, "a band encircling the earth, of the breadth of the base of the Great Pyramid, contains one hundred thousand million square feet."

Adapting this statement to our primal Pyramid feet, and to a form of expression suitable to bringing out the diameter of the earth, and in inches, we have

$$\frac{100{,}000{,}000{,}000}{\frac{762{\cdot}5}{12} \times 3{\cdot}14159} = 500{,}946{,}700 \text{ Primal or Pyramid inches.}$$

This resulting quantity, so much greater than the Polar diameter, may be referred, from the encircling-band manner of its derivation, to a mean latitude of 45°.

The second of the analogies is, that "the height of the Great Pyramid is $\frac{1}{270{,}000}$th of the earth's circumference."

Why, however, that particular fraction? Had it been $\frac{1}{250{,}000}$ or $\frac{1}{500{,}000}$, or even $\frac{1}{1{,}000{,}000}$, *i.e.*, anything easily made up of fives and times of five, there would have been a rude sort of "Pyramid reason" in it: but for 270,000 I could find no reason. Considering, however, the act of standing by the Pyramid on its base, in this inquiry where its height is to be measured against the earth which it is standing upon,—and finding that the area of its base in hundredths of feet has, when thrown into a circular shape, a circumference equal to 270,299,—I presumed that that might be accepted, if not as a reason, at least as an apology, for trying the case with that number. Arranging the expression accordingly so as to bring out the axis in inches, there appears,

$$\text{Pyramid height in inches} \times \frac{270{,}299}{3{\cdot}14159} = x;$$

$$\text{or } 5825{\cdot} \times 86038{\cdot}901 = 501{,}176{,}400 \text{ Primal or Pyramid inches.}$$

This diameter of the earth, obtained so directly from the height and act of standing of the Pyramid on its own base, must, if any one result could do so more than another, refer to the diameter in the Pyramid's own latitude: and the Pyramid's position is stated in the French maps, and most of their memoirs, to be, 29° 59′ 6″; but may be assumed in such an approximate case as this, to be 30°.

Computing next, from a theoretical polar diameter of 500,000,000, the values for latitudes 45° and 30°, with a compression of $\frac{1}{300}$, we have the following to compare with the Pyramid deductions:—

Computed Earth Diameters.	Pyramid Analogies.
Polar, . 500,000,000.	Polar, = 500,000,000.
Lat. 45° . 500,840,000.	45° = 500,946,700.
Lat. 30° . 501,257,000.	30° = 501,176,400.

Simple inspection of these numbers will show at once, that by far the greater portion of the whole difference from the Pyramid-base deduction, assumed polar,

is explained by the compression hypothesis, combined with the latitudes 45° and 30°. The amount of difference left outstanding, is indeed much smaller than the limits of errors of observation in the best modern measures of the earth's polar diameter: so that perhaps we ought therewith to be content. But of course the question will be asked, in which way do the residual differences point, *i.e.*, for more, or less, polar compression than that favourite quantity of $\frac{1}{300}$?

And then comes the answer; that they indicate the average quantity of $\frac{1}{300}$ to be exceedingly close to the truth; but accompanied by an irregularity, tending to produce a protrusion near Lat. 45°, at the expense probably of the neighbouring parts; and inclining to give the earth, in meridian section, something of that squarish look, which Sir WILLIAM HERSCHEL thought that he had detected telescopically in the planet Saturn. Mr MAIN'S Greenwich observations of Saturn are, indeed, considered in some quarters to have annihilated the Herschelian idea; and they have proved that there is no very large quantity of such squareness; but such a moderate amount as what is indicated by the Pyramid analogies, is far beyond the powers of astronomical micrometer observation, in our bad observing atmosphere and even in the present day, to hope to reveal.

(4.) *Latitude Markings.*

The grounds on which we have hitherto considered that the latitudes 45° and 30° were intended to be alluded to in the Pyramid, are certainly of a hypothetical character; and therefore, though the hypotheses be ever so sound, they will be improved and strengthened in the opinion of many persons, if proofs of a practical character can be found to support them.

Proofs too, of this order, do really appear to exist in the building still; though we must begin with something of hypothesis to arrive at them.

In figure 3 (Plate XXIII.), therefore, is a carefully drawn theoretical meridian section of the Pyramid, as in fig. 1, (p. 668); but in place of the large square of the base, there is now a small square, symmetrically placed with it. The size of this small square is 103·246 metrons in the side, and is obtained by computation as being equal to the area of the Pyramid's meridian section.

This small square, in itself, and by some very simple divisions, shown by the dotted lines in fig. 3, enables us to arrive quickly at the placings of all the few chambers and passages in the Pyramid.

To prove this important relation, fig. 4 (Plate XXIV.) contains a careful copy of Colonel HOWARD VYSE'S large meridian section of the Pyramid, in the 1st volume of his great work;* and to facilitate the comparison, I have added to it the chief lines of the hypothetical Plate XXIII. fig. 3, in rows of dots.

* This section, like most meridian sections published, since the time of Professor GREAVES in 1637, agrees to overlook the small distance by which the passages of the Pyramid, though truly in the plane of the meridian, are slightly to the east of the true central meridian section of the Pyramid. See fig. 7, Plate XXVII.

First, we may remark, of the chambers,—the top of the topmost chamber of construction is on the level of the upper side of the square EF. The floor of the so-called "King's chamber," is nearly on a level with the line LN, or $\frac{1}{3}$ the semi-diameter below EF. The floor of the "Queen's chamber," so called, is nearly on a level with the line OP, or $\frac{2}{3}$ the semi-diameter below EF; and the floor of the subterranean chamber is on a level with the line C′S, or $\frac{1}{2}$ below the centre; and these are all the chambers known to exist in the Pyramid.

Second, of the passages, principal. The entrance passage places itself exactly on our hypothetical line C′*′, until very near its lower termination. The ascending passage, with the floor of the grand gallery, also place themselves exactly on our hypothetical line, in fig. 3, QRL; and the horizontal passage places itself also on our line, RO.

Now, the angular direction of C′*′, (QRL being the same angle inverted), is made by construction the same as that of C*; and that again depends on the height of the Pyramid as radius, and the semi-side of the square of area as sine; and $=26^\circ\ 18'\ 10''$. This might be looked on at first as a pure geometrical result, but it depends on the measured height and base of the actual Pyramid; and, further, it agrees unexpectedly with the measured angle of the entrance passage, which has hitherto been universally allowed an *astronomical* signification; viz. to point to α Draconis, the Pole star of 2121 B.C., at its lower culmination, or, according to Sir JOHN HERSCHEL, $26^\circ\ 15'\ 45''$; and inasmuch as the best measures of the actual angle of the Pyramid passages are anywhere between $25^\circ\ 55'$ and 27°, our hypothesis is as fair as it need be.

That hypothesis next gives the line CE, as at an angle of 45°, of course; but it is further, and architecturally testified to in the Pyramid, according to Col. HOWARD VYSE's plate, by the line of the Southern air-channel being also at an angle of 45°; or one of the geodetic indications we are seeking.

The Northern air-channel on the contrary, his plate makes at an angle of 33° or 34°, for the drawing is not fully accurate; let us say, then, $33^\circ\ 42'$. If, then, the mean of this and the angle of the entrance passage, the only remaining communication from the Pyramid's interior to the outer air, on the same northern side be taken, it is equal to 30°; or the same as our hypothetical line C′ 30°′, which is by construction = to C 30°, and which depends on the semi-diameter of the area-square as sine, and the radius of a circle of equal area to the base of the Pyramid, as radius; and is the other geodetic indication being sought.

Strictly computed, with the 116·5 and 183 metrons to start from, this theoretical Goedetic or Latitude angle is

$$29^\circ\ 59'\ 59{\cdot}2'',$$

and reminds us that the observed Latitude of the Pyramid in 1799, was less than 30°, and more than $29^\circ\ 59'$.

On the one hand, however, it may be said, that, given a Pyramid built to realise

the abstract circular analogy, or Mr TAYLOR's first proposition,—an angle C 30° *must* come out very like what we have educed: no matter in what Latitude soever of the earth such a Pyramid were planted down.

That is quite true: but then, on the other hand, we are assured by the observations of the French Academicians, for the azimuth of the sides of the Pyramid, and which they found under 20′ of error,—that astronomy had a most important share in the foundation of this Pyramid: and therefore, when we find appended to the above-mentioned construction-angle of 29° 59′ 59·2″ a certain other angle of 26° 18′, which astronomers have already proved before the world, to have been intended for the lower culmination of the Pole star of the Pyramid-building period,—there seems hardly anything else that we may conclude, except that the first angle is astronomical also, and represents the latitude at the time.

For the mere purpose of checking a determination of the terrestrial polar compression, the difference of 53″, or less, in the ancient and modern latitude is of no great importance. But is that difference explainable by a slow change of the latitude of places on the earth, insensible from year to year, though notable in 4000 years? Perhaps it is so; for the latitude of Greenwich has been similarly *decreasing* under its three last Astronomers-Royal, to the extent of almost 2″ in 100 years; partly from other known or suspected causes, but not entirely.

It may again be objected, that if the latitude of the Pyramid is to be typified, as above, in the proportions of certain of its parts,—there is only one parallel of latitude, where such a Pyramid could be set up, and preserve alike both its geometric truth, and its astronomical and geographical indications.

To this, we may say, that the remark is perfectly just, and will prevent other Pyramids in Northern or Southern countries ever competing with the scientific importance, and fullness of meaning of the Great Pyramid; while if, as we may be presently able to show, the founders of the Great Pyramid were not native Egyptians, but came into Egypt from a country in another latitude, and went back to it again after finishing the Great Pyramid in Egypt, but built no similar Pyramids in their own country, it would almost appear that they understood the weight of the modern objection to any other parallel of latitude than that which they employed, and even went out of their way to seek.

In the meantime, we merely conclude this section with a few of Col. HOWARD VYSE's measures of the Pyramid, to check his drawing, or rather our small reproduction of it, before going forward to some more important consequences.

Approximate Pyramid Dimensions. By Col. HOWARD VYSE, 1839.

	Feet.	Inches.	Inches.
Former base,	764	0	9168
Present base,	746	0	8952
Present height, vertical,	450	9	5409
Perpendicular height from base to bottom of entrance,	49	0	588
Distance of centre of the entrance, eastward from centre of Pyramid,	24	6	294

Approximate Pyramid Dimensions—continued.

	Feet.	Inches.	Inches.
Angle of entrance passage = 26° 41′.			
Length from beginning of roof to the junction of ascending passage,	63	2	758
From thence to end of descending part,	= 257	8	3092
Depth from base of Pyramid to roof of subterranean chamber,	= 90	8	1088
First ascending passage, length,	= 124	4	1492
Angle = 26° 18′.			
Grand Gallery, length,	= 156	0	1872
Horizontal passage, length,	= 109	11	1319
Northern and southern air-channels,—			
Inclined height from base of Pyramid,	= 331	0	3972
Northern channel, length,	= 233	0	2796
Southern channel, length,	= 174	3	2091
From base of Pyramid to floor of King's Chamber,	= 138	9	1665
Height from floor of King's Chamber, to roof of Col. Campbell's chamber, or "topmost chamber of construction,"	= 69	3	831
Height from base of Pyramid, to floor of Queen's Chamber,	= 67	4	808

(5.) *Unique Interior.*

Though Pyramids are numerous in Egypt, and may be counted, some say by hundreds, there is not a second instance, to be found in any part of the land, of the full interior arrangements of the Great Pyramid of Jizeh.

In proof of this remarkable assertion, the reader may be referred either to Colonel Howard Vyse's and Mr Perring's very numerous plans and sections of all the principal pyramids in Lower and Middle Egypt, or to the "*theory of pyramid structure*" put forth by Dr Lepsius, and Mr Wild, architect, and testified to by Bonomi, Gliddon, and almost all other modern Egyptologists. The theory is, when abstracted to set forth the subject we are dealing with, that a pyramid is a king's tomb, built by himself: that he begins on his first accession to the throne, by excavating deep in the rocky soil a sepulchral chamber, reached only by a descending passage; and that he then goes on adding masonry every year round about a nucleus erected vertically over the said chamber, until he dies. When the upper pyramidal mass has spread at its base, by yearly additions, beyond the descending entrance-passage's mouth in the rocky soil,—it, the passage, is continued upwards, and at the same angle as before, through every additional layer of masonry. But when the king dies, his body is conveyed down that inclined passage, and deposited in a stone sarcophagus, in the underground chamber; after which the entrance to the passage is sealed up, and all the rough rectangular corners of the layers of masonry on the outside of the pile are bevelled off, *in situ*, to one, smooth, inclined surface.

Now that mere subterranean chamber with a descending passage, which is all the internal arrangement implied in the *theory*, and all that is found in general Egyptian practice; (except when one king has broken into the Pyramid cellar of another, and made himself a subterranean sepulchre and entrance-

passage from a different side, by digging into another's property; and even then it is still the same order of arrangement),—this one principle or feature, we say, exists also in the Great Pyramid (see fig. 4); and is exemplified in the one descending entrance-passage, and the subterranean chamber marked *c*.

That particular portion, and that portion only, of the internal arrangement of the Great Pyramid, is a common Egyptian institution; and there are proofs in fact that the Romans were once inside that chamber. (Col. HOWARD VYSE, 2d vol. p. 290.)

But all the upper parts of the arrangements of the interior (see the same fig. 4), all that is gained by the *ascending* passages, are absolutely peculiar to the Great Pyramid alone; and there are no proofs whatever that Romans, Greeks, Persians, or the Egyptians themselves, subsequent to 2130 B.C., knew anything whatever about the existence of that part of the interior. Its builders had in fact sealed it up carefully; and their stone-seal remained on, and kept its secret faithfully, until it fell off, of its own accord, or in a manner rather more than accidental, in the time of Caliph Al Mamoun, about 820 A.D.

Then it was that the first symptoms of there being an ascending passage were perceived: It could not, however, be rushed straight into by the eager beholders, because an unliftable portcullis of granite stopped the way; but by breaking a path through the smaller masonry, Al Mamoun and his people succeeded in entering the same ascending passage further on; and from thence pushing forward, fired by the hope of seizing "on the wealth deposited by the antediluvian kings of the earth," they passed through the "Grand Gallery," the little ante-chamber, and then entered the so-called King's Chamber, evidently the principal chamber of the whole Pyramid, the last of its series of rooms and passages, and the one for which the entire structure had been erected.

A magnificent room it was, 34 feet long, 17 broad, and 19 high, formed on every side, floor, walls, and ceiling, of enormous blocks of granite, perfectly flat, admirably polished, and fitting to each other so closely, that the smallest needle could not be introduced into any of the joints. But there was nothing in the chamber! The eager visitors looked about (by torchlight, for all of the interior of the Pyramid, save the entrance passage, is necessarily unvisited by the light of day), and could find absolutely nothing; unless indeed a stone trough, as they called it, or a granite chest; but if a chest, it had no lid and was entirely empty! The Caliph and his companions were thunderstruck; their treasure theory had so completely failed; and though they afterwards quarried holes in different parts of the exquisite building, they found no jewels of silver or jewels of gold, beyond the one particular pot of money which Al Mamoun himself had just previously buried, *to encourage the workmen*.

Previous to Al Mamoun's entrance into the Pyramid, the Arabian writers had indulged in every kind of rhapsody as to the extraordinary treasures which it contained; and soon after his signal failure, they recovered somewhat of their spirits, and began, in their patron's praise, to chaunt his astonishing findings;

but in terms that now prevent their being listened to with patience: while, seeing that some of them declare that the king's chamber was admirably sky-lighted, when we know the interior to be absolute darkness; and that its ceiling was covered with inscriptions, when the whole of the finished interior is positively without a stroke of inscription,—shows how little we can trust their statements, either that a dead body having a golden breastplate, was found in the stone box, with a sword of inestimable value, and a carbuncle the size of an egg, shining like the light of day; or, that the said box was full of gold in coin of very large size.

Ages passed after Al Mamoun's essay, European travellers began to look in at the Pyramids, and rather patronised the corpse notion: for the stone chest, or marble hot-bath, or porphyry coffer, as it was variously called, was very much in shape like the lower part of an Egyptian sarcophagus.

But then in that case, why was it, the coffer, so entirely without ornament; why so utterly without inscription, when that would be the precise place where Egyptians would have lavished their inscriptions; why without a cover, and without any fixing places to receive a cover; and then who ever heard of a corpse being buried above, and so much as 140 feet above, the level of the ground: and, lastly, if the room was intended for a corpse only, why was it so well ventilated, by the two remarkable and effective air-channels (see Plate XXIV.) discovered by Colonel Howard Vyse?

In short, the failings of the sepulchral theory were so many, as of themselves to give rise to the idea with some philosophic minds who weighed everything carefully (such as the French Academicians in 1799, and Sir Gardner Wilkinson in 1858), that no body of a king had ever been deposited in that porphyry coffer. The Academicians even boldly expressed a belief that the said coffer might probably have been intended for something entirely different; or, for a standard of linear measure. This idea, however, they were not able to prove; and even the very memory of it seemed to be lost within a few years after, by reason of the excessive applause which followed the discovery of hieroglyphical interpretation.

Hieroglyphics, however, have done nothing for the explanation of the porphyry coffer, because indeed it has no hieroglyphics, and in the meanwhile Mr Taylor comes out with a new and striking idea,—*

The porphyry coffer, says he, is a standard measure, not of length, but of capacity and weight; it was the original of all corn measures; the Pyramid itself receiving its name from πυρος wheat and μετρον measure; and the so-called "Quarters," in which the British farmer, up to the present day, measures his wheat, are accurately quarters of the cubical contents of the porphyry coffer in the King's Chamber of the Great Pyramid.

* While at press, I am informed of the existence of a rare pamphlet, but have not yet been able to see it. *The Origine and Antiquitie of our English Weights and Measures discovered by their near agreement with such standards, that are now found in one of the Egyptian Pyramids. London.* 1706. *Anonymous:* and reprinted in 1745, with the authorship attributed to Professor Greaves, of Oxford, who died in 1652.

Let us look into the accuracy of this very suggestive statement.

(6.) *The Porphyry Coffer; its Size and Material.*

The following table contains all the printed particulars which I have been able to collect, touching the measured size of the coffer; and it will be seen, notwithstanding the popular impression as to the most accurate admeasurements possible having been repeated again and again on every part of the Pyramid,—that these measures of the coffer are, on the whole, so bad, as to reflect not a little discredit on almost all those modern civilised nations they emanate from; and who, when at home, employed on their *own* standards, measure *them*, they say, to seven or eight decimals of an inch, true.

Some of the measurements may be in French inches, and therefore require an increase of their number to represent English inches, but that would not much improve the real diversities of the measures by different men.

MODERN MEASURES OF THE PYRAMID COFFER.

AUTHORS.	Date. A.D.	MATERIAL.	EXTERIOR.			INTERIOR.		
			Length.	Breadth.	Depth.	Length.	Breadth.	Depth.
			Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
Bellonius, . .	1553	Black marble	144·	72·	...	...	...	...
P. Alpinus, . .	1591	Black marble	144·	60·	60·	...	...	...
Sandys, . . .	1610	.....	84·	47·	...	...	...	...
De Villamont, .	1618	Black marble	102·	...	60·	...	...	...
Prof. Greaves, .	1638	Thebaic marble	87·5	39·75	39·75	77·856	26·616	34·320
De Monconys, .	1647	.....	86·	37·	40·	...	...	...
M. Thevenot, .	1655	Hard porphyry	86·	40·	40·	75·?	29·?	...
M. Lebrun, . .	1674	.....	74·	37·	40·	...	...	...
M. Maillet, . .	1692	Granite	90·	48·	48·	...	...	...
De Careri, . .	1693	Marble	86·	37·	39·	...	...	...
Lucas, . . .	1699	Like porphyry	84·	36·	42·	74·?	26·?	...
Egmont, . . .	1709	Thebaic marble	84·	...	42·	72·?	...	...
Père Sicard, .	1715	Granite	84·	42·	36·	...	...	. .
Dr Shaw, . .	1721	Granite	84·	36·	42·	72·?	24·?	...
Dr Perry, . .	1743	Granite	84·	30·	36·	...	...	...
M. Denon, . .	1799	?	84·	48·	38·	...	...	...
M. Jomard, . .	1800	Granite	90·592	39·450	44·765	77·836	26·694	37·285
Dr Clarke, . .	1801	Granite	87·5	39·75	39·75	...	...	...
Mr Hamilton, .	1801	Granite	90·	42·	42·	78·?	30·?	...
Dr Whitman, .	1801	.....	78·	38·75	41·5	66·?	26·75?	32·
Dr Wilson, . .	1805	.....	92·	38·	...	80·?	26·?	34·5
M. Caviglia, .	1817	.....	90·	39·	42·	78·?	27·?	...
Dr Richardson,	1817	Red granite	90·	39·	39·5	...	...	...
Sir G. Wilkinson,	1831	Red granite	88·	36·	37·	...	...	...
Col. Howard Vyse, .	1839	.....	90·5	39·0	41·0	78·0	26·5	34·5

When a note of interrogation has been applied to an interior measure, such measure has been obtained from the exterior one, through means of the *thickness* as given by the observer in question.

The three chief authorities who, for accuracy, distance all the rest, are

Professor GREAVES in 1638,
M. JOMARD in 1800,
And HOWARD VYSE in 1839.

Yet even on their measures it would be dangerous to speculate on what the *outside* dimensions of the coffer may be, to within an inch or two.

Fortunately, for a *capacity* measure, we have not to pay much attention to the exterior; and the *inside* dimensions are given with less uncertainty. Nevertheless, HOWARD VYSE's are rude beyond toleration in such a case (only to the nearest half inch); and the French measures are vitiated in the depth element, by a blunder of nearly three inches; *i. e.*, it seems so to my best judgment.

Hence, the ancient Oxford Professor is the only authority left outstanding; and happy it is that he so far transcended his age's ideas of accuracy in such admeasurements; and thought, too, so much more of the interior, than the exterior.

There is, indeed, some doubt as to the real value of the length of a foot on the scale employed by GREAVES, and that known as the English foot in the present day; and there, the French academicians have done good service; for GREAVES, immediately after his measure of the coffer in 1638, marked off the length of one foot of his scale on the unchanging granite of the walls of the chamber; and these marks being visible in 1799, M. JOMARD measured them and found them $=0{\cdot}3046$ of a French metre; whence, on the understanding that one French metre is equal to 39·37079 modern English inches, we have the following corrected statement of GREAVES' measure, viz.—

Interior length	$=77{\cdot}806$
,, breadth	$=26{\cdot}599$
,, depth	$=34{\cdot}298$

and these being multiplied together, yield

70,982·4

English cubic inches, as the observed contents of the Pyramid coffer.

If we then turn to the British Act of Parliament defining from the bushel the number of cubic inches in a "Quarter," and multiply that by 4, we have,

70,982·1;

a closeness of approach which is something more than startling.

As to the material of this remarkable vessel, it would be hazardous to venture much upon what it is exactly, seeing that it has been described as everything, from black marble to red granite; but we rather incline, from private information, to "porphyry;" and, in the event of the metrological character being fully established, it would be important to ascertain both precisely what sort of porphyry it is, and where it came from. At present, we must rest satisfied, with

its having apparently most admirable qualities for a standard measure, viz., extreme hardness, utter unoxidisability, perfect freedom from flaws, small expansion from heat and extraordinary great age.

(7.) *Why of that size?*

Given, agreeably with the observations detailed, that the cubical contents of the coffer *are*, 70,982·4 cubic inches,—let us inquire, why are they so? or, why was the coffer capacity measure, made of that particular size?

With the linear standard, if a similar reason be asked, the Pyramid answers at once, because that length is the one ten-millionth of the earth's axis of rotation: but if the question be asked an Englishman, touching his bushel,—he can only say, accident; and if the modern Frenchman, touching his "litre,"—he can only say, that it is the cube of an arbitrary fraction of his linear metre

Now it is the intellectual boast of the Frenchman that his linear metre is an integral fraction of a linear proportion of the earth itself, or rather, a quadrant of the meridian; then why did he not try to get a similar scientific merit for his capacity measure, viz., that it is an integral fraction of the capacity of the earth? But he did not do so; nor did he contrive for his weight measure, that it should be a similar fraction of that all-important physical characteristic, the weight, of the whole earth-ball; whose general figure, and one and only property, as implied by the French metrical system, is. that of being a curved line, like the boomerang of an Australian savage—a flaw, certainly, in the scientific recommendation of the French system. Then how does the same point fare in the Pyramid?

I had been quite at a loss to find out any reason there, for the size of the coffer; until certain features of the Pyramid itself led me to an idea, which was not in my mind before. Over the entrance door into the chamber containing the coffer, are five vertical parallel lines, or, as some say, a space divided into five equal parts. This should have reminded any one of the Pyramid standard, and kept him true to that, as indicating the foundation of what was within.

Now $50^3 = 125000$, and that is much more than the cubic contents of the coffer; besides indicating a rectangular figure for the earth. *Three* dimensions must be taken, but they belong to a spherical body, or, as the latest geodesists have shown, to an ellipsoid with *three axes*, viz., the Polar, and the major and minor of the Equator; and this seemed to be intimated by the three curved hollows in the antechamber, described by many authors, and pictured in our Plate XXV. fig. 5. Reducing the cube of 50 to a sphere's contents, will however not give the coffer's contents; it will give the same number of *places* of figures, but their value is too small.

Seeing, however, that weight-measure usually goes with capacity; and this is typified in the ante-chamber, by the suspended block of granite in the 4th groove, close to the semicircular hollows; and being reminded by the 5 chambers of

construction above the coffer room, of the fraction representing the earth's *mean density*,* I tried applying that to 50^3, and got the contents of the coffer at once, though in the form of being multiplied by 10. Striking off, however, one figure,—so as to have the number of places of figures as given by the pure capacity reduction,—then the precise value of the coffer's cubic contents in inches was obtained, to at least this very close degree of approach, viz.:

$$\frac{50^3}{10} \times 5{\cdot}672 = 70{,}900;$$

but 70,900 being Pyramid inches, must be increased to

$$70{,}970{\cdot}2$$

in order to represent the same in English inches. The measured quantity, in the same terms, it will be remembered, gave 70,982·4; but we shall now use the theoretical determination of 70,970·2 in preference.

This then forms a definite metrological quantity, derived thus simply from the one ten-millionth of the earth's axis of rotation, influenced only by the combined qualities of the earth's capacity power, and mean density, or weight-power; and has therefore eminent scientific recommendations for the grand standard of capacity and weight measures.

(8.) *Of what Weight?*

The capacity of the coffer is already given at 70,970·2 English cubic inches; but what weight shall be assigned to it?

The best plan is, to fill the vessel of that given size, with pure water, and weigh the contents. Now the Great Pyramid seems to have had in its normal state a deep well penetrating down to lower strata, soaked with and filtering the Nile water: while the so-called Queen's Chamber, with its peculiar depressed floor and air-tight masonry, served as a reservoir at once large and conveniently placed: water therefore could be had. But then, of what temperature would it be? for water alters its density rapidly on heating or cooling.

The mere mention of this scientific requirement of modern times, seemed at first to threaten ruin to the sufficiency of any arrangement descended from primeval days of the world, to meet the exacting demands of present physical science; but

* The history of the experimental determination of the earth's mean density, is a very interesting one, and its honours fall almost entirely to Great Britain. It has been tried by the attraction of the plumb line on mountains: by the effect on a pendulum at the top and bottom of a mine, and by the "Cavendish experiment," between the parts of a philosophical apparatus; and has varied in the first case from 4·5 to 5·4; in the second from 6 to 6·5; and in the last from 5·4 to 5·8; or in the latest, and most perfect trial of it, by FRANCIS BAILY, from 5·68 to 5·66. His own published mean is 5·675, but uncorrected for some circumstances which he himself thinks should be corrected, and which we have estimated, in accordance with his numerical indications, at −·003.—*See further at p.* 699.

the result of examination has been the crowning of the Pyramid system with some most unexpected features of practical success, and purposed intention.

To "correct for temperature," is a very interesting occupation in Natural Philosophy, whether accomplished by calculation, or by instrumental appliances of compensation; but in practice, and where extreme accuracy is required, it is found that heat is so excessively subtile an influence, and has so many various actions on bodies, sometimes with a secular, and at others with a periodical effect, —that the safest plan by far is, to reduce the amounts of temperature variations themselves to the lowest possible ebb; and then only have to deal with the effects of the very small residual quantities of heat so left. Hence at Pulkova, Paris, Edinburgh, and some of the most accurate Observatories in the world, great advantage has recently been found, by placing the sidereal clock of each establishment, though armed with a reputed temperature compensation-pendulum, under circumstances where variations could not happen so easily, quickly, or to so great an amount, as in the open room: and every increased degree of such protection has been attended by better performance of the clock.

The simple placing of the clock in a large closet, has been of sensible service; but much more good has been obtained by establishing it in a cellar, as at Pulkova; and more still, in a *cave* 95 feet under the surface of the ground, as at Paris. But at no Observatory in the world have they a room, like that of the King's Chamber containing the porphyry coffer or weight and capacity standard, in the Great Pyramid, protected by nearly 180 feet in depth, of solid stone on every side. In such a room, the semi-annual variations of atmospheric temperature may be cut down from 50° Fahr. to something less than ·01 of a degree.

Hence the interior Pyramid temperature, must, so far as depends on external natural causes (for of course it is not proof against numerous Arabs inside, with blazing torches), be practically constant from day to night, and summer to winter, and year after year. There is evidently, then, a peculiar temperature to the Pyramid; which, in Metrology would form a valuable constant.

What then, is, that temperature? Or, rather, what used it to be, when the building was in its normal state, throwing off the sun's hot rays from its polished white casing-stones, and having the dryness of its atmosphere corrected by watery vapour effused from its lower, open water-well? see Plate XXIV.

As the Pyramid has never been observed by moderns under these circumstances, we must seek our data from various older quarters; confined however practically to the French alone; amongst whom M. JOMARD states that he and his compatriots, in 1799, noted the temperature of the King's Chamber to be 22° cent.; of the lower part of the dry-well, 25° cent.; and of certain tombs outside the Pyramid, also, 25°.

Now these two last sites are nearly on the same level, and a few feet under the general surface of the ground; it is therefore right that they should have the

same temperature. But the other station, though inside the Pyramid, is 140 feet above the ground outside; it is therefore also right that its temperature should be less, perhaps about 22°.

But this temperature was raised unnaturally by the presence of Arabs and their torches, and by the absence of watery vapour. Referring therefore to the same philosopher's measure of the temperatures of the great Joseph well in the citadel of Cairo,—and where there was probably rather too much watery vapour,—and finding him give that temperature as 17° or 18° centigrade,—we may suspect that the mean between these two and what was observed in the King's Chamber, at least to the nearest even degree, would be the true result for that chamber under normal circumstances.

We conclude, therefore, from thence, that the Pyramid constant is 20° centigrade; or, 68° Fahrenheit. But the note worthy of remark about that particular point is, that it defines what may be called a temperature of $\frac{1}{5}$th; *i.e.*, $\frac{1}{5}$th the distance between freezing and boiling, upwards from the freezing point; exhibiting again in the most unexpected manner, the typical division of the Pyramid. Hence there can be little hesitation in adopting 68° Fahr.; and inasmuch as one English cubic inch of distilled water weighs 252·458 English grains at a temperature of 62° Fahr., and Barometer at 30 inches,—the coffer, measuring 70,970·2 English cubic inches, would weigh 17,917,000 English grains, with the contained water at that temperature. But reducing the weight of the water from its density at 62° to that at 68°, the quantity becomes 17,905,500 English grains.

This, therefore, is the total weight of the Pyramid's grand standard of weight measure; excepting my own possible errors of reduction.

(9.) *Pyramid Weights and Measures.*

With the capacity of the coffer, its water-weight, and the linear standard, as already determined, we have now to see what sort of a commercial and scientific system of measures they are capable of affording, and for small as well as for large quantities; especially, too, in how far such a Pyramid-derived system may agree, and in how far it may differ in the subsidiary items, from what is at present in use in Great Britain.

The chief point of difference will evidently be in the divisors; for the Pyramid can acknowledge of little beyond fives and times of five; and they are not found frequently in the British system. Nevertheless, it is remarkable to see in how many instances the two arrangements approach each other; and that in some cases, where a closer approach might have been desirable, it is found by appealing from the "Imperial System" of George IV. to the ancient British weights and measures.

Pyramid Capacity Measure.

Denomination.				Reference in Pyramid cubic inches.		Reference in Pyramid pounds of water-weight.
Unit	=	1 drop	=	0·002836	=	0·0001
100 drops	=	1 dram	=	0·2836	=	0·01
10 drams	=	1 oz.	=	2·836	=	0·1
10 oz.	=	1 pint	=	28·36	=	1·0
10 pints	=	1 gallon	=	283·6	=	10·0
10 gallons	=	1 bushel	=	2836·0	=	100·0
2·5 bushels	=	1 sack	=	7,090·0	=	250·0
*10 sacks	=	1 coffer	=	70,900·0	=	2,500·0

Or arranged in double entry,

Drops	100	=	dr. 1											
	1,000	=	10	=	oz. 1									
	10,000	=	100	=	10	=	pint 1							
	100,000	=	1,000	=	100	=	10	=	gall. 1					
	1,000,000	=	10,000	=	1,000	=	100	=	10	=	bush. 1			
	2,500,000	=	25,000	=	2,500	=	250	=	25	=	2·5	=	sack 1	
	25,000,000	=	250,000	=	25,000	=	2,500	=	250	=	25·	=	10	= coffer 1

And compared with the Imperial system, through the temporary medium of English cubic inches,

Pyramid Capacity Measures.		English Cubic Inches.
1 drop	=	0·0028388
1 dram	=	0·2838808
1 oz.	=	2·838808
1 pint	=	28·38808
1 gallon	=	283·8808
1 bushel	=	2,838·808
1 sack	=	7,097·02
1 coffer	=	70,970·2

British Capacity Measures.		English Cubic Inches.
1 drop Apoth.	=	0·0036103
1 dram Apoth.	=	0·2066187
1 ounce Apoth.	=	1·73295
1 pint Imperial	=	34·659
1 gallon Imp.	=	277·274
1 bushel Imp.	=	2,218·192
1 sack Imp.	=	6,654·576
4 quarters Imp.	=	70,982·144

Pyramid Weight Measure.

Denomination.				Reference to Pyramid Cubic Inches of Water.		Reference in Terms of Pyramid Pound.
Unit	=	1 grain	=	0·002836	=	0·0001
100 grains	=	1 dram	=	0·2836	=	0·01
10 drams	=	1 oz.	=	2·836	=	0·1
10 oz.	=	1 pound	=	28·36	=	1·0
10 lbs.	=	1 stone	=	283·6	=	10·0
10 stone	=	1 cwt.	=	2,836·0	=	100·0
2·5 cwt.	=	1 wey	=	7,090·0	=	250·0
10 weys	=	1 ton	=	70,900·0	=	2,500·0

And we have arranged for double entry,

* In place of 10 *sacks* = 1 *coffer*, there may be used, 2·5 *sacks* = 1 *quarter*; and, 4 *quarters* = 1 *coffer*.

Grains	100	=	dr. 1									
	1,000	=	10	=	oz. 1							
	10,000	=	100	=	10	=	lb. 1					
	100,000	=	1,000	=	100	=	10	=	st. 1			
	1,000,000	=	10,000	=	1,000	=	100	=	10	=	cwt. 1	
	2,500,000	=	25,000	=	2,500	=	250	=	25	=	2·5	= wey 1
	25,000,000	=	250,000	=	25,000	=	2,500	=	250	=	25·0	= 10 = ton 1.

And compared with British imperial weight measures, through the temporary medium of English grains,

Pyramid Weights.		English Grains.
1 grain	=	0·71622
1 dram	=	71·622
1 ounce	=	716·22
1 pound	=	7,162·2
1 stone	=	71,622·
1 cwt.	=	716,220·
1 wey	=	1,790,550·
1 ton	=	17,905,500·

	English Grains.
1 grain old English	= 0·75000
1 grain modern English	= 1·00000
1 dram Avoird.	= 26·71875
1 dram Apoth.	= 60·00000
1 ounce Avoird.	= 427·5
1 ounce Troy, Apoth.	= 480·0
1 pound Avoird.	= 7,000
1 pound old Engl. & Scot.	= 7,600
1 stone, meat	= 56,000
1 stone, coal	= 98,000
1 cwt. Avoird.	= 784,000
1 wey, English	= 1,274,000
1 ton Avoird.	= 15,680,000
1 ton shipping	= 18,816,000

Pyramid Linear Measure.

Denomination.				Reference to Earth's Axis of Rotation.		Earth's Radius of Rotation.
Unit	=	1 inch	=	$\frac{1}{500,000,000}$		
and 12 inches	=	1 foot	=	$\frac{1}{41,666,666·6}$ &c.		
but 25 inches	=	1 arm	=	.....	=	$\frac{1}{10,000,000}$
10 arms	=	1 rod	=	.....	=	$\frac{1}{1,000,000}$
10 rods	=	1 acre-side	=	.....	=	$\frac{1}{100,000}$
25 acre-sides	=	1 mile	=	.....	=	$\frac{1}{4,000}$
4 miles	=	1 league	=	.....	=	$\frac{1}{1,000}$

And arranged for double entry,

Inches	12	=	ft. 1											
	25	=	,,	=	arm 1									
	250	=	,,	=	10	=	rod 1							
	2,500	=	,,	=	100	=	10	=	acre-side 1					
	62,500	=	,,	=	2,500	=	250	=	25	=	mile 1			
	250,000	=	,,	=	10,000	=	1,000	=	100	=	4	=	1 league	

And compared with English linear measure, through the temporary medium of English linear inches,

Pyramid Linear Measures.		English Inches.	English Linear Measures.		English Inches.
1 inch	=	1·000992	1 inch	=	1·000
12 inches	=	12·011805	12 inches	=	12·000
1 arm	=	25·0248			
1 rod	=	250·248	1 rod	=	198·000
1 acre-side	=	2,502·480	1 acre-side	=	2,504·525
1 mile	=	62,562·000	1 mile	=	63,360·000
1 league	=	250,248·000	1 league	=	218,721·6

For the corollaries of linear measure, viz., square and cube, we have, for Pyramid surface measure,—

Pyramid Surface Measure.

144	Square Pyramid inches	=	1 Square Pyramid foot
625	Square Pyramid inches	=	1 Square Pyramid arm
100	Square Pyramid arms	=	1 Square Pyramid rod
100	Square Pyramid rods	=	1 Pyramid acre
625	Pyramid acres	=	1 Square Pyramid mile
16	Square Pyramid miles	=	1 Square Pyramid league.

Or,

Square inches	144	= sq. ft.	1					
	625	= „	=	sq. arm	1			
	62,500	= „	=	100	= square rod	1		
	6,250,000	= „	=	10,000	= 100	= acre	1	
	.....	„	=	6,250,000	= 62,500	= 625	= 1	sq. mile
	.....	„	=	1,000,000,000	= 1,000,000	= 10,000	= 16 = 1	sq. leag.

And Pyramid cubic measure,

Pyramid Cubic Measure.

1,728	cubic Pyramid inches	=	1	cubic Pyramid foot	
15,625	„ inches	=	1	„	arm
1,000	„ arms	=	1	„	rod
1,000	„ rods	=	1	„	acre
15,625	„ acres	=	1	„	mile
64	„ miles	=	1	„	league

Cubic Inches.								
1,728	= cub. ft.	1						
15,625	„	= cubic arm	1					
15,625,000	„	=	1,000	= cubic rod	1			
15,625,000,000	„	=	1,000,000	=	1,000	= cub. arm	1	
.....	„	=	15,625,000,000	=	15,625,000	=	15,625	= 1 cubic mile.
.....	„	=	1,000,000,000,000	=	1,000,000,000	=	1,000,000	= 64 = 1 cub. league.

Pyramid Heat Measure.

While for heat, the Pyramid scale makes water freezing = 0,
boiling = 250;

The standard Pyramid temperature being = 50 of that scale, and the heat at which iron begins to give out light = 1000 of the same, according to the Fahrenheit statement of the Diffusion of Useful Knowledge Society's Natural Philosophy, under head of Thermometer and Pyrometer.

Now all these subdivisions may be called arbitrary, inasmuch as they are so to a certain extent; for I have ventured to employ my judgment to some small degree as to when to use a divisor of 5, and when of twice 5; but have been guided therein; firstly, by what appeared to be the more important and primeval stand-points in the British system of weights and measures: and, secondly, by what seemed to make the resulting apparent, or so-called Pyramid system, most perfect and complete according to its own innate features;—the result spontaneously setting forth the chief elements of the only sound system for a national metrology, viz., to satisfy science by a thoroughly pervading and consistent reference to appropriate ruling features in nature; to satisfy commerce by the correctness of large standards, as well as easy numerical relations to all others; and to satisfy the poor and working men by small units easily identifiable for any one seeking under difficulties for only a moderate degree of accuracy.

It would extend the limits of this paper too far, to say everything that might be said in favour of the ancient system of the Pyramid, under all these heads; for it *is* that ancient system of 4000 years ago, whose standards are still preserved (which is more than our Parliament can say of their standards of only 40 years since),—and all that I hope *I* may have been allowed to do in the case, is merely to disentangle some minor modifications from the confusion of human affairs,—and to assist in showing that the Great Pyramid Metrology is—more scientific than the French,—more convenient than the British,—and yet at the same time altered only very slightly from the British: so slightly in all material points, that until Mr TAYLOR led the way in this research, no one could have imagined what an amount of material improvement and arithmetical facilitation could be made by such small and unpretending alterations. At least we may conclude that no one could have thought of it, for it has not been independently proposed anywhere else; and a very numerous political party has recently moved in Parliament, to introduce scientific improvements, by no means so great, by bringing in the French metrical system into this country, and abolishing the whole of the English hereditary measures, root and branch, by force of law; a more extensive sweep of despotic power, and pregnant with more confusion as well as chance of sapping the patriotism of the masses, than anything which has been attempted in England or Scotland for the last 800 years.

In *Capacity-measure*, therefore, we may briefly point out, that the coffer, or large end of the scale, is scientifically connected with the earth's capacity; and is a large standard, admirably constructed, and uniquely, if not also miraculously preserved to our times:—while the unit for small purposes, is a drop of water; correctly defined by a Pyramidal proportion to the coffer, and approximately in any one's hands at a moment's notice, to test wherewithal a small capacity measure.

The facility of testing measures, is the most important feature they can have. The British Imperial capacity system only has one such helping point, viz., the

gallon of water = 10 lbs.; and that is accompanied by the draw-back, that in a British Imperial Weight system, there is no 10 lb. weight there.

With the Pyramid capacity system on the contrary, every single step of the capacity can be tested, by one or more weights which will be found as a matter of course wherever Pyramid weights are kept. The system restores moreover that ancient hereditary maxim of Great Britain,

A pint's a pound
All the world round,

which was only abrogated by unfortunate legislation so late as George IV.'s reign; and finally, the Pyramid capacity system extends from the greatest bulks ever dealt with by commerce, to the smallest that are required by science; while the British Imperial, stops half way; and makes exceeding anomalies when apothecaries, bound by law to use nothing but Troy weight, find themselves compelled to prepare a small capacity measure for themselves founded on avoirdupois weight.

In *Weight measure*, similarly, the Pyramid ton, which is the coffer full of water, is scientifically connected with the most important and unalterable of all the physical qualities of the earth, viz., its weight, which is more than can be said of any human devised weights whatever; it is, moreover, a standard of large size, admirably constructed, and wonderfully preserved; and the unit for small purposes is a grain; correctly defined by a Pyramidal proportion to its standard; and approximately defined and possible to every man, by an ordinary grain of wheat. In bringing up now this well-known seed as a hasty, working unit of weight for the poor, the Pyramid system only brings up again what was the original grain-weight of this country, until an apparently needless law declared that 32 of these real grains, should henceforward be divided into 24 artificial grains; and that these enlarged grains, which no poor man could test very readily, should be, from that time forward, the grain of weight measure. Furthermore, it will be observed how readily the Pyramid weights may be tested by water-measure; and how complete the whole system is from tons to grains without any break or inconvenient divisions.

In *Linear measure*, it is almost needless to point out the inimitable scientific reference of the Pyramid's chief standard; the lasting nature of that great structure, the Pyramid itself, with the prepared surface of solid rock on which it stands, showing its ancient fiducial marks existing still; and the convenience of the small unit, the *inch*; correctly defined by reference to its standard on the earth; and approximately in every one's hand, literally as well as figuratively, in a thumb-breadth; by which any workman may at a moment's notice, ensure within some limits, the truth of his work. There is, however, a further advantage which the Pyramid Linear system has, in making the length of the side of an acre, that all-important land measure, evenly commensurable with linear mea-

sures, larger and smaller, and with the earth itself; yet without sensibly, to the people, altering its present absolute dimensions.

The result of this arrangement tells with thrilling effect, on the last and most scientific movement which the Government has made, for a grand Map of Great Britain. It is hardly a dozen years since this new survey for a map was begun, on the scale of $\frac{1}{2500}$th of nature; and there may be required 50 to 100 years for its completion. It is a splendid map, the most accurate and perfect thing of the kind ever attempted either in this or any other country; but the only draw-back is, that its scale of $\frac{1}{2500}$th of nature, is absolutely unconformable to any existing legal British measures, either Linear or Square. In linear, it is a map of 25·344 inches British to the mile; and in square, of 1·018 square British inch to the acre: and these fractions are found so annoying to practical working men,—that we recently heard an eminent Indian surveyor condemn the arrangement without mercy; and prefer, if there must be an odd fraction anywhere, to have it in place of the $\frac{1}{2500}$th, which only scientific men care about. Government seem in fact to be in a rather awkward position: for, to please some few politico-scientific men, they have adopted a scale for the national survey which is the annoyance of all working men throughout the land; and they do not like to alter back again to please the working, and affront the political men.

Yet see how the Great Pyramid system, if Government would only embrace it, could relieve them of the difficulty in a moment: for the map can remain as it is, viz., of the $\frac{1}{2500}$th of nature, to please the said powerful agitators of the community; and it will nevertheless be, in *Pyramid* linear measure, 25 inches, even, to 1 mile; and in *Pyramid* square measure, 1 inch, even, to an acre; to the immense satisfaction of the numbers who compose the large industrial part of that community itself.

In *Heat* measure, there is only occasion to remark, that the Pyramid system gets rid of the generally confessed and unnatural anomaly as to the *place* of zero in Fahrenheit's scale; and allows working men to speak of winter temperatures, in the way they find it most serviceable to speak of them, viz., as so many degrees of frost or heat, whenever the temperature is below or above the freezing of water; agreeably with the — and + readings of scientific continental thermometers. At the same time, the Pyramid scale, having a greater number of degrees between freezing and boiling, than either Fahrenheit, Centigrade, or Reaumur, enables a greater number of different temperatures to be alluded to, without the inconvenient employment of fractions, than any other known scale.

(10.) *The Sacred Cubit of the Jews.*

The cubit is, by name, one of the earliest of all measures, and has been most extensively employed by all the great nations of antiquity. As usually explained, it is the distance from the elbow to the end of the middle finger, a space with

most men, equal to about 18 or 19 inches,—and the length, very nearly of the cubit as employed amongst the Greeks and Romans. But amongst the Egyptians, Assyrians, and Phœnicians, the cubit was more nearly = 20·7 inches; and was seldom found more than a few tenths of an inch different therefrom; while in Egypt more particularly, it was kept and is still kept, very close thereto, by being engraved on the column of the Nilometers; no departure from the scale of which can take place, without introducing immense confusion into the agriculture of the country. Whatever, therefore, was the derivation, or reason for originally fixing on that particular length for a working scale, or measure,—there seems no doubt amongst all authors, ancient and modern, that the chief Egyptian cubit both is, and always was, close upon 20·7 inches in length.

Amongst other authors, Sir Isaac Newton deduces this size for it, from some of the passage measures of the Great Pyramid; which we know, from other sources, must have been built by Egyptian workmen. And, in his "Dissertation on Cubits," he shows that its, (the 20·7 inch cubit), use was introduced among the Israelites, during their long captivity in Egypt, and their cruel bondage to building work, *which required constant attention to measures.* He points out, however, in strangely clear and express terms, that they looked on it invariably as "a profane and adventitious cubit," as "the cubit of a man," merely; and employed it only for ordinary, social, and week-day purposes: while, for sacred occupations, they employed a certain other and larger cubit, which he has reason to believe that the leaders of their race had received, or adopted in very early times indeed, or *long before they went down to Egypt.* In such case, the preserving of that measure through the Egyptian captivity, and its after restriction to sacred purposes only, and its continued employment therein up to the time of Josephus,—form very notable features in its history; and Sir Isaac endeavours to arrive at a precise knowledge of its size. By continued approximations from many facts, he reduces the limits within which its length must be contained, from variations of 2 or 3 inches, to hardly as many tenths; and at last arrives at the absolute length of 24·9 English inches as the most probable result. He guards, however, against this determination being considered final, and refers its improvement to those *who shall hereafter measure more stones in the Great Pyramid and the Temple at Jerusalem:* pending which, but noting the usual shortening of standard scales in long ages, and that much of the above determination rests on a scale preserved by the Talmudists, and which they confess was shorter than two ancient copies of the sacred cubit engraved on the walls of Susan and Babylon,—there seems fully sufficient reason to increase the 24·9 English, to 25·0 Pyramid, inches.

And now comes the question, why should a measuring rod of that length have been considered as "sacred;" and one so little differing from it, as 20·7, be considered "profane, and amongst a people frequently receiving divine revelation of knowledge?" We have no reason given in the Bible to guide us, and none

is reported to exist among Jewish traditions: but this we now know from modern science and Mr TAYLOR'S Great Pyramid deductions combined, viz., that a length of 20·7 inches, is wretchedly incommensurable, while 25·0 inches, is admirably, and even inimitably commensurable with the one, and only true metrical reference which the earth possesses, viz., its axis of rotation; it is in fact $\frac{1}{10,000,000}$th of the earth's Polar semi-axis, or radius. Now this length is also the length of our "smaller" standard of the Pyramid; and although that has been deduced hitherto, from the larger standard halved, for practical convenience; it is also to be derived directly from the Pyramid, by taking that particular chronological fraction, or $\frac{1}{366}$th of *one* side of the Pyramid's base; and we are then reminded, by the four sides of the Pyramid (in such case representing four years), of the period during which the incommensurability of the earth's day, and year, is practically restored. And this further leads on to an inquiry, whether there may possibly be any indication within the Pyramid, of that division of time also, which the Jews held as sacred, and to have been commanded from the most primitive times,—viz., the seven days to form a week.

(11.) *Time Measures in the Pyramid.*

In his discussion of GREAVES' measures of the Pyramid, Sir ISAAC NEWTON remarked much upon the clearness with which some particular standard of length had been adhered to throughout the whole of the *inclined passages;* and he appeared to think that that standard was the profane cubit of Memphis, or a multiple of it. If the measures be taken in the shortest transverse directions, this is often the case; but if we compute the vertical height, under the Pole-star inclinations of 26° 18′, these measures of the working men of Egypt, are at once turned,—with quite enough approximation to distinguish what measure of space is intended, in a case where *time* is the real element concerned,—into the scientific standard of the Pyramid, or the linear measure intended by the directors of the building of the Pyramid, and who, there is reason to believe, were completely apart from the Egyptians themselves.

Now, the transverse heights of the small inclined passages being given as follows, with large errors of observation, apparently from the observers not adhering very rigidly to the true *transverse* measure,—

PASSAGE NAME.	OBSERVER.	TRANSVERSE HEIGHT.
		Inches.
Entrance, ...	Howard Vyse,	47·0
„	Caviglia,	49·0
Ascending, ...	Howard Vyse,	47·0
„	Jomard,	43·2
Ante-chamber (level), .	Howard Vyse,	44·0
„ „ . .	Jomard,	43·7
„ „ . .	Caviglia,	43·0

the space of 44·8 inches may be considered very close to the true transverse height, and becomes, under the angle of 26° 18′, = 50 inches. In a similar manner the height of the Grand Gallery, given by various observers, without much specification, as anywhere between 270 and 360 inches, may be concluded equal to 314 of transverse, and 350 of vertical height. The Grand Gallery then, is seven times the height of a small gallery, similarly inclined; and is constructed in integral terms of both the scientific standard of the Pyramid, and the sacred standard of the Jews. The only question, therefore, now to settle, is, what does the larger linear standard of the Pyramid,—which has already had its metrical character and proportions settled in a manner that requires no repetition,—signify when in such a position?

To this end, recourse may be had to our diagram, Plate XXVI. fig. 6, representing the passages in the *Northern lower half*, of the Meridian section through the Great Pyramid, according to the hypothesis partly given in Plate XXIII. fig. 3; the data therefore stand thus—

Semi-base breadth of Pyramid,	=	91·5 Metrons.
Height of Pyramid,	=	116·5 ,,
Semi-side of small area-square,	=	51·624 ,,
One-half semi-side, ,,	=	25·812 ,,
One-third semi-side, ,,	=	17·208 ,,
Angle of lower culmination of old Pole-star,	=	26° 18′
Place of Pole at Pyramid,	=	30 0
Upper culmination of old Pole-star,	=	33 42
Angle of Equator at the Pyramid,	=	60 0
26° 18′ tangent of, with radius of 91·5,	=	43·222
30 0 ,, ,,	=	52·828
33 42 ,, ,,	=	61·023
Transverse height of small passage,	=	44·8 inches.

Of all these data, the last and the two first are the only ones derived from the Pyramid itself, by measure of observers; all the rest being conclusions from the theoretical Pyramid, confirmed, however, more or less by reference to measure, and produced in the first instance for the extremely different questions discussed in the earlier sections of the paper.

It will be seen then, on inspecting the diagram, that the increased height given to the Grand Gallery over an ordinary small passage, depends entirely upon Latitude and pole-star-angles, into the size of the Pyramid; and comes out, as mentioned above, 7 times the breadth of a small passage. In the next place, at the lower Northern entrance point, D, into said Grand Gallery, there is a remarkable meeting together of three passage lines, and four astronomical lines; which all intersect there within the breadth of a pen's stroke: constituting it the most important point, both theoretical and practical in the whole Pyramid. And amongst these lines, the *Equator-line* of the Pyramid enters the Grand Gallery, and makes it, as it were, peculiarly its own; *i.e.*, makes it the Equatorial and

Zodiacal, and therefore the chronological part of the interior structure, as contrasted with all those Polar-pointings we have previously met with on the Northern side of the Pyramid. While the gallery itself subtends from a typical point (See Plate XXVI.), the Zodiac angle of the time.

If then *chronology* be intended in the Grand Gallery, and its height represent 7 units of that species, these units must be days, for in no department of metrology, is the natural unit marked out so clearly, accurately, and lastingly, as the time unit is by the rotation of the earth on its axis. The Grand Gallery then, as its first symbolization, represents a week of days; or embodies in the most magnificent masonry in the Pyramid, the Divine arrangement of days into periods of weeks; an arrangement not followed by the Egyptians, but considered most eminently and primitively sacred by Moses and the Jewish nation.

Nor is this the only support which the Pyramid gives to so remarkable a conclusion; for, besides the Grand Gallery, another passage leads off from that chronological point D, on Plate XXVI., viz., the horizontal passage communicating with the Queen's Chamber, and bears a similar import. The distance from D, for instance, to the Great Step,—which rises from the floor of the horizontal passage to that of the Grand Gallery, and, corrected for the angle = 100 inches, or $= \frac{1}{366}$th of the entire perimeter of the Pyramid, and therefore signifies a day,—as also the depressed portion of the floor of the passage at its southern end; each of these specified parts goes seven times into the whole length of the passage: approximately perhaps only, but very nearly indeed, if we take the mean of Colonel Howard Vyse's measures and the hypothesis of our Plate XXVI. While the Queen's Chamber itself, whose floor-measures are commensurable with those marked portions of the passage, adds a third testimony to the same end, and is safe from the small errors of measurement, being, in fact, a room with *seven* sides; and the only room in the whole Pyramid of which the same can be said.

After this it is hardly worth while to delay over the circumstance of the 7 overlappings of each side wall of the Grand Gallery (see Plate XXVII. fig. 8), combined with the 26 remarkable holes in the ramp of the Western wall, and 26 + 2 in the Eastern wall, indicating an *intention* of chronology still further, by showing a year divided into two *weeks of months; i.e.*, even months of 26 days each, excepting the last one, which requires *one* day to be added in an ordinary year, and *two* in a leap-year: the three hollow years and one full one of a leap-year period being further indicated by the fittings, or fillings of the ante-chamber at the head of said Grand Gallery; or, to the 30 overlapping stones of the ceiling, and the 30 times and a portion (if the French, and Howard Vyse, numbers can be depended on), with which the long step at the head of the Grand Gallery goes into the length of that passage, as typifying also the ordinary secular months of 12 to the year.

It is hardly necessary, indeed, to allude to these additional confirmations,

because what was mentioned previously admits of easier investigation as to its truth, and is perfectly sufficient for its purpose,—viz., to show that the first and chief sacred Jewish measure of time, as well as their sacred measure of space, "which they had received long before they went down to Egypt," is embodied, though in terms unintelligible and unmeaning to Egypt, in the Great Pyramid.

Date of Pyramid.

In the matter, however, of time, on the large scale, attention may be profitably drawn to some further points connected with fig. 6, Plate XXVI.

The angle of the passage entrance, measured by Col. HOWARD VYSE, at 26° 41′, has been settled by so excellent an authority as Sir JOHN HERSCHEL in the astronomical world, as meaning the lower culmination of α Draconis, the pole star of the time; and which Sir JOHN has computed at 26° 15′ 45″, for the year 2121 B.C.; whence, if the Colonel's angle be well measured, we may deduce from the annual precession in North Polar Distance for that part of the sky, viz., + 18″ per annum, that the date of the Pyramid passage in question is earlier by about eighty-three years, or is to be fixed at 2204 B.C.

Yet, if there be force in our general conclusions, as to the less value for scientific ends in the descending, than the ascending portion of the interior,—it will be more proper to take either the theoretical angle, which gives the intersections at D, (Plate XXVI.); or, the practical angle of the Grand Gallery, measured by the French at 25° 55′ and 26° 0′, but by Col. HOWARD VYSE at 26° 18′, and computed by myself at 26° 18′ 10″; and then we have for the date, 2129 B.C.*

* In further elucidation of the note on page 686, and the number 5·672, chosen as the best resulting value from BAILY's Experiments for the Mean Density of the Earth, it may be mentioned, that in the last page of his valuable memoir, he gives the following several numbers as the exact foundations for his more popular announcement of 5·675, and desires his future readers to form their own idea of the real mean amongst them; viz.:—

Mean of all the observations without exception,	5·6747
The same, when one series of doubtful observations has been abstracted,	5·6754
The same, with a second doubtful series also abstracted,	5·6666
The same, with a third doubtful series also abstracted,	5·6683
The same, with a fourth doubtful series also abstracted,	5·6604

Now, the simple mean of all these comes out 5·66908, but as that evidently gives too much importance to the latter results, which Mr BAILY did not allow to appear at all in the 5·675,—let us take a mean of the two, and we have 5·67204.

Again, if it be agreed that the first result is five times the weight of any one of the others, and then take the mean accordingly, we have 5·67158. Or, if it be further settled that the same reasons which make the first more weighty than the second, make the second somewhat more weighty than the third, and so on to the fifth; that is, that every subsequent given result represents a less number of observations,—let us multiply the first by 10, and the others by 4, 3, 2, and 1, when we have for this form of the mean, 5·67227. And putting all these three probable means together, 5·67204, 5·67158, and 5·67227,—there appears for the final mean, 5·67196: sufficiently represented by our 5·672.

(12.) *The Final Argument.**

We are now approaching the point when it becomes necessary to attempt to institute an inquiry as to whence, at that early age of the world, more than 200 years before Abraham, so much knowledge of difficult secrets in physical science, allied with some of the earlier Divine commands in the Pentateuch, originally came; and why, or under what circumstances such particulars were so carefully, lastingly, and expensively embodied in the Great Pyramid. Contracted space, however, forbids my doing more at present than referring to the discussion of this very difficult, but most extraordinarily important, division of the subject in Parts IV. and V. of my recently published book, "Our Inheritance in the Great Pyramid;" and concluding with the two following appendixed notes.

APPENDIX I.

Chronology corrected by the Great Pyramid.

In the table of approximate chronology given in the work above mentioned, I was content to follow for the *absolute* dates those published by some of the best modern hierologists, as being not only in all probability very close to the truth, but also specially able to prove beyond hieroglyphic doubt, that the Great Pyramid, if built in the reigns of Kings Shofo and Nou-Shofo, of the 4th Egyptian Dynasty, as attested by the quarry marks on the rough stones, must have been a *pre-Abrahamic* monument.

But the hierologists, though exceedingly trustworthy when speaking of successional order, have few or no means in their science which enable them to fix absolute dates with great exactness. What, then, does the Great Pyramid's absolute date, alluded to on p. 699, say upon this important question?

It seems to assert this, that the early hierologist dates are rather too large; or, in other words, that they are evidently of the school of Josephus and the Septuagint version of the Scriptures; while the Great Pyramid's metrical and builded date, astronomically computed, agrees rather with the chronology of the Hebrew version of the Scriptures. Which very notable and perhaps desirable additional testimony, in the present disputed state of the two chronologies (see "Kitto's Cyclopædia of Biblical Literature"), may be best illustrated by the two columns of rival dates in the subjoined table:—

* Written in September 1864.

TABLE OF COMPARATIVE CHRONOLOGY, APPROXIMATELY.

Scripture Contemporary Names.	Septuagint and Hieroglphic years.	Hebrew and Pyramido-Astronomical Years.	Egyptian Dynasties. Principal.	Egyptian Dynasties. Subordinate.	Egyptian Monuments.	General Events.
	B. C.	B. C.				
Noah, ...	3200	2500	...	...	...	...
...	3100	2450	...	...	...	...
...	3000	2400	...	...	...	...
...	2900	2350	...	...	...	...
...	2800	2300	...	...	...	...
...	2700	2250	1. Thinite, . . .	...	...	Animals "appointed" to be gods in Egypt.
...	2600	2200	3. Memphite, . .	2. Thinites, . . .	Pyramids of Lower Egypt,	
...	2500	2150	4. Memphite, . .	...	Jizeh Pyramids and tombs,	Kings Shofo and Nou-Shofo repudiate and repress Egyptian idolatry.
...	2400	2100	...	5. Elephantines, .	...	
...	2300	2050	11. Theban, . . .	9. Hermanthites, .	Sinai and other Quarries, .	Animal worship in Egypt restored.
...	2200	2000	6. Memphite, . .	...	...	
...	2100	1950	12. Theban, . . .	14. Xoites, . . .	...	...
Abraham, . . .	2000	1900	7. Memphite, . .	15. Theban, . . .	Memphite Pyramids,	...
Isaac, ...	1900	1850	13. Theban, . . .	10. ?	Tombs, &c., ...	...
Jacob, ...	1800	1800	8. Memphite, . .	16. Theban, . . .	Obelisks, ...	...
Joseph, ...	1700	1700	...	17. Shepherds, Hyksos	Temples, ...	All Egypt under one King.
...	1600	1600	18. Thebans, . .	returned ?	...	...
Moses, ...	1500	1500	...	...	...	...
Joshua, ...	1400	1400	...	...	Brick Arches, ...	Rameses the Great.
...	1300	1300	19. Thebans, . .	...	Great Sphinx, ...	Troy taken.
...	1200	1200	20. Thebans, . .	...	Colossi, ...	...
Samuel—David, .	1100	1100	21. Tanites, . .	...	Temple-Palace, ...	...
Solomon, ...	1000	1000	22. Bubastites, . .	...	Tombs, ...	...
...	900	900	...	...	Karnak-Temple, ...	Olympiads began.
Isaiah, ...	800	800	23. Tanites, . . .	...	Palaces, ...	Captivity of Israel.
...	700	700	24. Saïte; 25. Ethiop.	...	Syene Quarries, ...	Nebuchadnezzar.
Jeremiah, . . .	600	600	26. Saïte, . . .	...	Stone Arches, ...	Cambyses conquers Egypt.
...	500	500	27. Persians, . . .	...	Tombs, ...	Xenophon.
Malachi, ...	400	400	28. Saïte, 29, 30, & 31.	...	Meroe Pyramids, . . .	Alexander the Great.
...	300	300	Greeks, ...	...	Luxor completed, . . .	...
...	200	200	...	...	Philæ Temple, ...	...
...	100	100	...	...	Esneh Zodiac, ...	Julius Cæsar.
...	30	30	Romans, ...	...	Dendera Zodiac, ...	...
	A. D.	A. D.			Gebel Dokhan quarries, .	
...	620	620	Arabians, ...	...		Casing stones removed from Great Pyramid.
...	1517	1517	Turks, ...	...	...	
...	1798	1798	French, ...	...	...	The Porphyry Coffer, mischievously, and irretrievably injured.
...	1801	1801	Turks and local Pachas,	...	...	

APPENDIX II.

Colonel Strange's Measure of the Exchequer Queen Elizabeth's Standard Ell.

The extreme closeness of the present legal British inch to the ancient Pyramid, and truly earth-commensurable inch, is one of the best proved, as well as most interesting features of the preceding inquiry; yet, rather astonishing to say, the approximation was once, and might have been now much closer, except for an accident, or rather an erroneous step among certain high parties; the case being this:—

The present legal inch is merely the $\frac{1}{36}$th part of the Parliamentary standard and unit yard. Originally, the inch itself seems to have been the unit; and the yard, or any other standard, merely such and such a number of the units strung together for some temporary purpose of convenience.

The smallness of this inch unit, and its easy approximate identification by a thumb's breadth, were eminently appropriate to the conditions which should be preserved in metrical units for the poor and working classes, while its even earth's axis commensurability was the most admirable feature for all religious and intellectual minded men to contemplate. In the days of Queen ELIZABETH, something of this belief appears still to have lingered among the officers of Government; and, therefore, when desired to deposit in the Exchequer an example of the national linear measure, the officers concerned placed there two long standards, one of them composed of 36, and the other of 45, of the unit inches, under the names of the yard and the ell; the *mutual* incommensurability of which, except through the medium of inches, seemed almost intended to call attention to the inch as the unit, though the yard and the ell themselves might be useful as standards.

These standards remain at the Exchequer to this day; and one of them, the yard, has been the parent, through means of a series of copies begun in 1742, of the present standard yard of Parliament. Why the gentlemen employed by Government, from 1742 downwards, chose to be guided entirely by one of the Elizabeth standards, and to throw the other overboard, although scientifically, from its greater length, and historically also, it ought to have been even more attended to than the other, I have never heard explained; but at present we have only to do with the fact, and endeavour to ascertain what would have been the result to the nation, if the larger and, in early times, the more popular standard, *i.e.*, the ell, had been employed to give the length of the present parliamentary inch.

To this end reference may be made to Mr BAILY's excellent historic paper in

the "Royal Astronomical Society's Memoirs," vol. ix. p. 40, wherein the comparison made by GRAHAM, in 1743, before deputations from the Government and the Royal Society, is described as having shown that the Queen Elizabeth ell's 45 inches exceeded the quantity of 45 *such* inches as the same Queen's yard contained 36 of, by 0·0494 inch. But our present parliamentary yard, though descended from Queen Elizabeth's yard, is not identical with what it is, or was, in that day, because, for one reason, it is copied immediately from "Bird's yard," and that from the Royal Society's yard, and that from "the Tower yard," and that only from Queen Elizabeth's standard measure, and so badly, that all these subsequent yards are about 0·0075 inch larger than their intended prototype. Applying which correction to the former statement for the ell, we conclude that measure to have its 45 inches equal to 45·0419 of *such* inches as the present British standard yard contains 36 of.

Now, inasmuch as 45 pyramid, or truly earth-commensurable inches = 45·0446 of these present standard inches, our first determination, so far as it can be trusted for the ell's inches, shows that they were as nearly earth-commensurable as anything that modern science could well have desired.

This is surely a strange additional episode to the many curious ones already cited in the history of the Great Pyramid, and our own national metrology; and has appeared, therefore, to be worthy of further research, especially as certain subsequent comparisons made about fifty years after GRAHAM's, and cited by Mr BAILY, at page 48 of the same Essay, seem to throw doubt upon it. In short, nothing but a modern measure of the Elizabeth ell could now be looked on as fully satisfactory; and, being unable to visit London myself, I requested my friend Colonel STRANGE, late astronomical-assistant on the Indian Trigonometrical Survey, and gifted with remarkable understanding in all practical instrumental affairs, to kindly undertake the task for me. This labour the Colonel entered on immediately after having obtained, by formal application to H. M. Exchequer and Treasury, the requisite leave to examine their ancient standards.

The process which he adopted—aided partly by Mr CHISHOLM, chief clerk, and by Mr CHANY, junior clerk and official comparer for the Exchequer, both of whom, in their respective departments, he speaks of in high terms, as well for their zeal in the experiment, as for practical skill—seems to have been equivalent to laying off the length of the modern Government standard yard on the old Elizabeth ell, from one end of it; and then transferring to a slip of brass the outstanding portion of the other end of the ell, viz., 9 inches, more or less. This method is excellent in principle, because it throws all the anomalies of the 45 inches into 9 only, where they can be easily measured, and in a state subject to very little thermometrical expansion; while in practice, if I may judge from the veritable brass slip which Colonel STRANGE has sent me, with the fiducial lines almost microscopically delicate, the method has been carried out so well, that I

have only to add the Colonel's own account of the proceedings, and here give his final result: viz., that 45 inches of the Elizabeth ell are equal to 45·0386 inches of the modern legalized, but erroneously derived, British standard yard. That is, Colonel STRANGE confirms GRAHAM's result within ·003 of an inch: and shows, that each of the ell's inches still come within so small a quantity as 0·00013 of an inch to one of the original Pyramid and earth-commensurable inches of the early ages of the world; of which very trifling difference too, one-half may be due to shrinking of the old scale—made in a material which standard scales never should be made in—between 1743 and 1864; while something more should be allowed for the time preceding.

REPORT ABOVE ALLUDED TO, AS RECEIVED FROM COLONEL STRANGE, F.R.S.

EXCHEQUER, *August* 3, 1864.

Experiment 1.

Compared Exchequer secondary standard yard No. 45 with Simms's horizontal comparator, by means of a T square, and found no difference equal to thickness of a line of the graduation on the comparator. Thermometer 69° Fahr.

The comparator is a fixed graduated brass scale, with adjustable gun-metal table underneath it. It is taken as a standard yard at 62°. It is not intended for the most refined purposes. Messrs Simms say it can certainly be depended on to less than (0·001) one thousandth of an inch.

The secondary yard No. 45 is an end measure of Baily's metal, with spherical agate extremities. It is taken as standard $-0{\cdot}0000639$ inch at 62°.

The above experiment was tried in order to test our means (the T square) of comparing an end with a line measure.

Experiment 2.

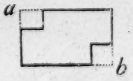
Queen Elizabeth's yard was compared with No. 45 (above referred to) in Simms's vertical comparator, and found = No. 45 $-(\frac{1}{50,000})$ one fifty thousandth of an inch. Thermometer 69°.

The yard has been broken and mended, the joint being loose. It is a very rough piece of workmanship, and the above close result of course was due to the accidental use of a particular spot on the extremities.

The vertical comparator supports the yard measure under trial in a vertical position. A fine spirit-level resting on the upper end of two yards alternately tried, indicates their relative lengths in terms of the level scale, the value of whose divisions is known.

Experiment 3.

An oak rod (specially made for the purpose), provided with a fixed brass stud at one end, and longitudinal spherical-headed screw at the other, was adjusted (by moving the screw) to fit the bed of Elizabeth's ell. The rod and ell were then laid side by side, and compared by means of two T squares. No difference could be detected with the help of a hand magnifier.

The bed is of some copper alloy. Its dimensions are, long 4 feet 1·2 inch, wide 1·5 inch, thick 1·1 inch. A transverse section is of this form 

The excavations *a* and *b* (nominally rectangular) constitute the beds of the ell and yard respectively. They are of the roughest workmanship, apparently executed with a cold chisel.

The ell fits its bed tightly. The yard was not tried.

Each bed is marked at both ends with the crown and letter E, the former device as close as possible to the terminal edge of the recess, in order, apparently, to denote that it is a standard at that point. But the rudeness of the workmanship renders the beds quite valueless for such purposes.

On the edge of the yard recess there are very coarse divisions (filed probably), intended to represent 12 inches (2 of which are halved); the total 12 inches being 0·12 inch too long.

Experiment 4.

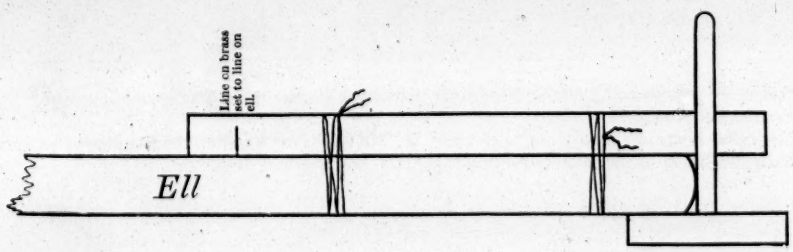
Queen Elizabeth's ell was laid on the horizontal comparator. One of its extremities was brought into the same vertical plane with O of the comparator scale by means of two T squares, one placed vertically against the ell, the other horizontally on division O. The two adjacent edges of the squares were then cautiously brought into contact by means of the longitudinal screw of the comparator.

A T square was now laid horizontally to coincide with the 36 inch division of the comparator, and a fine line drawn on the ell with a steel point. [I have used the word "horizontal," in speaking of the position of the T squares, for simplicity's sake. The divisions of the comparator are in truth cut on a plane inclined about 20° to the horizon. One surface of the ell was brought into the same plane by means of the two wooden angular supports on which it rested.]

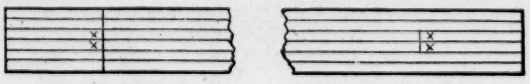
There now remained to be determined the value of the nominal 9 inches contained between the line drawn (as above) on the ell and its extremity.

A piece of brass, prepared for the purpose, had a fine line drawn across it near one end. This piece of brass was tied edgeways along the marked side of the ell; in section, thus

. The line on the piece of brass was brought, with a magnifier, into as exact coincidence as possible with the line on the ell. A T square was now applied to the end of the ell, and a fine line drawn across the piece of brass with a steel point. The following diagram will perhaps help to explain the arrangement:



The two lines on the piece of brass may now be taken to represent the nominal 9 inches by which the ell exceeds the yard of the comparator. The value of this space is a matter of future determination. There are longitudinal lines on the brass, and the measure is to be taken between those marked with crosses, thus



The T squares used were obtained from Messrs Hotzapffel, expressly for the purpose, and were of excellent workmanship, appreciably correct.

The whole of the above operations for obtaining the length of the ell were performed, independently, by Mr Chany, an officer of the Exchequer accustomed to such work, and an excellent manipulator, and by myself; every adjustment was disturbed between the two determinations. Neither in the line drawn on the ell, nor on that drawn on the piece of brass, did Mr Chany and I differ perceptibly, a hand magnifier being used for the examination. The thermometer in the room stood steadily at 69° throughout; but no attempt was made to obtain the true temperature of the ell, comparator, &c., which were necessarily handled a good deal.

My own conviction is, that this comparison of Queen Elizabeth's ell with the Exchequer standard, cannot differ so much as (0·005 in.) five thousandth of an inch from the truth.

Queen Elizabeth's ell is a half-inch square prism, made of some red metal, very dark generally from oxidation, but showing a colour like soft gun-metal in places. It is very roughly made, no surface approaching a plane, and no corner a right angle. The two extremities are stamped thus The ends, defining the measure, are tolerably smooth, and irregularly convex. The stamped side has coarse graduation on it. The divisions (filed apparently) are at the following distances from one end, viz., 2·75, 5·65, 11·30, and 22·50 inches. Intended, as these evidently are, as half, quarter, &c. of the ell, their errors may perhaps afford some criterion of the dependence to be placed on the standard as a whole.

Experiment 5, *at Messrs Troughton and Simms's.*

August 4, 1864.

The piece of brass was compared with 9 inches of Messrs Troughton and Simms's brass scale, by means of their comparing apparatus furnished with two micrometer microscopes. Independent observations were made by Mr William Simms (W. S.) and myself (A. S.), as follows:—

	W. S. Rev.	W. S. Div.	A. S. Rev.	A. S. Div.
Between one pair of parallel longitudinal lines on the piece of brass, . . .	9	62·0	9	59·5
Between the other pair, ...	9	58·5	9	59·7
	9	59·0		

Thermometer, 71·5.

Messrs Troughton and Simms's brass scale is taken as exceeding the Exchequer standard (0·001) one thousandth of an inch *per yard.*

The value of one division of the micrometer is (0·00004) four hundred thousandths of an inch.

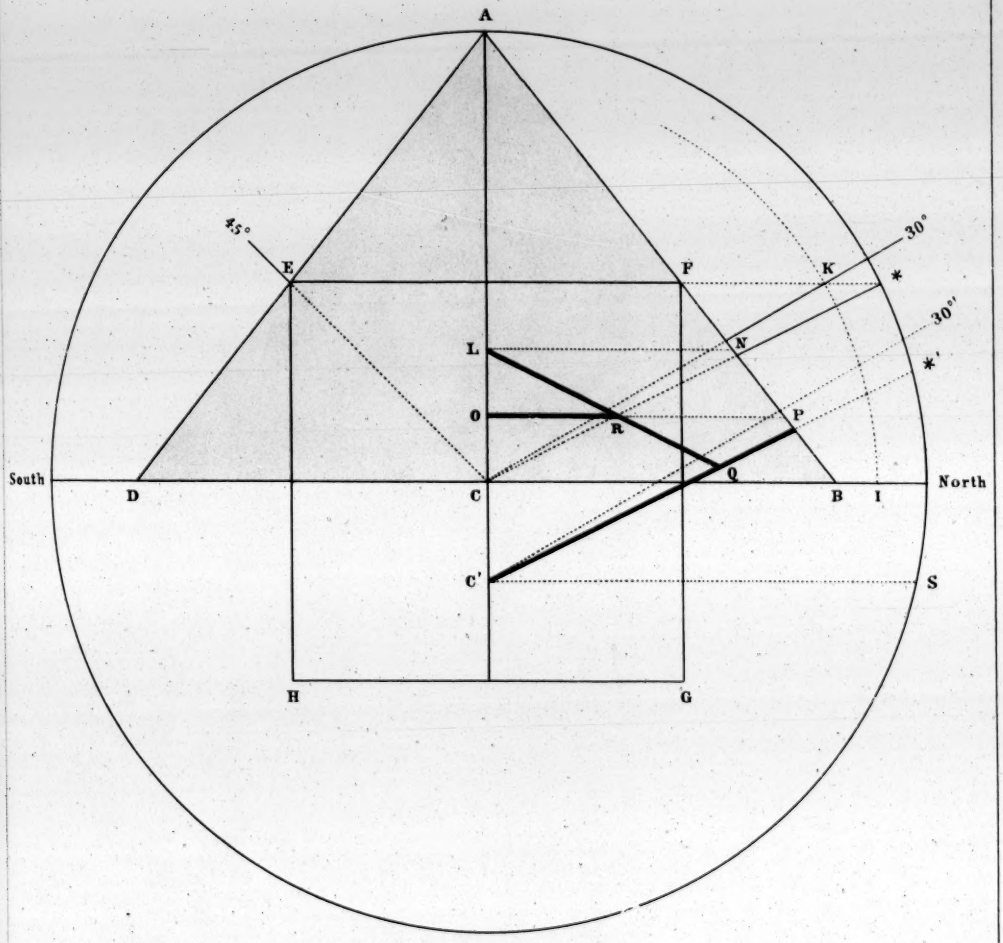
Each revolution of the micrometer contains (100) one hundred divisions.

I deduce from the above experiments, as an approximate result, *uncorrected for temperature,* that Queen Elizabeth's ell = Exchequer yard + 9·0386 inches.

(Signed) A. Strange.

see p.p. 677, 678.

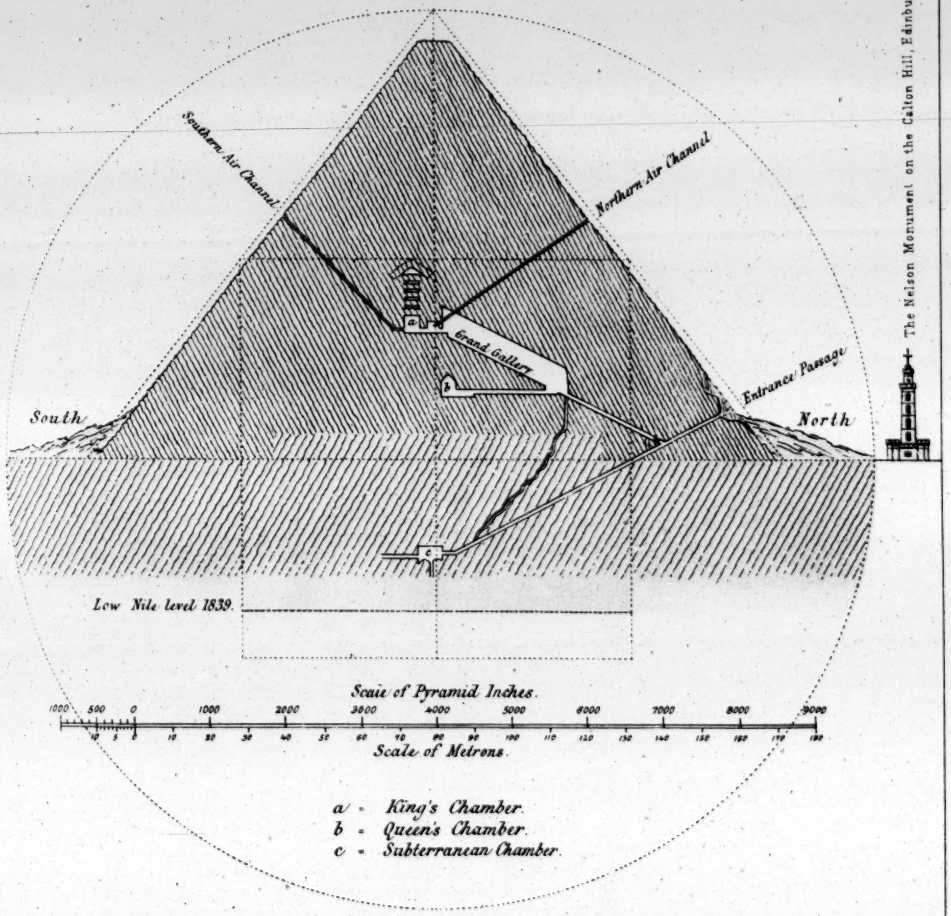
Fig. 3.



LATITUDE MARKINGS AND PASSAGE-PLACINGS BY HYPOTHESIS.

see p.p. 677, 678, 681, 687.

Fig. 4.



MERIDIAN SECTION OF GREAT PYRAMID,

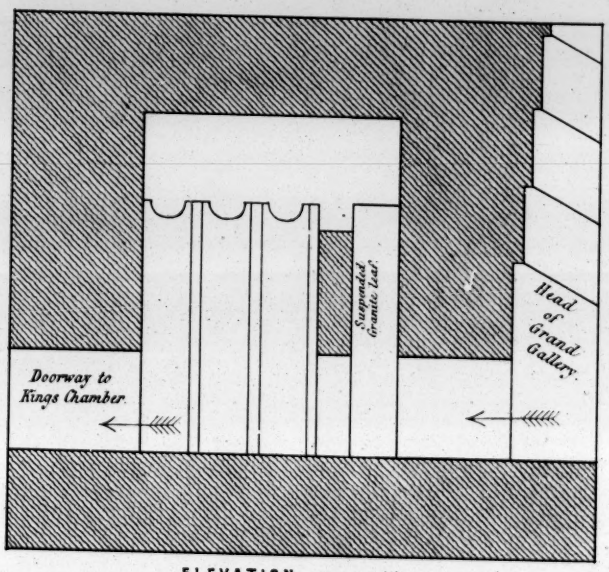
after **HOWARD VYSE.**

with the principal hypothetical lines of Fig. 3. marked in dots.

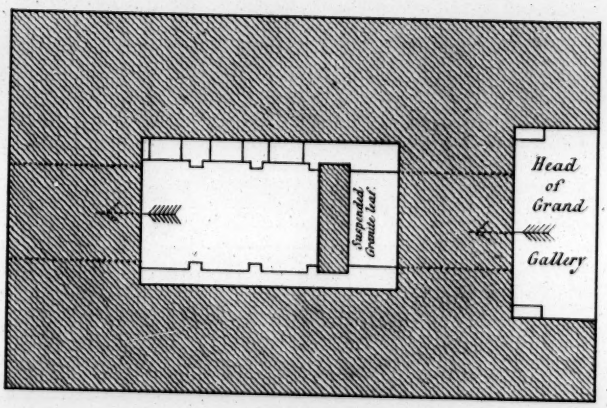
see p. 685.

Fig. 5.

ANTECHAMBER IN SECTIONAL ELEVATION & PLAN.



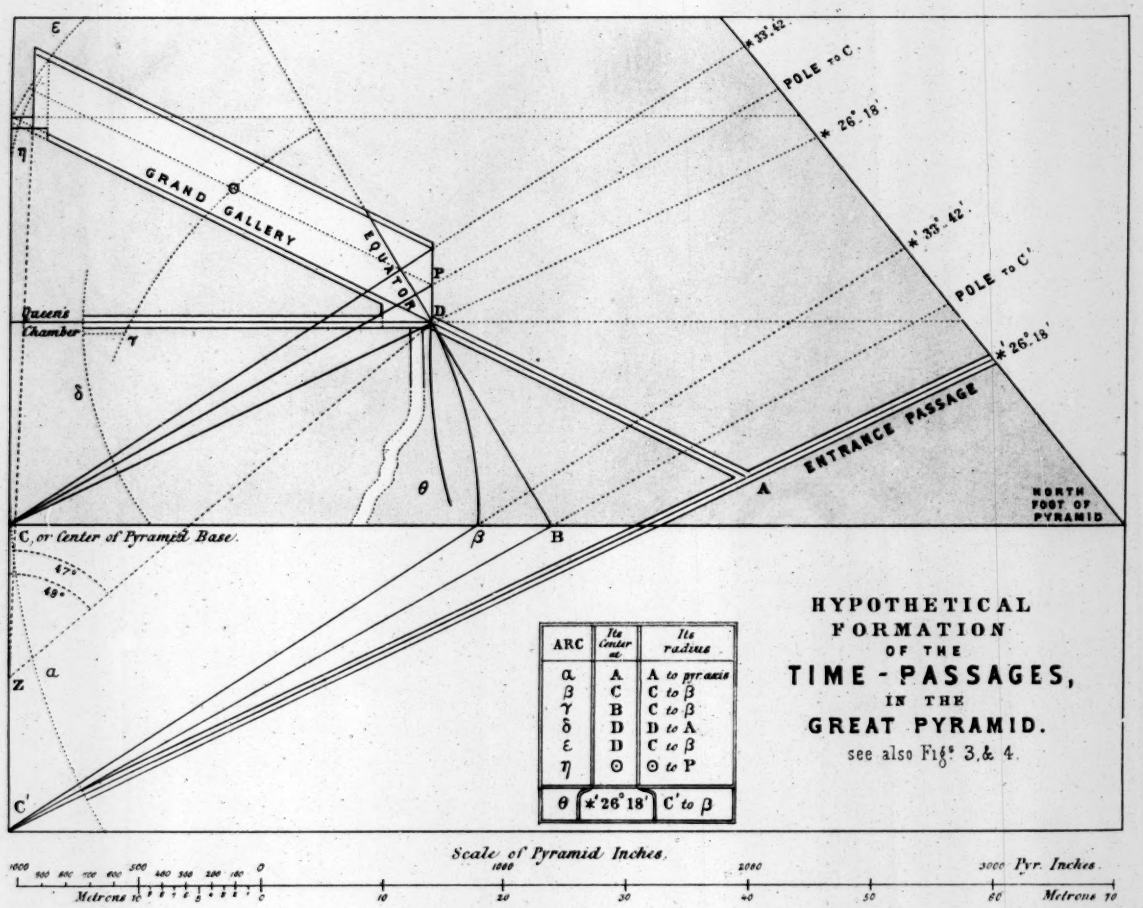
ELEVATION, LOOKING WEST.



PLAN

Scale of Pyramid Inches.

Fig. 6.



ARC	Its Center at	Its radius
α	A	A to pyr axis
β	C	C to β
γ	B	C to β
δ	D	D to A
ε	D	C to β
η	⊙	⊙ to P
θ	*'26°18'	C' to β

HYPOTHETICAL FORMATION OF THE TIME-PASSAGES, IN THE GREAT PYRAMID.

see also Figs. 3, & 4.

see p.p. 697, 698, 699.

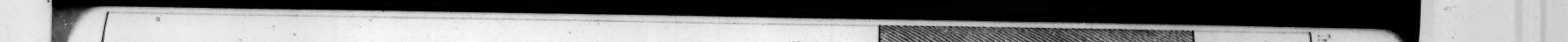
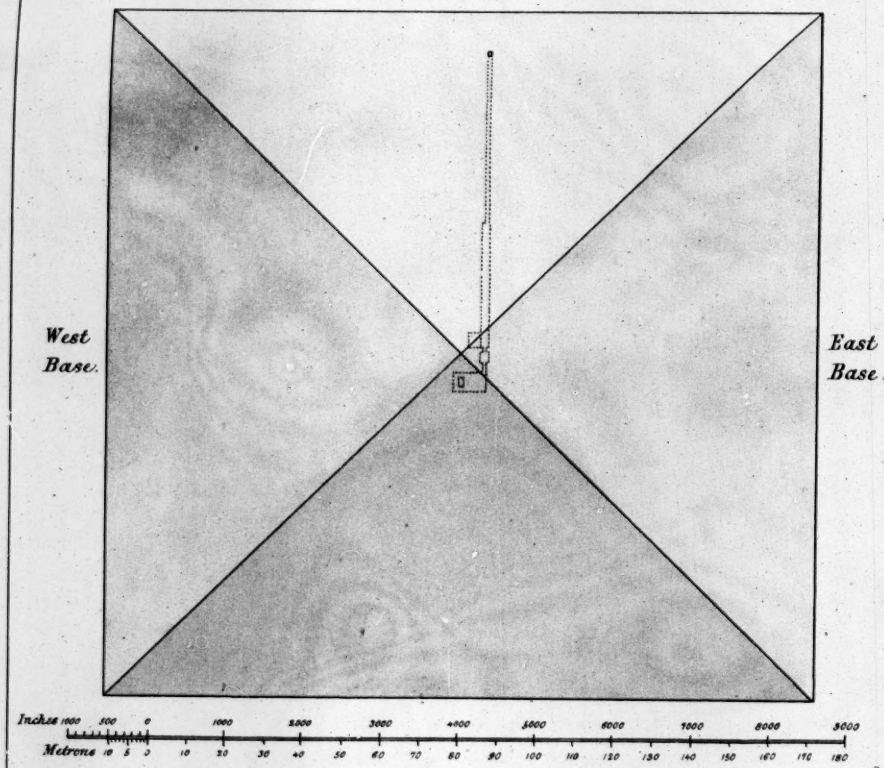


Fig. 7.



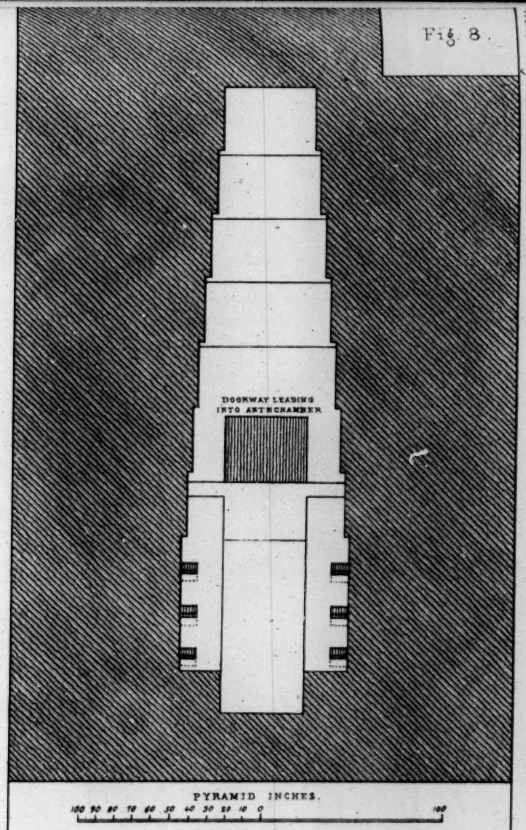
GROUND PLAN OF THE GREAT PYRAMID

IN ITS ORIGINAL COMPLETENESS OF THE CASING-STONE SURFACE:

shewing the internal passages by dotted lines.

see p. 667

Fig. 8.



VERTICAL CROSS-SECTION

OF THE GRAND GALLERY

looking South.

see p. 698

XLIV.—*On the Theory of Isomeric Compounds.* By Dr A. Crum Brown.

(Read May 2, 1864.)

In the following remarks I intend to confine myself to the consideration of those compounds which have not only the same composition per cent., and the same molecular weight, but also the same constitutional formula. Such compounds may be termed absolutely isomeric. As the constitutional formula of few substances is fully known, this class is of course a small one, or rather there are few substances of which we can certainly say that they belong to this class.

The following are the principal pairs of substances which are or have been supposed to be absolutely isomeric. I shall first enumerate them, and then proceed to discuss the nature of their isomerism.

1. The hydrides of the alcohol radicals, and the so-called alcohol radicals, as hydride of ethyl and methyl gas; hydride of propyl and methyl-ethyl.
2. Chloride of ethyl and the product of the action of chlorine on hydride of ethyl. Also the chlorides of other alcohol radicals and the mono-chloro derivatives from the corresponding hydrides.
3. Chloride of vinyl and chloracetene.
4. Fumaric and maleic acids.
5. Two of the three acids citraconic, itaconic, and mesaconic.
6. Bromo-maleic and isobromo-maleic acids.
7. Bibromo-succinic and isobibromo-succinic acids.
8. Active and inactive malic acid.
9. Active and inactive aspartic acid.
10. Two of the varieties of tartaric acid.
11. The two series of bodies known as the compounds of ethylene and of ethylidene.
12. Lactic and paralactic acids.
13. The two series of alcohols derived, the one by fermentation from sugar, and the other by the addition of water to the olefines, as Berthelot's and Friedel's propylic alcohol, Wurtz's hydrate of amylene, Erlenmeyer and Wanklyn's β hexylic alcohol.

To the same class belong of course the ethers derived from these alcohols.

14. The isomeric acids and alcohols having the general formula $C_nH_{2(n-3)}O$, as kressylic acid, and benzoic alcohol.

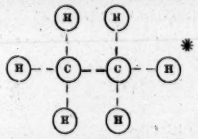
I now proceed to inquire whether these pairs of substances are absolutely isomeric or not.

In order to do so, we must determine whether they are really different, whether they are not perhaps identical; and secondly, whether they are or are not

metameric; in other words, whether the theory of atomicity is insufficient alone to explain their difference.

1. Are there two isomeric series having the common formula $C_nH_{2(n+1)}$? Or are these bodies identical? Till lately, most chemists would have replied unhesitatingly that these substances were different. According to the observations of FRANKLAND, hydride of ethyl and methyl gas, otherwise so like one another, show a very different reaction when treated with chlorine in diffused day-light. Were this observation confirmed, it would be quite sufficient to prove that the two bodies were not identical. Several chemists (CARIUS, SCHORLEMMER) have, however, lately expressed doubts as to the correctness of FRANKLAND'S observation, and it would certainly be satisfactory that it should be repeated with special precautions.

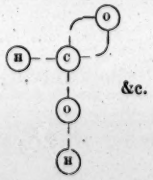
That these bodies are not metameric, that is, that the theory of atomicity is incapable of explaining the difference between them (assuming, as we may do, till FRANKLAND'S observation be found incorrect, that there is a difference), is evident. For there is on that theory only one possible constitutional formula for a substance having the composition and molecular weight expressed by the empirical formula C_2H_6, viz.,



The same reasoning is sufficient to show that the other pairs in this series, as methyl-ethyl and hydride of propyl, ethyl and hydride of butyl, &c., are not metameric. The question of the *identity* of these bodies must, however, be still regarded as an open one.

2. There can be little doubt that chloride of ethyl and the substance produced by the reaction of chlorine on hydride of ethyl are essentially different. The

* I may here shortly explain the graphic notation which I employ to express constitutional formulæ, and by which, it is scarcely necessary to remark, I do not mean to indicate the physical, but merely the chemical position of the atoms. An atom is represented by its usual symbol, surrounded by a circle with as many lines proceeding from it as the atom contains equivalents, thus an unequivalent atom is represented by (A)–, a biequivalent atom by –(B)– or (B)<, and so on of the others. When equivalents mutually saturate one another, the two lines representing the equivalents are made continuations of one another, thus water is (H)–(O)–(H). Formic acid

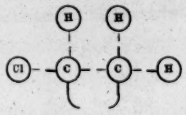


&c.

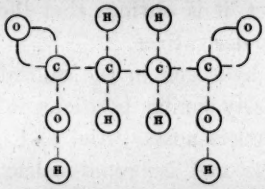
This method seems to me to present advantages over the methods used by Professors KEKULÉ and ERLENMEYER; and while it is no doubt liable, when not explained, to be mistaken for a representation of the physical position of the atoms, this misunderstanding can easily be prevented.

former boils at $+11°$ C, the latter is a gas not condensible by a cold of $-18°$ C. There is also a great difference between them as to solubility in water. They cannot be metameric, as there are not two possible constitutional formulæ for C_2H_3Cl. They are therefore absolutely isomeric. The same argument does not hold in the case of any of the homologues of chloride of ethyl except chloride of methyl. For there may be two formulæ for the molecule C_3H_7Cl, two for C_4H_9Cl, three for $C_5H_{11}Cl$, &c., without varying the mode of arrangement of the carbon atoms *inter se*.

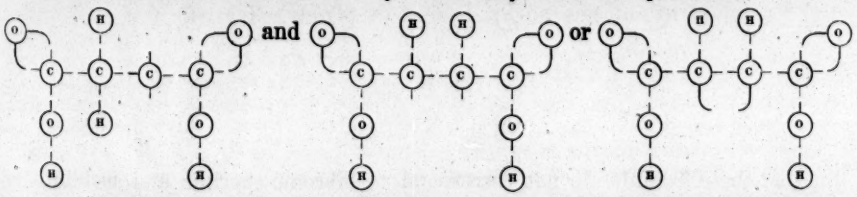
3. Chloride of vinyl and chloracetene are undoubtedly different. They resemble one another in nothing but composition and molecular weight. If we exclude the possibility of the existence of diatomic carbon* in such a molecule, there is only one constitutional formula to represent them both,



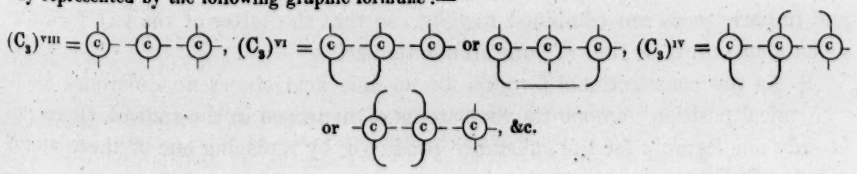
4. The constitutional formula of succinic acid is



Maleic and fumaric acids, each form succinic acid by the addition of two atoms of hydrogen. And as both are dibasic, these two atoms of hydrogen cannot be contained in either of the groups HO in succinic acid. They must therefore be two of the hydrogen atoms directly combined with carbon. Now, there are two conceivable constitutional formulæ by which they could be represented,

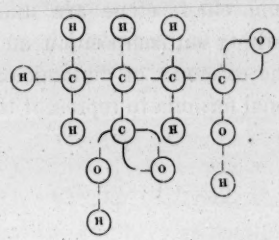


* I do not intend to deny the possibility of this, but all we know of such "non-saturated" substances leads to the belief that the atomicity of the carbon radical C_n is reduced, not by one or more of the carbon atoms becoming diatomic, but by the union of the carbon atoms taking place in the way represented by the following graphic formulæ :—



Of these the latter only is admissible, for the theory of atomicity taken strictly does not admit of free affinities in a molecule. This formula then must be common to the two acids.

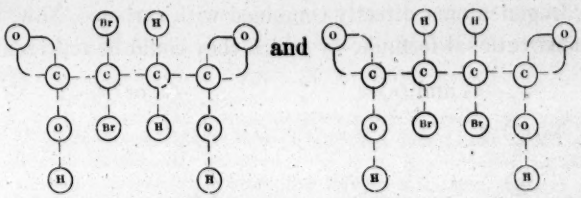
5. In the abstract of this paper, published in the Society's "Proceedings," the dehydrogenates and the bibromo derivatives of pyrotartaric acid were inadvertently included in the list of bodies probably absolutely isomeric. From the relation of pyrotartaric acid to propylene, and of the latter substance to FRIEDEL'S alcohol, we may deduce the following formula for pyrotartaric acid :—



If this formula be correct, it is obvious that there *may* be three metameric dehydrogenates and bibromo derivatives.

6. As the two atoms of hydrogen in the radical of maleic acid are, on the theory of atomicity, in precisely similar positions, it is obvious that there cannot be two metameric bromo-maleic acids. And as KEKULÉ has shown that there are two acids, bromo-maleic and isobromo-maleic, these must be absolutely isomeric.

7. Two perfectly admissible formulæ can be constructed to represent bibromo-succinic acid :—



But that these formulæ do not correspond to bibromo-succinic and isobibromo-succinic acids is plain from the following considerations. These acids are formed by the direct addition of bromine to maleic and fumaric acids. The bromine must therefore be combined with the two carbon equivalents, which in maleic and fumaric acids are combined together, so that the latter of the two formulæ given above must be that of both brominated acids.

8. As the constitutional formula for succinic acid shows no difference as to "chemical position" among the four atoms of hydrogen in the radical, there can be only one formula for the substance produced, by replacing one of these atoms

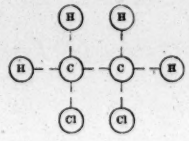
by the water residue; but we have two varieties of malic acid, active and inactive; these must therefore be absolutely isomeric.

9. The same argument applies to the case of active and inactive aspartic acids; aspartic acid being succinic acid in which one atom of radical hydrogen has been replaced by the ammonia residue NH_2.

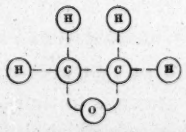
10. As there are two possible formulæ for bibromo-succinic acid, there are also two for tartaric, or dioxy-succinic acid. We have not the same reason for excluding either of these in this case as in that of the bibrominated acids;* but as we have three undoubtedly different varieties of tartaric acid (excluding racemic acid), two of these at least must have one of these formulæ in common.

So far we have been concerned with pairs of substances which seem to be really absolutely isomeric. The remaining substances in our list are more probably metameric.

11. If we consider the various reactions of the ethylene compounds, particularly the formation of glycollic acid from glycol, we are forced to the conclusion that the two unsaturated equivalents of the radical ethylene belong to two different carbon atoms. This conclusion may also be arrived at in another, perhaps less satisfactory way. As chloride of ethylene is formed by the direct union of chlorine and ethylene, the chlorine must be combined with those carbon equivalents which in ethylene gas are combined with one another; but equivalents of the same atom cannot be combined with one another, therefore in chloride of ethylene the two atoms of chlorine must be combined with different carbon atoms. The constitutional formula of chloride of ethylene is therefore



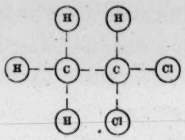
and that of oxide of ethylene



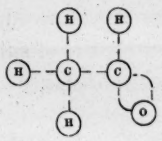
Again, the reactions of aldehyde, the oxide of ethylidene, lead to the view of its constitution first proposed by KOLBE, and now almost universally adopted, $\left.\begin{matrix}CH_3\\H\end{matrix}\right\}CO$,

* It would be interesting to compare the properties of the tartaric acid formed from isobibromo-succinic acid with that from bibromo-succinic, and with the varieties obtained from the grape.

in which both oxygen equivalents are combined with the *same* carbon atom. The constitutional formula of chloride of ethylidene is therefore



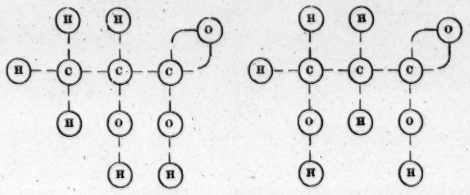
and that of aldehyde



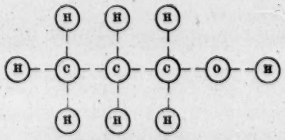
These two series of substances are therefore metameric.

12. The researches of WISLICENUS (An. Ph. cxxviii. 1) and LIPPMANN (An. Ph. cxxix. 81) prove that lactic acid and paralactic acid stand to one another in a relation similar to that of chloride of ethylidene and chloride of ethylene; that, in fact, the former is a compound of ethylidene, with the water residue and the group $\left(\acute{H}O\right\}\acute{C}O\right)'$ and the latter of ethylene with the same radicals. They have therefore the following constitutional formulæ, and are metameric :—

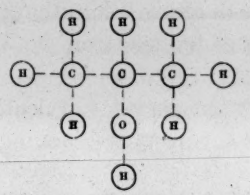
Lactic. Paralactic.



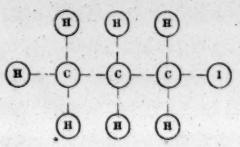
13. With regard to the two series of alcohols—the alcohols proper and the hydrates of the olefines—we know, at least, that FRIEDEL'S alcohol is not absolutely isomeric with propylic alcohol, but, as suggested by KOLBE, only metameric. If we consider the relation of these alcohols to their aldehydes—propionic aldehyde, and acetone, the aldehyde of FRIEDEL'S alcohol—we easily see that the formula of propylic alcohol is



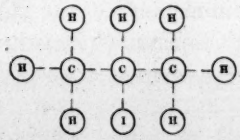
while that of FRIEDEL's alcohol is



As FRIEDEL's alcohol is identical with that obtained by BERTHELOT from propylene, it is highly probable that the same difference exists in the case of all the other members of the two series. In the same way the iodide of propyl is



and the hydriodate of propylene



14. As to the aromatic alcohols and the isomeric acids, we know too little of their constitution to speak definitely with regard to their relation. They may, as suggested by ERLENMEYER (Zeitschrift, vii. 12), be absolutely isomeric, or, on the other hand, they may be related to one another as the two series of alcohols last mentioned are.

Having enumerated the bodies which may without hesitation be called absolutely isomeric, I shall now consider the bearing which the existence of such substances has upon the theory of atomicity.

If we examine the fundamental definitions of that theory, we shall see that there is a point of importance left undecided. We define a multequivalent atom as an atom having two or more equivalents, by means of which it may unite with the equivalents of other atoms, but it is not decided whether these equivalents are similar to one another or not. On the former supposition, there can be only one substance corresponding to each constitutional formula, and absolutely isomeric compounds are impossible. It must therefore be rejected, as such compounds exist. We must then assume that some of the equivalents of at least some multequivalent atoms are different from other equivalents of the same atoms.

This assumption may take one of two forms,—1. We may suppose that the difference is an essential and unchangeable one; that, for instance, the two equi-

valents of a diatomic atom differ from one another as chlorine does from bromine; and that the one can no more be changed into the other, than an atom of chlorine can be changed into an atom of bromine; or, 2. We may suppose that such a change is possible.

Our knowledge of facts is not as yet sufficiently extensive to enable us to decide definitely between these two hypotheses, but it may be of some use to examine their consequences.

The second is, as yet, obviously too vague and indefinite to admit of this, I shall therefore confine my remarks to the first. The principal advocate of this hypothesis is Professor ERLENMEYER of Heidelberg. Professor BUTLEROW of Kasan, has also published some speculations in the same direction; and in his paper on Organic Acids in the supplementary volumes of LIEBIG's "Annalen," Professor KEKULÉ of Ghent treats shortly on the same subject. The only attempts, however, to apply this hypothesis in a definite way to the explanation of particular cases of absolute isomerism are, as far as I am aware, 1. That of Professor KOLBE, who applies a form of this hypothesis to the case of the isomerism of oxide of ethylene and aldehyde. I have already given my reasons for believing that these substances are metameric, and shall therefore not discuss the point further here. 2. That of BUTLEROW (Zeitschrift, v. 301), who endeavours by means of it to explain the isomerism of hydride of ethyl and methyl gas; and 3. That of KEKULÉ (Ann. Ch. Ph., Supp. b. ii. 111), in his explanation of the isomerism of maleic and fumaric acids, and of citraconic, itaconic, and mesaconic acids.*

I shall examine the 2d and 3d of these examples in detail, and think I shall be able to show a certain degree of inconsequence in both. Professor BUTLEROW argues, that in methyl gas the two atoms of carbon are combined by two affinities of the same kind (secondary affinities), each being the affinity which in iodide of methyl is combined with iodine. In hydride of ethyl, the two carbon atoms are combined in the same way as in the other members of the ethylic series, therefore, probably in the same way as in the members of the acetic series, one of which is acetonitrile, which is cyanide of methyl, the one is therefore the free affinity of methyl (secondary), the other the free affinity of cyanogen. These must be different, because hydride of ethyl is not identical with methyl gas. BUTLEROW indicates this, by calling the free affinity of cyanogen primary. We have thus in methyl gas two primary affinities united together, and in hydride of ethyl a primary united to a secondary.

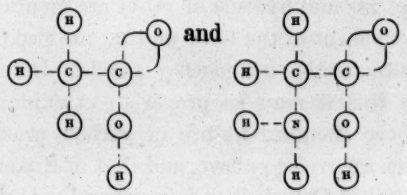
In this reasoning we have three assumptions,—1. That the nature of the carbon affinities is unchangeable; 2. That the carbon atoms continue united together by the same affinities through a series of chemical reactions, such as the

* I do not here notice the remarks of Professor KOLBE (Zeitschrift, vi. 13) on the same subject, as his object is rather to prove the metamerism than to explain the isomerism of these bodies.

transition from acetonitrile to hydride of ethyl; and, 3. That hydride of ethyl is not identical with methyl gas.

By carrying this argument a little further, and making use of no additional assumption, we arrive at an absurdity,—thus, the carbon radical of the acetic series is the same as that of oxyacetic (glycollic) acid, that again is the same as that of oxalic acid, therefore as that of oxalic nitrile or cyanogen gas; but in cyanogen gas we have the two carbon atoms united by two *primary* affinities; but we have before proved, that in the acetic series they are united by a primary affinity of the one, and a secondary affinity of the other. It is obvious, then, that at least one of our assumptions is false. And when we closely examine the two general assumptions (1. and 2.), we shall see reason to believe, that neither of them is rigidly true.

It is well known that the replacement of one equivalent in a compound by another, while it leaves the "chemical structure," or "chemical position" of the other atoms unchanged, exerts an influence on the *intensity* of the chemical attraction, not only of the equivalents directly concerned in the replacement, but of all the equivalents in the molecule.* To see this we have only to compare the nature of the force, uniting H to O in acetic acid, and in Glycocoll,



We here see the hydrogen and the ammonia residue NH_2 exerting a "disturbing" influence on the relation of oxygen to hydrogen through two carbon atoms. Many other examples will at once occur to every chemist. The nature of the equivalents, that is, of the force they exert, is thus seen to be variable, but the facts of absolute isomerism force us to admit, that this variation in the character and intensity of the chemical force exerted by the different equivalents of one atom depends upon something else, as well as upon the nature of the other equivalents with which that atom is united.

We find, then, that although the nature of one equivalent of an atom does change, as the other equivalents are united with different substances, there must be some original difference between them, which renders absolute isomerism possible. In what this original difference consists, whether it is essential or merely accidental (using the word in its strictly logical sense), we cannot as yet say.

We may thus divide the force, uniting any two equivalents, into two com-

* Butlerow notices this disturbing influence (Zeitschrift, vi. 516) as opposing an obstacle, which he seems to regard as for the present insuperable, in the way of determining whether a difference exist or not among the equivalents of a multequivalent atom.

ponents, one depending on the structure of the molecule, and the position of the equivalents in question in it, and the other independent of these. For convenience we may call the first the molecular, and the second the atomic component.*

From all that we know of the disturbing effect of an equivalent on the relations of the other equivalents in the molecule, we may safely assume that the *molecular* component of the force uniting the two carbon atoms to form the hexatomic carbon radical $(C_2)^{VI}$, is not the same in any two compounds. And if there be more than one body having the formula C_2H_6, we are forced to the conclusion, that in these cases at least, the atomic component is different also.

The question in reference to the second assumption mentioned above may now be stated thus,—Does the atomic component of the force uniting the two carbon atoms remain the same through such a series of transformations as that connecting acetonitrile and hydride of ethyl? There is every reason to suppose that it does, if it is always the same for the same pair of equivalents; for there is nothing in any of these transformations which would lead us to suppose that one carbon equivalent has changed places with another. We are then brought to the dilemma, either methyl gas and hydride of ethyl are identical, or a change takes place in the atomic component of the force uniting the two carbon atoms in some of the transformations connecting cyanogen gas and hydride of ethyl.

In connection with this, it may be proper to examine shortly the relation between the view of the chemical nature of carbon provisionally adopted by BUTLEROW, in the paper referred to above, and that of KOLBE, as explained in his "Lehrbuch der Organischen Chemie," and in several valuable papers in LIEBIG'S "Annalen," and in ERLENMEYER'S "Zeitschrift."

Professor KOLBE considers carbon (carbonyl $=(C_2)$, $C=6$) as a tetratomic element (as indeed it is now admitted by every one to be), and holds that in a large number of organic compounds, the four equivalents united to the carbonyl atom may be divided into two groups,—the intra-radical and the extra-radical,—so far his view resembles that of BUTLEROW; but it differs from it in the following respects:—BUTLEROW assumes that it is an essential property of the carbon atom to combine in this way, that carbon *always* combines with two equivalents in one way, and with two others in another way. KOLBE only admits this in the case of bodies derived from dibasic carbonic acid. In methylic compounds, for instance, he regards the three atoms of hydrogen as precisely similar. When we examine the theories more closely, we see another reason to doubt their identity. If we try to compare them, we find it difficult to decide whether BUTLEROW'S

* The term component is, of course, not used here in its strictly dynamical sense, what is meant is, that the total force uniting a pair of equivalents, is a function of two quantities, the one depending on the structure of the molecule, and the position in it of the two equivalents, and the other on the chemical nature of the two equivalents.

primary affinities correspond to the intra-radical or extra-radical affinities* of KOLBE; for instance, in formic acid, $-C\begin{cases} H \\ O'' \text{ or } C_2\begin{cases} O_2 \\ H \\ O\cdot HO \end{cases} \\ OH \end{cases}$, the H and the OH (or $O\cdot HO$) are united to the extra-radical affinities, but by carrying out BUTLEROW'S reasoning, we are led to the conclusion, that the H is united to a primary, and the OH to a secondary affinity, unless we suppose that the free affinity of methyl is different from that of formyl.

It is not impossible that such a difference may exist, but till this point is settled, and till we know whether the two free affinities of $(CO)''$ are similar or not, it is impossible satisfactorily to compare the two theories.

The other attempt mentioned above to explain cases of absolute isomerism, in harmony with the theory of atomicity, is that of Professor KEKULÉ.† After shortly recounting the principal facts, more minutely described in his admirable researches on organic acids, he says,—"All these facts find, in my opinion, to a certain extent, their explanation in the following considerations:—According to the views on the atomicity of the elements which I communicated some time since, succinic acid and its homologue pyrotartaric acid may be regarded as closed molecules, that is, all the affinities of the atoms composing the molecule are saturated by other atoms. Both acids contain two atoms of hydrogen replaceable by radicals, because two atoms of hydrogen are united to the carbon group by means of oxygen. These two replaceable (typical) atoms of hydrogen may be easily exchanged for metals, because, besides the two atoms of typical oxygen (*i.e.*, oxygen united by only one affinity to carbon), there are other two atoms of oxygen united to carbon by both affinities, which, therefore, in the language of the typical theory, belong to the radical. If these two atoms of hydrogen are represented apart from the rest, as is done by means of the typical formulæ,—

$$\left.\begin{matrix} C_4\,H_4\,O_2 \\ H_2 \end{matrix}\right\} O_2 \qquad\qquad \left.\begin{matrix} C_5\,H_6\,O_2 \\ H_2 \end{matrix}\right\} O_2$$

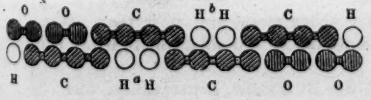
or still more clearly by the graphic representations which I have made use of more than once in another place,‡ it will be readily seen, that in succinic acid

* I use the terms intra- and extra- radical affinities as abbreviations for the carbon affinities, with which the intra- and extra- radical oxygen atoms are combined.

† *Loc. cit.*

‡ In order to elucidate this passage as much as possible, I append the graphic representations referred to:—

Succinic Acid.



Pyrotartaric Acid.

there are four, and in pyrotartaric acid six, other atoms of hydrogen present. This hydrogen is considered, according to the typical theory, to belong to the radical, according to the theory of atomicity, to be directly united to the carbon, and in such a way that there are always two atoms of hydrogen united to the same carbon atom. If we now suppose, that in the one or the other of these two normal acids two such hydrogen atoms are wanting, we have, on the one hand, the composition of fumaric and maleic acids, on the other, the formula of citraconic itaconic, and mesaconic acids. Now, as there are in succinic acid *two* pairs of hydrogen united in this way to carbon, we easily see the possibility of the existence of two dehydrogenated acids, as the one or the other of these pairs of hydrogen atoms is absent.

"Similarly, in the case of pyrotartaric acid, the existence of three isomeric dehydrogenated acids is intelligible, in each of which another of the three pairs of hydrogen atoms directly united to carbon is absent. At that place in the molecule where the two hydrogen atoms are wanting, there are two carbon affinities unsaturated, there is at that place, so to speak, a blank."*

There is some difficulty in understanding this last statement. For what can be meant by two affinities of the same carbon atom uniting together? unless the definition of either "atoms" or "combination" be completely changed. Or if we take the natural meaning of the sentence last quoted, and suppose two carbon atoms pushed together, so that two affinities of each previously united to hydrogen come to be united together, the two wanting hydrogen atoms do not come from the same, but from two different carbon atoms.†

But leaving this preliminary difficulty out of consideration, and also granting the existence in such a molecule of diatomic carbon, a glance at the diagrams is sufficient to show, that this view does not give us two but only one formula for fumaric and maleic acids, unless a difference be admitted among the affinities of carbon. The pairs of hydrogen atoms, which I have marked *a* and *b*, have perfectly similar positions, the one being related to one end of the diagram, exactly as the other is to the other end. In the same way the graphic representation of pyrotartaric acid suggests only two formulæ for citraconic, itaconic, and mesaconic acids, there being no apparent difference between the position of the pairs *c* and *e*.‡ This explanation is insufficient, not to say unintelligible, unless we suppose a functional difference between the two carbon atoms, with which the pairs of atoms of hydrogen are united; and if we make this assumption, it follows as a necessary consequence, that there is a difference between the two groups of HO. It is no doubt possible, that such a difference may exist; but it

* "It may of course equally well be assumed, that the carbon atoms are, as it were, pushed together (zusammengeschoben), so that two carbon atoms are united by two affinities of each. This is only another form of the same idea."

† See Erlenmeyer, Zeitschrift, vi. 21.

‡ See Butlerow, Zeitschrift, vi. 524.

seems very unlikely that such a property of a substance so well known as succinic acid should not have been observed.

We thus see, that the attempts to apply to the explanation of particular cases the principle of a difference between the equivalents of multequivalent atoms have failed, not, however, as far as can at present be seen, from any absurdity in the principle itself, but rather from a want of well-observed facts to guide us in its application. These we may expect before long, from the labours of BUTLEROW, SCHORLEMMER, and others, and we shall then be in a position to form a definite opinion as to the form which this hypothesis should assume.

XLV.—*On the Theory of Commensurables.* By Edward Sang, Esq.

(Read 7th March 1864.)

The general proposition in the theory of commensurables is to determine the conditions under which lines, surfaces, or solidities, connected with prescribed figures or forms, may have their ratios expressible by integer numbers.

The attention of geometers must have been drawn to this subject by the contemplation of incommensurable lines: the altitude of an equilateral trigon is incommensurable with the base; the diagonal of a square incommensurable with the side, and so on. And, when the two sides of a right angle are expressed by two numbers, the hypotenuse is, in the great multitude of cases, incommensurable with the sides: thus, if the sides be 5 and 7 inches respectively, the length of the subtense cannot be accurately expressed either in integers or fractions. However, when the sides are 3 and 4 units, the hypotenuse is exactly 5 of the same units.

This circumstance seems to have turned the inquiries of geometers at a very early period to the discovery of those cases in which the three sides of a right-angled trigon are all commensurable; and the computation of Pythagorean numbers was to them a very interesting problem.

Many problems of a similar nature may be proposed: thus we may require that the three sides of a trigon, and the line bisecting one of the angles, be all commensurable; that the four sides and the two diagonals of a tetragon be all expressed by integer numbers, and so on.

The methods of solving such problems may be said to constitute the theory of commensurables.

Section 1.—*On the Right-Angled Trigon.*

1. If A and B represent the sides of a right-angled trigon, of which the hypotenuse is C, we must have the equation

$$A^2 + B^2 = C^2,$$

and the question is to discover integer values of A, B, and C, which may satisfy this equation.

When it happens that two of these three numbers have a common divisor, the third number must have the same divisor; for if A and B had the common divisor n, so that $A = na$, $B = nb$, $A^2 + B^2$, that is C^2 would be divisible by n^2; in other words, C would be divisible by n, and thus the case A, B, C might be reduced by division to the lower numbers a, b, c.

And conversely, when we have obtained one solution, as 3, 4, 5, we can thence

deduce others, as 6, 8, 10; 30, 40, 50, by multiplication. Hence it is almost enough to consider only those cases in which the three numbers are prime to each other. For convenience we shall indicate that the solution is in the lowest terms, by the use of the small letters; solutions in general being indicated by capitals.

2. Any odd prime number may represent the side of a rational right-angled trigon, but of only one.

For the equation $A^2+B^2=C^2$ may be put under the form $A^2=C^2-B^2=(C+B)(C-B)$, while A^2 may be regarded as the product of unit by A^2; wherefore we may put

$$A^2=C+B; \quad 1=C-B, \text{ whence}$$
$$B=\tfrac{1}{2}(A^2-1); \quad C=\tfrac{1}{2}(A^2+1)$$

so that if A be an odd number, A^2-1 and A^2+1 are both even, and therefore B and C both integers.

Representing prime numbers by the Greek letters, if $a=a$, we have $b=\frac{1}{2}(a^2-1)$; $c=\frac{1}{2}(a^2+1)$, whence the solutions

$$3, 4, 5; \quad 5, 12, 13; \quad 7, 24, 25; \quad 11, 60, 61, \quad \&c.$$

Since a is a prime number, the only decompositions of a^2 are $a^2\times 1$ and $a\times a$. The former of these has just been considered; the latter would give $a=C-B$, $a=C+B$, whence $B=o$, $C=a$, showing that the trigon has collapsed into a straight line.

3. Any odd number which is the product of *two* unequal prime factors may represent the side of *four* distinct rational right-angled trigons: of these two are reducible and two are in their lowest terms.

If A be the product of two primes, α, β (neither of which is 2), we may put A^2 as the product of two factors in five ways, viz., $\alpha^2\beta^2\times 1$; $\alpha^2\beta\times\beta$; $\alpha\beta^2\times\alpha$; $\alpha^2\times\beta^2$; and $\alpha\beta\times\alpha\beta$; the last of these can give no trigon, so that there remain the four solutions

$b=\frac{1}{2}(\alpha^2\beta^2-1)$	$B=\frac{1}{2}(\alpha^2\beta-\beta)$	$B=\frac{1}{2}(\alpha\beta^2-\alpha)$	$b=\frac{1}{2}(\alpha^2-\beta^2)$
$c=\frac{1}{2}(\alpha^2\beta^2+1)$	$C=\frac{1}{2}(\alpha^2\beta+\beta)$	$C=\frac{1}{2}(\alpha\beta^2+\alpha)$	$c=\frac{1}{2}(\alpha^2+\beta^2)$

In the second of these, each side is divisible by β, in the third by α, so that these two solutions are included among those of the preceding article: but in the first and last, the sides are all prime to each other.

Hence the solutions 15, 112, 113; 15, 8, 17; 33, 544, 545; 33, 56, 65, &c.

4. Any odd composite number of the form $\alpha^p.\beta^q.\gamma=A$ may be the side of half as many distinct trigons as there are units in the product $(1+2p)(1+2q)(1+2r)\ldots$ less one.

For the continued product $\beta^2=\alpha^{2p}.\beta^{2q}.\gamma^{2r}\ldots$ &c., has, inclusive of unit and A^2 itself, divisors to the number $(1+2p)(1+2q)(1+2r)\ldots$ of which A itself is

one. Omitting the case A × A, which cannot give a trigon, it follows that A^2 may be represented as the product of two factors in $\frac{1}{2}\{(1+2p)\ (1+2q)\ (1+2r) \ldots -1\}$ ways. Each one of these gives a distinct solution, for if $A^2 = P \cdot Q$ be one of these decompositions,

$$A = \sqrt{P.Q};\ B = \tfrac{1}{2}(P-Q);\ C = \tfrac{1}{2}(P+Q).$$

Of the total number of these solutions, only those are in their lowest terms in which P and Q are prime to each other. Therefore, if n be the number of separate primes which enter into A as factor, the number of cases in their lowest terms is 2^{n-1}, and is irrespective altogether of the exponents p, q, r, &c.

5. No double of an odd number can be the side of a rational right-angled trigon in its lowest terms.

For if A be the double of an odd number itself prime, or the product of two or more prime factors, α, β, its square is $4\ \alpha^2\ \beta^2$, which can be resolved into unequal factors prime to each other only of the general forms $4\ \alpha^2\beta^2 \times 1$; $4\ \alpha^2 \times \beta^2$, one of which is even and the other odd; wherefore the sums and the differences of these factors are all odd, so that in the solutions

$$A = 2\alpha\beta;\quad B = \tfrac{1}{2}(4\alpha^2\beta^2 - 1);\quad C = \tfrac{1}{2}(4\alpha^2\beta^2 + 1)$$
$$A = 2\alpha\beta;\quad B = \tfrac{1}{2}(4\alpha^2 - \beta^2);\quad C = \tfrac{1}{2}(4\alpha^2 + \beta^2)\ \&c.$$

the fraction $\frac{1}{2}$ must occur. To remove this fraction we must double all, and then

$$A = 4\alpha\beta;\quad B = 4\alpha^2\beta^2 - 1;\quad C = 4\alpha^2\beta^2 + 1:$$
$$A = 4\alpha\beta;\quad B = 4\alpha^2 - \beta^2;\quad C = 4\alpha^2 + \beta^2:\ \&c.$$

6. Any number of the general form $2^t \lambda\mu$ in which t exceeds unit, λ and μ being odd numbers prime to each other, may be the side of a rational right-angled trigon in its lowest terms.

For $A^2 = 2^{2t}\lambda^2\mu^2$ and may be resolved into the two factors $P = 2^{2t-1}\lambda^2$; $Q = 2\mu^2$, whence $A = 2^t\lambda\mu$; $B = 2^{2t-2}\lambda^2 - \mu^2$; $C = 2^{2t-2}\lambda^2 + \mu^2$, which are all integer and prime to each other.

If n be the number of odd primes which enter into A as factors, the total number of cases in their lowest terms is 2^{n-1}.

7. l and m being any two numbers whatever, $l^2 + m^2$, $2lm$ and $l^2 - m^2$ represent the three sides of a right-angled trigon.

For if $A = 2lm$, $B = l^2 - m^2$ and $C = l^2 + m^2$, we have $A^2 + B^2 = C^2$.

If l and m have a common divisor, A, B, C, can all be divided by the square of that divisor. If l and m be prime to each other and *both odd*, the numbers in the above solution may be halved; but if one be odd, and the other even, the solution as given is in its lowest terms.

8. Of a right-angled trigon in integers, one of the two sides is divisible by 3.

In this investigation we may suppose that the trigon is in its lowest terms.

In the equation $a^2 + b^2 = c^2$, if the number a be not divisible by 3, it must be of one of the forms $3n+1$, $3n-1$, so that its square must be of the form $3n+1$.

If now b be also indivisible by 3 its square must be of the same form, so that a^2+b^2 would be of the form $3n+2$, which cannot possibly be that of a square number. And thus one of the two must be divisible by 3.

9. Of a right-angled trigon in integers, one of the two sides is divisible by 4.

The square of every odd number is of the form $8n+1$, wherefore if a and b were both odd numbers, a^2+b^2 would be of the form $8n+2$, which cannot be that of a square; thus one or other of the two must be even. Now if a be even, b and c must be both odd, wherefore a^2 being the difference of two odd squares, viz., c^2-b^2 must be divisible by 8; but no square can be divisible by 8 unless its root be divisible by 4, wherefore the even side a must be divisible by 4.

10. Of a right-angled trigon in integers, one of the three sides is divisible by 5.

All numbers not divisible by 5 are of the forms $5n\pm1$, $5n\pm2$, the squares of which are contained in the forms $5n+1$, $5n-1$. If neither a nor b be divisible by 5, their squares cannot be both of the form $5n+1$, nor both of the form $5n-1$, for then a^2+b^2 would be of the form $5n+2$, or $5n-2$, neither of which belongs to a square. Wherefore, if a be of one of the forms $5n\pm1$, b must be of one of the forms $5n\pm2$, in which case a^2+b^2 is of the form $5n$, and therefore may be a square.

Again, if the hypotenuse c be not divisible by 5, $c^2 \backsim 5n\pm1$; and similarly, if b be not divisible by 5, $b^2 \backsim 5n\pm1$. If now $b^2 \backsim 5n+1$ while $c^2 \backsim 5n-1$, the difference c^2-b^2 would be of the form $5n-2$; or if $b^2 \backsim 5n-1$ while $c^2 \backsim 5n+1$, c^2-b^2 would be of the form $5n+2$; neither of these can be square. The other two combinations $b^2 \backsim 5n+1$, $c^2 \backsim 5n+1$, and $b^2 \backsim 5n-1$, $c^2 \backsim 5n-1$, both give $c^2-b^2 \backsim 5n$ which can be a square, and then a must be divisible by 5.

11. The hypotenuse of a right-angled trigon in its lowest terms can never be divisible by 7.

For if c were divisible by 7, neither a nor b can be so, as then all three would be divisible by 7. Hence a and b must be of some of the forms $7n\pm1$, $7n\pm2$, $7n\pm3$, and therefore their squares must belong to some of the forms $7n+1, 7n+4$, $7n+2$. Now no two of the remainders 1, 1; 4, 4; 2, 2 can make up 7, and therefore a^2+b^2 can never be divisible by 7.

The same argument may be used to show that 19, 23, 31, &c., cannot be divisors of the hypotenuse of a right-angled trigon in its lowest terms.

12. *Lemma.* If two numbers be each the sum of two squares, their product may be decomposed into two squares at least in two ways.

Let $d=s^2+t^2$, and $e=u^2+v^2$, then, on multiplying we obtain

$$de=s^2u^2+s^2v^2+t^2u^2+t^2v^2,$$

which may be put under either of the forms,

$$de=s^2u^2+2stuv+t^2v^2+s^2v^2-2stuv+t^2u^2;$$
$$de=s^2u^2-2stuv+t^2v^2+s^2v^2+2stuv+t^2u^2;$$

which are equivalent to

$$de=(su+tv)^2+(sv-tu)^2;$$
$$de=(su-tv)^2+(sv+tu)^2;$$

and thus the product de is shown to be the sum of two squares in two different ways.

Cor. The square of the sum of two squares is also the sum of two squares

For if

$$\begin{aligned} d &= s^2+t^2 \\ d^2 &= s^4+2s^2t^2+t^4 \\ &= s^4-2s^2t^2+t^4+4s^2t^2 \\ &= (s^2-t^2)^2+(2st)^2 \end{aligned}$$

13. If a number be divisible into two squares prime to each other in two different ways, it is the product of two numbers, each of which is the sum of two squares.

Let c be the sum of s^2 and t^2, and also the sum of u^2 and v^2, that is, let

$$C=s^2+t^2=u^2+v^2;$$

then

$$s^2-u^2=v^2-t^2, \text{ or } s+u:v+t::v-t:s-u$$

wherefore the ratio $s+u:v+t$ is not in its lowest terms; that is, $s+u$ and $v+t$ must have a common divisor, which we may suppose to be e, put then $s+u=ex$, $v+t=ey$; and then $x:y::v-t:s-u$, so that we may put $v-t=fx$, $s-u=fy$, in which f may have the value *unit*.

We thus obtain

$$\begin{aligned} s+u&=ex; \quad v+t=fx \\ s-u&=fy; \quad v-t=ey \end{aligned}$$

whence

$$s=\tfrac{1}{2}(ex+fy),\ t=\tfrac{1}{2}(fx-ey)$$

so that

$$\begin{aligned} s^2+t^2 &= \tfrac{1}{4}\{e^2x^2+2efxy+f^2y^2+f^2x^2-2efxy+e^2y^2\} \\ &= \tfrac{1}{2}(e^2+f^2)(x^2+y^2) \end{aligned}$$

and thus c is the product of two factors, each of which is the sum of two squares.

Cor. Hence no prime number can be divided into two squares in more than one way.

14. If an even number be the sum of two squares, its half is also the sum of two squares; and conversely.

If the even number $2n$ be the sum of two squares s^2 and t^2, its half n is also the sum of two squares.

It is evident that the numbers s and t must either be both even or both odd, wherefore $\frac{1}{2}(s+t)$ and $\frac{1}{2}(s-t)$ must be integers; now $\frac{1}{4}(s+t)^2+\frac{1}{4}(s-t)^2=\frac{1}{2}(s^2+t^2)=n$, therefore n is the sum of two squares.

15. If the successive square numbers be divided by any odd prime a, the remainders, including zero, recur in groups of a terms; and of the $a-1$ terms

between 0 and 0, the one half is the converse of the other: farther, no two remainders in the half group are alike.

The import of these assertions may be seen by taking any prime number, as 11, and dividing the successive square numbers 0, 1, 4, 9, 16, 25, 36, 49, &c. by it. The remainders are found to be 0, 1, 4, 9, 5, 3, 3, 5, 9, 4, 1, 0, 1, 4, 9, 5, 3, 3, 5, 9, 4, 1, 0, and so on.

In reference to the divisor a, the natural numbers 0, 1, 2, 3, &c., belong to the forms

$$\left.\begin{array}{l} na+0 \\ na+1 \\ na+2 \\ \quad\cdots \\ \quad\cdots \\ na+\frac{a-3}{2} \\ na+\frac{a-1}{2} \\ na-\frac{a-1}{2} \\ na-\frac{a-3}{2} \\ \quad\cdots \\ \quad\cdots \\ na-2 \\ na-1 \\ na-0 \end{array}\right\} \text{of which the squares are of the same forms with} \left\{\begin{array}{l} na+0 \\ na+1 \\ na+4 \\ \quad\cdots \\ \quad\cdots \\ na+\frac{a^2-6a+9}{4} \\ na+\frac{a^2-2a+1}{4} \\ na+\frac{a^2-2a+1}{4} \\ na+\frac{a^2-6a+9}{4} \\ \quad\cdots \\ \quad\cdots \\ na+4 \\ na+1 \\ na+0 \end{array}\right.$$

From which it is obvious that the order of the remainders in the latter half of the group is inverse of the order in the former half.

Also the same remainder cannot occur twice in the half group. For if any two squares, as k^2 and l^2, k and l being each less than $\frac{1}{2}a$, had the same remainder, their difference k^2-l^2 would be divisible by a. Now both $k+l$ and $k-l$, the factors of k^2-l^2, are less than a, and their product cannot possibly be divisible by a.

16. The sum of two squares which are prime to each other is not divisible by any number of the form $4n-1$.

Every composite number which is of the form $4n-1$, must have an odd number of factors of the same form combined with some or no factors of the form $4n+1$. It will be enough to show that the sum of two squares prime to each other cannot be divided by any prime number of the form $4n-1$.

If the two numbers s and t be both odd, s^2+t^2 is even, and its half is the sum of two squares and also odd; and if s^2+t^2 be divisible by any prime β, its half must also be divisible by β. Wherefore we have only to show that the sum of an odd and an even square can never be divisible by $\beta = 4n-1$.

Let us suppose that $s=p\beta\pm k$, $t=q\beta\pm l$, then

$$s^2+t^2=(p^2+q^2)\beta^2\pm 2(pk+ql)+k^2+l^2$$

k and l being each less than $\frac{1}{2}\beta$. Wherefore if the sum s^2+t^2 be divisible by a

prime number β, we can always find two numbers k and l each less than $\frac{1}{2}\beta$ also divisible by β.

In regard to k and l being odd or even, there are only three possible combinations; one may be odd and the other even, in which case k^2+l^2 is of the form $4n+1$; they may be both odd, in which case k^2+l^2 is double of an odd number, which odd number is the sum of an odd and even square, and still divisible by β; lastly, they may be both even, in which case we can halve them, and continue to halve, until one of the quotients be even and the other odd, without affecting the divisibility of the sum of their squares by β.

Therefore, universally, if the prime number β be a divisor of s^2+t^2, s and t being prime to each other, it must also be the divisor of k^2+l^2, in which k and l are each less than $\frac{1}{2}\beta$, the one being even and the other odd.

Now k^2+l^2 being of the form $4n+1$, cannot be divisible by any number β of the form $4n-1$, unless the quotient be also of the form $4n-1$; but k^2+l^2 is less than $\frac{1}{2}\beta^2$, wherefore that quotient must be less than $\frac{1}{2}\beta$. And thus if any prime number β of the form $4n-1$ can be a divisor of the sum of an odd and an even square, some number less than its half, and consequently some prime number less than its half, and of the same form $4n-1$, must also be a divisor.

If it were possible, then, that a prime such as 103 could divide the sum of an odd and an even square, it would follow that some other prime of the same form, and less than 51, would also be a divisor. The greatest prime of the form $4n-1$, under 51 is 47; if it, or any prime of the same form less than it, were a divisor of k^2+l^2; it would follow that some other prime (23 or under) would also be a divisor. In this way we must ultimately arrive at the smallest prime of this form, which is 3. Now there is no even number less than the half of 3, so that 3 cannot be a divisor of the sum of an odd and even square, and therefore we conclude that no prime number, nor any other number, of the form $4n-1$ can be the divisor of the sum of two squares prime to each other.

It is obvious that, whatever β may be, it is always a divisor of $(\beta s)^2+(\beta t)^2$, but then it is also the common divisor of βs and βt.

17 Every prime number of the form $4n+1$ is the hypotenuse of one sole right angled trigon in integer numbers.

Every prime number of the form $4n+1$ can be resolved into two square numbers, one of which is odd and one even, but only into one pair. Let, then

$$\gamma=(2p)^2+q^2,$$

and

$$\alpha^2=16p^4+8p^2q^2+9^4=(4p^2-q^2)^2+4pq,$$

wherefore γ is the hypotenuse of a right-angled trigon, of which $4p^2-q^2$ and $4pq$ are the two sides.

Here it may be observed that one of the sides is always divisible by 4.

18. Every product of two prime numbers γ, δ, each of the form $4n+1$ is the

hypotenuse of four right-angled trigons in integers; of which two are in their lowest terms, and two are reducible by the divisors γ, δ.

For, since every prime number of the form $4n+1$ is the sum of two squares, we may put

$$\gamma^2 = p^2 + q^2, \quad \delta^2 = r^2 + s^2,$$

whence we can form the four equations

$$\begin{aligned}
\gamma^2\delta^2 &= (pr+qs)^2 + (ps-qr)^2 \\
\gamma^2\delta^2 &= (pr-qs)^2 + (ps+qr)^2 \\
\gamma^2\delta^2 &= \gamma^2r^2 + \gamma^2s^2 \\
\gamma^2\delta^2 &= \delta^2p^2 + \delta^2q^2
\end{aligned}$$

giving the four trigons

$$\begin{array}{ccc}
pr+qs, & ps-qr, & \gamma\delta \\
pr-qs, & ps+qr, & \gamma\delta \\
\gamma r, & \gamma s, & \gamma\delta \\
\delta p, & \delta q, & \gamma\delta
\end{array}$$

of which the two first are in the lowest terms, while the two latter are reducible.

19. The product of n primes, each of the form $4n+1$, is the hypotenuse of 2^{n-1} right-angled trigons in their lowest terms.

For with one prime γ we can have one trigon; with the product of two primes, $\gamma\delta$, we can have two trigons. On introducing a new prime factor ϵ, we can combine the sides belonging to it with each of the two former trigons in two ways, thus obtaining four cases. With a fourth factor ζ, we can obtain two for each of these four, and thus we proceed doubling the number of trigons at each accession of a new factor.

20. Every power of a prime number of the form $4n+1$ is the hypotenuse of one sole right-angled trigon in its lowest terms.

If a, b, γ, represent the three sides of a right-angled trigon; that is, let

$$a^2 + b^2 = \gamma^2.$$

Also put A, B, γ^n, for those of a trigon having γ^n for its hypotenuse, or,

$$A_n^2 + B_n^2 = \gamma^{2n}.$$

Multiply these two equations and we obtain

$$A_n^2a^2 + A_n^2b^2 + B_n^2a^2 + B_n^2b^2 = \gamma^{2n+2},$$

or,

$$(A_na + B_nb)^2 + (A_nb - B_na)^2 = (\gamma^{n+1})^2,$$

or,

$$(A_na - B_nb)^2 + (A_nb + B_na)^2 = (\gamma^{n+1})^2,$$

from which it would appear that for every trigon with the hypotenuse γ^n we have *two* with the hypotenuse γ^{n+1}. One of these, however, is reducible, the other is not.

Taking the operation in detail, we have, on supposing

A_n, B_n, γ^n;

A_{n+1}, B_{n+1}, γ^{n+1}, &c. to have been deduced in succession from

a, b, γ,

we have

$$A_{n+1}=aA_n+bB_n;\ B_{n+1}=aB_n-bA_n.$$

If now we deduce from these,

$$A_{n+2}=aA_{n+1}+bB_{n+1};\ B_{n+2}=aB_{n+1}-bA_{n+1},$$

the results are,

$$A_{n+2}=(a^2-b^2)A_n+2abB_n;\ B_{n+2}=-2abA_n+(a^2-b^2)B_n,$$

which, if A_n and B_n be prime to each other, have no common divisor.

But if we take the second combination, and put

$$A_{n+2}=aA_{n+1}-bB_{n+1};\ B_{n+2}=aB_{n+1}+bA_{n+1},$$

we obtain

$$A_{n+2}=(a^2+b^2)A_n;\ B_{n+2}=(a^2+b^2)B_n;\ \gamma^{n+2},$$

which are all divisible by a^2+b^2 or γ^2, and bring us back to

$$A_n,\ B_n,\ \gamma^n.$$

And thus we see that in forming the successive trigons A_2, B_2, γ^2; A_3, B_3, γ^3, &c. from a, b, γ; and confining ourselves to those only which are in their lowest terms, we form only one series.

21. Every number which is the product of n primes of the form $4n+1$, or of any powers of those primes, may be the hypotenuse of 2^{n-1} right-angled trigons in their lowest terms.

In regard to the obtaining of trigons in their lowest terms, the power γ^n gives only one as γ itself does; wherefore the combinations $\gamma^n\delta^p\epsilon^q$, &c. give just as many cases as the product $\gamma\delta\epsilon$, &c.

General Scholium.

From these propositions we obtain a convenient method of computing and tabulating the cases of right-angled trigons in the lowest terms.

We form a list of the prime numbers of the form $4n+1$; these are 5, 13, 17, 29, 37, &c., and we decompose each of these into two squares; thus, $2^2+1^2=5$; $3^2+2^2=13$; $4^2+1^2=17$, and so on, the general form being $p^2+q^2=\gamma$. From these decompositions we then obtain, according to the formulæ

$$a=2pq,\ b=p^2-q^2,$$

the sides of the trigons of which these prime numbers are the hypotenuses.

Having thus constructed, to whatever extent we may desire, the table of trigons with prime hypotenuses, we combine them according to the formulæ of articles 18, 19, 20, and obtain the cases in which the hypotenuses are composite.

In table I. are given the roots of the component squares, the prime numbers, and the sides, for all prime hypotenuses under one thousand, and also the factors of the hypotenuse with the corresponding sides for all composite numbers up to the same limit.

Section 2.—*On Muarif Angles.*

22. If the sine and cosine of an angle be both commensurable with the radius, all other functions, as the tangent, secant, cotangent, cosecant, of that angle are also commensurable.

If the sine of any angle, as θ be expressed by the fraction $\frac{a}{c}$, a and c being prime to each other, the cosine of that angle is $\frac{1}{c}\sqrt{c^2-a^2}$, wherefore, in order that the cosine may be rational, c^2-a^2 must be a square number, say b^2, that is to say, $a^2+b^2=c^2$, and thus it appears that all such angles belong to rational right-angled trigons, so that we may write

$$\sin\theta=\frac{a}{c},\ \cos\theta=\frac{b}{c};\ \text{whence}\ \tan\theta=\frac{a}{b},$$

$$\cot\theta=\frac{b}{a},\ \sec\theta=\frac{c}{b},\ \operatorname{cosec}\theta=\frac{c}{a}.$$

Definition.—We shall see immediately that these angles indicate or make known the solutions of many problems in commensurables, and therefore I propose to designate them by the title معرف, *muarif*, formed from the same root as the word *tarif* in common use.

23. All the trigonometrical functions of the sum and of the difference of two muarif angles are also rational.

If $\sin\theta=\frac{a}{c}$, $\cos\theta=\frac{b}{c}$ while $\sin\phi=\frac{\mathrm{A}}{c}$, $\cos\phi=\frac{\mathrm{B}}{c}$, we have, according to the fundamental propositions of trigonometry,—

$$\sin(\theta+\phi)=\frac{a\mathrm{B}+\mathrm{A}b}{c\mathrm{C}};\ \cos(\theta+\phi)=\frac{b\mathrm{B}-a\mathrm{A}}{c\mathrm{C}}$$

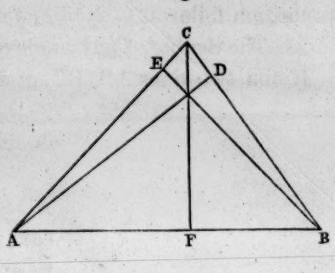
$$\sin(\theta-\phi)=\frac{a\mathrm{B}-\mathrm{A}b}{c\mathrm{C}};\ \cos(\theta+\phi)=\frac{b\mathrm{B}+a\mathrm{A}}{c\mathrm{C}}$$

which are all rational, and therefore all the other trigometrical functions of the same angles are rational.

N.B.—In Table I. the values of the lesser angle of each trigon is given in ancient degrees.

24. Every trigon of which two angles are muarif has its altitudes, its areas, the radius of the circumscribed circle, and those of the four circles of contact, all rational.

If the angles at A and at B be both muarif, the sides of the trigons AFC, BFC are commensurable; wherefore the area of the trigon is commensurable with the squares of any of the lines AF, FC, FB, BC, CA, also the trigons ADB, AEB, are similar to CFB and AFC respectively; wherefore their sides also are commensurable.



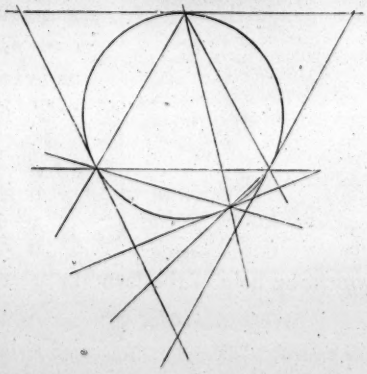
Or in general, all the angles shown in the figure are muarif, and therefore all the trigons have commensurable sides and areas.

If a, b, c, be the three sides of any trigon, and S the surface, the radius of the inscribed circle is $\frac{2S}{a+b+c}$, those of the three circles of external contact are $\frac{2S}{a+b-c}$, $\frac{2S}{a-b+c}$, $\frac{2S}{-a+b+c}$, and that of the circumscribed circle is $\frac{abc}{4S}$ and those, obviously, are all rational if S, a, b, and c be so.

25. If any straight line be assumed as a base, and if, at each of its extremities any number of muarif angles be made, the sides of these being indefinitely extended, all the distances intercepted on them are commensurable with the base, and all the included areas are commensurable with the square of the base.

It is evident that all the angles obtained by this construction are muarif, and that, therefore, the areas of all the trigons formed on the base are commensurable with its square, while their sides are all commensurable with the base itself. Now, all the segments are differences or sums of the sides of these trigons, and all the areas are differences or sums of their areas; wherefore, all of these are commensurable with the square of the base, and all of those with the base itself.

26. If, at any of the points of intersection of the preceding article, muarif angles be made, the segments and areas so obtained are all commensurable with the base, and with its square.



The truth of this assertion follows at once from that of the preceding.

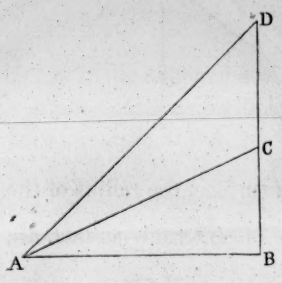
27. If, at the point of contact of a straight line with a circle muarif angles be made, if the extremities of the chords so obtained be joined and continued indefinitely, and if tangents be applied at their extremities, all the intercepted straight lines are commensurable with the diameter, and all the areas with the square of the diameter.

Since the angles which a chord makes with a tangent at its extremity are equal to those in the alternate segments of the

circle, every angle obtained in this way is muarif, and therefore the truth of the proposition follows.

28. The double of any angle of which the tangent is rational, is a muarif angle.

If the two sides AB, BC, of the right angle ABC be commensurable; that is, if the tangent of the angle BAC be rational, BAD double of BAC is muarif.



Since AC bisects the angle BAD, we have BA : AD : : BC : CD and $AC^2 = BA \cdot AD - BC \cdot CD$, whence easily $BA^2 - AC^2 : BA^2 + AC^2 : : BA : AD : : BC : CD$ wherefore AD and CD are both rational, and consequently BD is rational, that is to say, the angle BAD is muarif.

Otherwise if $\tan \theta = \frac{b}{a}$, whence $\sin \theta = \frac{a}{(a^2+b^2)}$, $\cos \theta = \frac{b}{\sqrt{(a^2+b^2)}}$ and $\sin 2\theta = \frac{2ab}{a^2+b^2}$; $\cos 2\theta = \frac{a^2-b^2}{a^2+b^2}$; that is both $\sin 2\theta$ and $\cos 2\theta$ are rational.

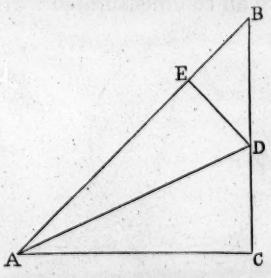
29. To find a muarif angle which may approximate as closely as may be desired to a given angle.

The general solution of this problem follows at once from the preceding theorem; we have to compute the series of fractions which approach to the tangent of the half of the given angle, and from these to deduce the corresponding muarif angles.

In particular cases, however, special solutions may be obtained.

Example 1.

30. To construct a rational right-angled trigon of which the angle may be nearly half a right angle.



Having constructed the right-angled isoskeles trigon ABC and bisected the angle at A by the line AD, draw DE perpendicular to the hypotenuse AB, then, as is easily shown, CD = DE = EB, and CD : DB : : 1 : $\sqrt{2}$; or, CD : AC : : 1 : $1+\sqrt{2}$, that is $\tan CAD = \frac{1}{1+\sqrt{2}}$, the Brounckerian approximations to which are

$$\frac{0}{1}\ \frac{2}{0}\ \frac{2}{1}\ \frac{2}{2}\ \frac{2}{5}\ \frac{5}{12}\ \frac{2}{29}\ \frac{2}{70}\ \frac{2}{169}\ \frac{70}{408}\ \frac{,2}{985}, \text{ &c.}$$

which, by help of the formula $A = 2ab$, $B = a^2 - b^2$, $C = a^2 + b^2$, give the cases,—

5,	4,	3,
29,	20,	21,
169,	119,	120,
985,	697,	696,
5741,	4059,	4060, &c.

This proposition may also be put in the form "to find two numbers which may differ by unit, and the sum of whose squares may be a square,"

Putting $B=A+1$ in the equation $C^2=B^2+A^2$, we have $C^2=2A^2+2A+1$, or $2C^2=(2A+1)^2+1$; that is to say, we must resolve the indeterminate equation $D=\sqrt{(2C^2-1)}$ in integers. For this purpose we put $\sqrt{2}$ in the form of a continued fraction, and obtain the progression,—

	1	2	2	2	2	2	2	2	2	
0	1	1	3	7	17	41	99	239	577	1393
1	0	1	2	5	12	29	70	169	408	985

&c.

of which the alternate terms $\frac{1}{1}, \frac{7}{5}, \frac{41}{29}$ belonging to our equation, the intermediates $\frac{3}{2}, \frac{17}{12}$, &c., belonging to the twin equation $D=\sqrt{2C^2+1}$. The former follow the law $(-1, 6)$, that is to say, if the progression of the numerators be $D_1, D_2, D_3, \ldots D_n, D_{n+1} \ldots$ we have $D_{n+2}=6D_{n+1}-D_n$, and so also of the denominators; the progression thus formed being,—

$$\frac{-1}{1}, \frac{1}{1}, \frac{7}{5}, \frac{41}{29}, \frac{239}{169}, \frac{1393}{985}, \frac{8219}{5741}, \text{ &c.}$$

in which the denominator of each fraction gives a hypotenuse, while the integers immediately above and below the half of the numerator represent the two sides.

Example 2.

31. To construct a right-angled trigon in integers, such that its lesser angle may be the tenth part of a revolution.

The tangent of $18°=20^g$ is $\sqrt{1-\frac{2}{5}\sqrt{5}}$; or ·3249 1969 6232 907, which, when resolved into a chain fraction, gives the quotients,—

3, 12, 1, 6, 1, 6, 25, 4, 1, 1, 15, 1, 1, 1, &c.,

whence the approximations,—

	3	12	1	6	1	1	
1	0	1	12	13	90	103	708
0	1	3	37	40	277	317	2179

&c.

which give the trigons,—

10, 8, 6, or 5, 4, 3; 1513, 1225, 888; 1769, 1431, 1040; 84829, 68629, 49860, &c.; the third of which gives an angle 36° 00′ 30″.

Example 3.

32. To construct a right-angled trigon in integers of which the lesser angle may be nearly 23° 27′ 26″, the obliquity of the ecliptic.

The half of this angle, viz., 11° 43′ 43″ has for its tangent ·207 6118, to which we have the successive approximations,—

$$\begin{array}{cccccccc} & 4 & 1 & 4 & 2 & 5 & 18 & \\ \frac{1}{0} & \frac{0}{1} & \frac{1}{4} & \frac{1}{5} & \frac{5}{24} & \frac{11}{53} & \frac{60}{289}, & \text{&c., which again give} \end{array}$$

the trigons 17, 15, 8; 26, 24, 10; or 13, 12, 5; 601, 551, 240; 2930, 2688, 1166; or 1465, 1344, 583; &c.

33. To find a trigon having its sides rational and its area commensurable with the squares of those sides, and which shall have, approximately, the shape of a given trigon.

The general solution of this problem follows at once from the preceding. We must find two muarif angles approximating to two angles of the trigon, and with these construct a rational trigon.

But some of the special cases admit of peculiar solutions.

Example 1.

34. To construct a trigon of which the three sides may be represented by three contiguous numbers while the area is a multiple of the square unit.

Let $a-1$, a, $a+1$ be the three sides, S being the area, then—

$$16S^2=3a(a-2)a(a+2)=3a^2(a^2-4)$$

wherefore $3(a^2-4)$ must be a square number, or a^2-4 must be the triple of a square. Let, then, $a^2-4=3x^2$. This equation cannot be satisfied when x is odd, for then a also would be odd, and the first member of the equation would be of the form $4n+1$, while the second would be of the form $4n+3$; hence both a and x must be even, or we may put—

$$a=2a,\ x=2z, \text{ whence}$$
$$a^2-1=3z^2;\ a^2=3z^2+1.$$

Now, when we develope $\sqrt{3}$ in the form of a chain fraction we obtain the approximations,—

$$\begin{array}{cccccccccc} & 1; & 1, & 2; & 1, & 2; & 1, & 2; & 1, & 2; \\ \frac{0}{1} & \frac{1}{0} & \frac{1}{1} & \frac{2}{1} & \frac{5}{3} & \frac{7}{4} & \frac{19}{11} & \frac{26}{15} & \frac{71}{41} & \frac{97}{56}, \text{ &c.} \end{array}$$

which belong, alternately, to the equations $a^2=3z^2-2$ and $a^2=3z^2+1$; the values of a thus obtained are 1, 2, 7, 26, 97, &c., the next term being four times the last term less the penult. Hence the trigons are—

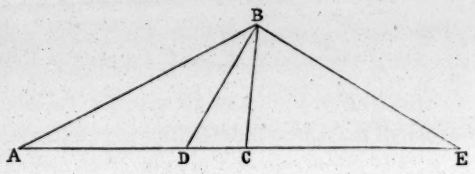
$$\left.\begin{array}{lllllll} 3, & 13, & 51, & 193, & 723, & 2701, & 10083, \\ 4, & 14, & 52, & 194, & 724, & 2702, & 10084, \\ 5, & 15, & 53, & 195, & 725, & 2703, & 10085, \end{array}\right\} \text{&c.}$$

35. In a given circle to inscribe a polygon having all its sides, diagonals, segments, commensurable with the diameter, and their areas with the square of the diameter; and which may approximate to a given inscribed polygon.

The general solution of this problem is another variation of No. 29; we have only to seek the muarif angle approximating to the angle at the circumference subtended by each of the sides of the given polygon, less one.

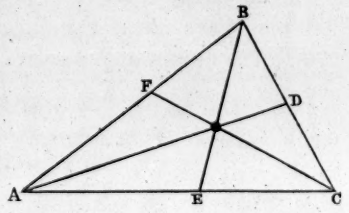
36. If, of a rational trigon, one of the angles be double of a muarif angle, the lines bisecting that angle internally and externally are commensurable with the sides.

If the angle ABC be double of a muarif angle, and if the angle at A also be muarif, the two trigons ABD, DBC, into which ABC is divided by the line BD drawn to bisect the angle ABC, have all their angles muarif, and so has CBE formed by drawing BE perpendicular to BD. Hence the line AE is cut harmonically and rationally.



37. If, of a rational trigon, two of the angles be doubles of two muarif angles, the six lines bisecting internally and externally the three angles intersect each other and the sides of the trigon produced, so that the segments are all rational, and the areas commensurable with their squares.

For, if each of the angles OAC, ACO, be muarif, their sum FOA must also be muarif, wherefore its complement FBO has all its trigonometrical functions rational; so that, all the angles in the figure being muarif, all the segments of the lines must be commensurable, and all the areas commensurable with the squares of the lines.



SECTION 3.—*On Co-ordinates.*

38. If any system of points have their co-ordinates from two rectangular axes, all rational, their co-ordinates referred to another pair of rectangular axes, making a muarif angle with the former, are also all rational.

If x and y be the co-ordinates of a point, when referred to one system of axes, u and v its co-ordinates when referred to another system with the same origin, and if θ be inclination of the one system to the other, we have —

$$u = x \cos\theta - y \sin\theta\,;\; v = x \sin\theta + y \cos\theta\,;$$

wherefore, if x and y be rational, and the angle θ muarif, u and v must also be rational.

39. To construct a tetragon which may have the co-ordinates of its corners and its sides all rational; subject to the condition that its sides have, nearly, given inclinations to the axes.

Since the inclinations of the sides of the actual polygon to the axes must be muarif, we must select such muarif angles as may be suitable; and, moreover, we may always transform the co-ordinates, so that one of the axes may be parallel to one of the proposed sides.

If, then, l_1, l_2, l_3, l_4 be the lengths, and θ_1, θ_2, θ_3, θ_4 the bearings of the four sides taken in order, we must have—

$$l_1 \sin \theta_1 + l_2 \sin \theta_2 + l_3 \sin \theta_3 + l_4 \sin \theta_4 = 0$$
$$l_1 \cos \theta_1 + l_2 \cos \theta_2 + l_3 \cos \theta_3 + l_4 \cos \theta_4 = 0$$

in which, if we put the appropriate rational fractions for the sines and cosines, we shall form two indeterminate equations, involving four unknown quantities.

By placing one of the axes of co-ordinates parallel to (say) the fourth side, we cause the equations to take the form—

$$l_1 \sin \theta_1 + l_2 \sin \theta_2 + l_3 \sin \theta_3 = l_4$$
$$l_1 \cos \theta_1 + l_2 \cos \theta_2 + l_3 \cos \theta_3 = 0$$

in which we have only to consider the latter.

As an example, let it be proposed to construct a tetragon ABCD, such that AB may have about the direction 22°, BC about 74°, CD about 168°, and DA 270°. The muarif angles corresponding to these directions are,—

22° .. 37′ .. 11″	13	12	5
73 .. 44 .. 23	25	7	24
167 .. 19 .. 21	41	−40	9
270 .. 00 .. 00	1	0	−1

and thus if $13a$, $25b$, $41c$, and d be assumed as the four sides, we must have,—

$$12a + 7b - 40c = 0\,;\quad 5a + 24b + 9c = d$$

in which, if the values of a, b, c, be obtained in integers, that of d must also be integer, so that we have only to consider the indeterminate equation $12a + 7b - 40c = 0$; this may be put under the form,—

$$\frac{12a + 7b}{40} = c$$

that is to say, we have to obtain such values of a and b as may make $12a + 7b$ divisible by 40. The lowest solution of this indeterminate problem is $a = 1$, $b = 4$, $c = 1$; whence the sides are 13, 100, 41, 110.

By an extension of the same process we may obtain polygons of any number of sides, having all their sides and ordinates commensurable. The area of such a polygon is also commensurable with the square of the linear unit; but it does not follow that the diagonals drawn from one corner to another are rational.

Section 4.—*On Trigonal Areas.*

40. The only regular figures which can be used to cover surface are the trigon, the tetragon, and the hexagon; and any one of these may be used in the measurement of surface. Thus we may as well say, that the area of a figure is so many trigonal inches, as that it is so many square inches. If we were accustomed to it, the one mode of expression would be as intelligible as the other; and by confining ourselves to the rectangular or tessular method of denoting areas, we may fail to discover all the relations of geometrical magnitudes; or even to apprehend the cause of the superior convenience of the actual system.

Since the regular hexagon contains exactly six regular trigons, there is no need for considering the trigonal and hexagonal systems separately; it will be enough for us to assume the surface of the equilateral trigon, constructed on the linear unit as the unit of surface. For the sake of brevity, we shall use the expression *trigonal area*, as an equivalent for the *area measured in equilateral trigons constructed on the linear unit.*

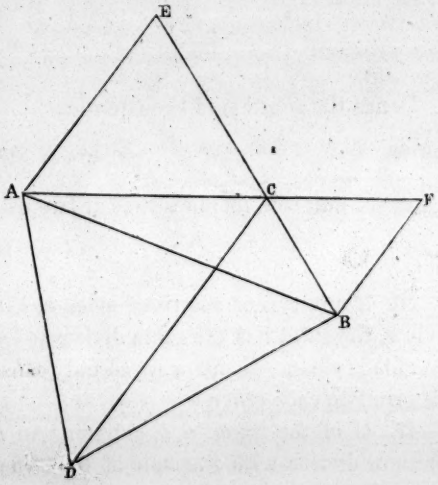
41. If one angle of a trigon be 60° or 120°, that is $\frac{1}{3}\pi$ or $\frac{2}{3}\pi$, its trigonal area is expressed by the product of the numbers representing the containing sides.

42. The trigonal area of any equilateral trigon is represented by the second power of the number of units in its side.

The truth of these two theorems is apparent; they are merely quoted for reference.

43. If a trigon have one angle 120°, the equilateral trigon, constructed on the subtense, is equivalent to the sum of the equilateral trigons, constructed on the two containing sides, together with the original trigon.

If the angle ACB be the third part of a revolution, the equilateral trigon ADB, constructed on the subtense AB, is equivalent to AEC and CFB, constructed on the containing sides AC, CB, together with the trigon ACB.



For ACF and ECB are straight lines; join DC. Then it is easy to show that the trigon DAC is equal to BAE, and DBC to ABF, wherefore the tetragon DACB is equivalent to the pentagon AECFB, together with the trigon ACB. That is, the trigon ABD is equivalent to the pentagon AECFB.

If a, b, c denote the three sides of such a trigon, c being the subtense of 120°, we have,—

$$a^2 + ab + b^2 = c^2.$$

This well-known theorem is easily deduced from the tetragonal system of areas, and is usually given in this form,—"the square of the subtense of 120° is equivalent to the squares of the containing sides, together with their rectangle;" but it is more consistent to put it in the above form; and we must observe, that a^2 (the second power of the number a) does not represent the square of the side BC, but the equilateral trigon on BC.

44. The equilateral trigon on the subtense of 60° is less than the sum of those on the containing sides by the area of the trigon.

The proof of this assertion is as easily obtained as that of the preceding. If a, b, c be the three sides of such a trigon, c being the subtense of 60° we have,—

$$a^2 - ab + b^2 = c^2$$

45. Every number which is of the form a^2+ab+b^2 may, unless a be equal to b, be put in two ways under the form A^2-AB+B^2.

This arithmetical proposition may also be stated thus,—"every number which is the quotient of the difference of two cubes by the difference of their roots, is also in two ways, the quotient of the sum of two cubes by the sum of their roots."

The preceding diagram at once illustrates the truth of this assertion, for AB, which is the subtense of 120°, with the containing sides AE, CB, is the subtense of 60°, with AE, EB, and also with AF, FB, for containing sides. Or, algebraically,

$$\begin{aligned} a^2+ab+b^2 &= (a+b)^2-(a+b)b+b^2 \\ &= (a+b)^2-(a+b)a+a^2 . \end{aligned}$$

otherwise

$$\frac{a^3-b^3}{a-b} = \frac{(a+b)^3+b^3}{(a+b)+b} = \frac{(a+b)^3+a^3}{(a+b)+a} .$$

Hence the solution of the equation

$$a^2+ab+b^2=c^2$$

in integer numbers includes that of two others of the form

$$a^2-ab+b^2=c^2$$

46. If any two of the three sides a, b, c, of a trigon of 120° have a common divisor, the third has the same divisor.

This is clear. Hence we need only consider those cases in which all the three are prime to each other.

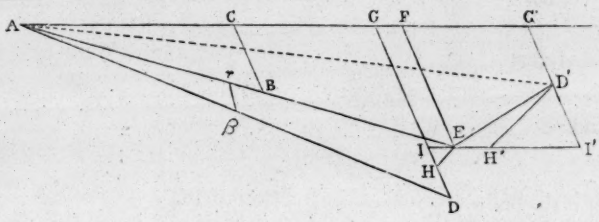
47. If of the trigon a, b, c, having an angle of 120°, the subtense c have a common divisor with the sum of the two sides, the sides themselves have that divisor.

For if AB and AF have a common divisor, FB must have the same divisor.

48. The product of two numbers each of the form a^2+ab+b^2 may be put, in two ways, in the same form.

This theorem (as is also the corresponding one of squares) is only a case of a more general theorem; instead of repeating the arithmetical proof which is to be found in treatises on the Theory of Numbers, I shall give its geometrical illustration.

Let there be two numbers P and Q, such that $P=a^2+ab+b^2$, while $Q=a^2+a\beta+\beta^2$, in which we may suppose $b>a$, $\beta>a$, and also the quotient $\frac{\beta}{a}$ to be greater than $\frac{b}{a}$.



From the point A in any indefinite straight line, measure off AC equal to b, make the angle ACB one-third of a revolution, and CB equal to a linear units. Join AB, then the second power of the number of units in AB is equal to a^2+ab+b^2, that is $AB^2=P$. Also, since AC is greater than CB, the angle CAB is less than 30°. In AB make Aγ equal to β, the angle A$\gamma\beta$, 120°, and cut off $\gamma\beta$ equal to a, join Aβ. Then $A\beta^2=a^2+a\beta+\beta^2=Q$. Also since the ratio A$\gamma$: $\gamma\beta$ is more unequal than that of AC : CB, the angle γAβ is less than CAB. And this disposition of the parts can always be made, unless the two ratios be identic. The angle CAβ is thus less than 60°.

From the line Aβ continued, cut off a distance AD to represent the product of the roots $\sqrt{P}$ and $\sqrt{Q}$, so that AD^2 may represent the product P . Q : draw DG parallel to BC, and we shall have $P.Q=AD^2=DG^2+DG\cdot GA+GA^2$; therefore the product P . Q will be decomposed as asserted, provided we can show that DG and GA are expressed by integer numbers.

Draw DE parallel to $\beta\gamma$, EF parallel to BC, EI parallel to FG, and having cut off IH equal to IE, join EH. EIH is evidently an equilateral trigon, while HDE is similar to CAB.

From the similarity of the trigons, we obtain the six proportions following, with their results:—

Aβ : $\beta\gamma$: : AD : DE	or $\sqrt{Q} : a :: \sqrt{P.Q} : DE=a\sqrt{P}$
Aβ : Aγ : : AD : AE	$\sqrt{Q} : \beta :: \sqrt{P.Q} : AE=\beta\sqrt{P}$
AB : BC : : AE : EF	$\sqrt{P} : a :: \beta\sqrt{P} : EF=a\beta$
AB : AC : : AE : AF	$\sqrt{P} : b :: \beta\sqrt{P} : AF=b\beta$
AB : BC : : DE : EH	$\sqrt{P} : a :: a\sqrt{P} : EH=aa$
AB : AC : : DE : DH	$\sqrt{P} : b :: a\sqrt{P} : DH=ba$.

But

$$DG=DH+HE+EF \; ; \; AG=AF-EH,$$

wherefore

$$DG=b\alpha+a\alpha+a\beta \quad ; \; AG=b\beta-a\alpha,$$

and

$$P.Q=(b\alpha+a\alpha+a\beta)^2+(b\alpha+a\alpha+a\beta)(b\beta-a\alpha)+(b\beta-a\alpha)^2.$$

Again; make the angle EAD′ equal to EAD, AD′ equal to AD; join ED′, draw G′D′I′ parallel to GD, EI′ parallel to FG′ and D′H′ to EH; then it is easy to show that D′I′H′ is equilateral, and that ED′H′ is equal to EDH.

Now

$$D'G'=EF-D'H', \; AG'=AF+EH+HD,$$

wherefore

$$D'G'=a\beta-b\alpha \; ; \quad AG'=b\beta+a\alpha+b\alpha,$$

and

$$P.Q=(a\beta-b\alpha)^2+(a\beta-b\alpha)(b\beta+a\alpha+b\alpha)+(b\beta+a\alpha+b\alpha)^2,$$

thus giving a second decomposition of PQ.

By interchanging a for b and α for β, in the above expressions, we obtain other two, viz.,—

$$P.Q=(b\alpha-a\beta)^2+(b\alpha-a\beta)(b\beta+a\beta+a\alpha)+(b\beta+a\beta+a\alpha)^2$$
$$P.Q=(a\alpha-b\beta)^2+(a\alpha-b\beta)(b\alpha+a\beta+b\beta)+(b\alpha+a\beta+b\beta)^2 \; .$$

and at first we might suppose that there are four decompositions: but if we take notice that if $b\alpha-a\beta$ be positive in the one it is negative in the other, so that two of the four must belong to the form A^2-AB+B^2.

If we regard the trigon ABC as analogous to the right-angled trigon of the previous part, we may, taking AB as the unit or radius, consider BC as coming in place of the sine, AC in place of the cosine; and, for the moment, we may call the former the *opposite*, the latter the *adjacent* of the angle: that is we may define

$$\frac{BC}{AB} \text{ as the } \textit{opposite} \text{ of CAB}$$
$$\frac{AC}{AB} \text{ as the } \textit{adjacent} \text{ of CAB};$$

and then we form four theorems for the functions of the sum and difference of two angles, analogous to the fundamental theorems of trigonometry. These are, putting ϕ and θ for the angles BAC, $\beta A\gamma$,

$$\text{opp. } (\phi+\theta)=\text{opp. } \phi.\text{ adj. } \theta+\text{adj. } \phi.\text{ opp. } \theta+\text{opp. } \phi.\text{ opp. } \theta$$
$$\text{adj. } (\phi+\theta)=\text{adj. } \phi.\text{ adj. } \theta-\text{opp. } \phi.\text{ opp. } \theta$$
$$\text{opp. } (\phi-\theta)=\text{opp. } \phi.\text{ adj. } \theta-\text{adj. } \phi.\text{ opp. } \theta$$
$$\text{adj. } (\phi-\theta)=\text{adj. } \phi.\text{ adj. } \theta+\text{opp. } \phi.\text{ opp. } \theta+\text{adj. } \phi.\text{ opp. } \theta.$$

in which it is to be remarked that we cannot pass from the function of $\phi+\theta$, to

the corresponding function of $\phi-\theta$ by a change of sign, as we can in orthogonal trigonometry.

49. If a number be of the form a^2+ab+b^2, its square is of the same form.

In order to decompose P^2 into A^2+AB+B^2, we have only to suppose that α, β of the preceding article are equal to a and b; whence,

$$\begin{aligned} P^2 &= (a^2-b^2)^2+(a^2-b^2)\,(2ab+a^2)+(2ab+a^2)^2 \\ &= (b^2-a^2)\ +(b^2-a^2)\,(2ab+b^2)+(2ab+b^2)^2 \end{aligned}$$

only one of which belongs to the prescribed form, the other belonging to A^2-AB+B^2.

50. The number 3 is of the form $1^2+1.1+{}^21$, it is the only number of that form whose square cannot be put in the same form.

If we make, in the preceding formulæ, $a=1$, $b=1$, we obtain

$$9=0^2+0\cdot3+3^2$$
$$9=0^2-0\cdot3+3^2$$

Our previous demonstration proceeded on the assumption that a and b are prime to and different from each other.

In the present case the trigon is collapsed into a line, while the conjugate trigons take the forms 0, 3, 3, and 3, 3, 3, the one being a line and the other an equilateral trigon.

51. When the number $P=a^2+ab+b^2$ is multiplied by 3, we have, on substituting 1 and 1 for α, β of No. 48,

$$3\,P=(2a+b)^2+(2a+b)\,(b-a)+(b-a)^2$$

of which the two conjugate forms are

$$3\,P=(a+2b)^2-(a+2b)\,(b-a)+(b-a)^2$$
$$3\,P=(a+2b)^2-(a+2b)\,(2a+b)+(2a+b)^2$$

52. When the sides of a trigon having one angle 120° are in their lowest terms, the subtense cannot be a multiple of 3.

If the subtense were divisible by 3, the sides must necessarily be of the forms $3n\pm1$; these admit of three distinct classes of combinations. First, a and b may be both of the form $3n+1$; secondly, they may be one of the form $3n+1$, the other of the form $3n-1$; and thirdly, they may be both of the form $3n-1$.

In the first case, if we put $a=3\alpha+1$, $b=3\beta+1$, we have,

$$9(\alpha^2+\alpha\beta+\beta^2)+9(\alpha+\beta)+3=c^2$$

now, no square can be divisible by 3 without being also divisible by 9, therefore this combination is impossible.

The same argument applies to the third case, for if $a=3\alpha-1$, $b=3\beta-1$, we have,

$$9(\alpha^2+\alpha\beta+\beta^2)-9(\alpha+\beta)+3=c^2.$$

In the second case, if we put $a=3\alpha+1$, $b=3\beta-1$, the sum of a and b is

divisible by 3, and therefore the subtense AB and the side AF being both divisible by 3, the other side BF must also be so, and consequently the trigon could not have been in its lowest terms.

53. The subtense of 120°, in its lowest terms, cannot be even.

For if both a and b were odd, we might put $a=2\alpha+1$, $b=2\beta+1$ which give

$$4(\alpha^2+\alpha\beta+\beta^2)+6(\alpha+\beta)+3=c^2,$$

which is inconsistent with c being even.

54. If one of the sides of 120° be even, it must be divisible by 4.

For if we put $a=2\alpha$, $b=2\beta+1$, we have $4\ (\alpha^2+\alpha\beta+\beta^2)+2\alpha+4\beta+1=c^2$; now, in all cases of c being odd, c^2-1 is divisible by 4, wherefore 2α, that is a, must be divisible by 4.

55. The subtense of 120°, in its lowest terms, cannot be a multiple of *five*.

For if c were divisible by 5, the side a must belong to some of the forms $5\alpha\pm1$, $5\alpha\pm2$, while b must also be of some one of these forms. Now if we conjoin the supposition that $a=5\alpha+1$, with each of the four $b=5\beta+1$, $b=5\beta+2$, $b=5\beta+3$, $b=5\beta+4$, we find that the sum a^2+ab+b^2 is of the form $5n+3$ in the first and third case, and $5n+2$ in the second case, neither of which is a possible form for a square number; but in the fourth case we find $c^2 \leftrightarrows 5n+1$, which is possible; but then a being $5\alpha+1$, and b, $5\beta+4$, their sum is divisible by 5, so that (No. 49) the trigon cannot have been in its lowest terms. Next, combining the form $a=5\alpha+2$, with $b=5\beta+2$, $5\beta+3$, $5\beta+4$, we find that the first and last give the forms $5n+2$ and $5n+3$, which are impossible, while the intermediate case gives the possible form $5n-1$, but then as $a=5\alpha+2$, $b=5\beta+3$, $a+b$ would be a multiple of 5. Combining now the form $a=5\alpha+3$, with $b=5\beta+3$, and $b=5\beta+4$, we find the forms $5n+2$, while lastly, combining $5\alpha+4$, with $5\beta+4$, we obtain the form $5n+3$, which is impossible for a square number.

Hence in no combination of numbers for a and b not divisible by 5, can we obtain a form possibly a square, for a^2+ab+b^2.

56. If a and b be two numbers prime to each other, a^2+ab+b^2 is either divisible by 3 or of the form $6n+1$.

In reference to the divisor 6, all numbers leave the remainders 0, 1, 2, 3, 4, 5, and the only possible combinations of two of these, not giving numbers having 2 or 3 for a common division are,—

$a=6\alpha+0$, $b=6\beta+1$	P $\leftrightarrows 6n+1$
$b=6\beta+5$	P $\leftrightarrows 6n+1$
$a=6\alpha+2$, $b=6\beta+1$	P $\leftrightarrows 6n+1$
$b=6\beta+3$	P $\leftrightarrows 6n+1$
$b=6\beta+5$	P $\leftrightarrows 6n+3$
$a=6\alpha+4$, $b=6\beta+1$	P $\leftrightarrows 6n+3$
$b=6\beta+3$	P $\leftrightarrows 6n+1$
$b=6\beta+5$	P $\leftrightarrows 6n+1$

$a=6\alpha+1, b=6\beta+1$	$P \backsim 6n+3$
$b=6\beta+3$	$P \backsim 6n+1$
$b=6\beta+5$	$P \backsim 6n+1$
$a=6\alpha+3, b=6\beta+5$	$P \backsim 6n+1$

57. The subtense of 120°, when the three sides are in the lowest terms, is of the form $6n+1$.

For, according to the preceding theorem, every number P is of one of the two forms $6n+1$, $6n+3$; now it cannot be of the form $6n+3$ when the three sides are prime to each other; wherefore, the subtense must always be of the form $6n+1$.

58. If a prime number a divide any number P of the form a^2+ab+b^2, a and b, being prime to each other, another number of the same form, but less than a^2, may be found also divisible by a.

If a and b be greater than a, we may put them under the forms $a=na\pm c$, $b=ta\pm d$, in which c and d are each less than the half of a. Inserting these values in the equation,

$$P=a^2+ab+b^2$$

we find,—

$$P=n^2a^2\pm 2nca+c^2+nta^2\pm tca\pm nda\pm cd+t^2a^2\pm 2tda+d^2$$

wherefore, if P be divisible by a, $c^2\pm cd+d^2$ must also be divisible by it; now $c^2\pm cd+d^2 \prec \frac{3}{4}a^2$.

59. If a and b be prime to each other, no prime number of the form $6n-1$ can be a divisor of $a^2+ab+b=P$.

For if some prime number a divide P, we may find some other c^2+cd+d^2, less than $\frac{3}{4}a^2$, which is divisible by a. Now, we have shown that c^2+cd+d^2 is either divisible by 3 or of the form $6n+1$; if it be divisible by 3, its third part is also of the same form, so that eventually we arrive at a number of the form $6n+1$. But no number of this form can be divided by $a \backsim 6n-1$ unless the quotient be of the form $6n-1$; hence if $c^2+cd+d^2 \backsim a$, the quotient β less than $\frac{3}{4}a$ must be of the same form. By proceeding in the same way, we can show that some other number γ, less than $\frac{3}{4}\beta$, must be a divisor of some number of the form e^2+ef+f^2; and so we can continue. By this process we must at last arrive at the number 5, the least of this class: now, we have shown that 5 cannot be a divisor; wherefore no number of the form $6n-1$ can divide the subtense of 120°.

60. The subtense of 120°, when the numbers are in their lowest terms, can only be a prime number of the form $6n+1$, or the product of two or more of such primes.

This follows immediately from the preceding theorem. Hence, in order to tabulate trigons of this kind, we must decompose the primes 7, 13, 19, &c., into three parts, a^2, ab, b^2. Having found, for example, that $7=1+2+4$, we obtain $7^2=3^2+3.5+5^2$ from the formula given in article 49.

In the appendix there is given a list of all primes of the form $6n+1$, under 1000, with their decompositions, and the resulting trigons, as also the values of the lesser angles.

Thus there are muarif angles belonging to the trigonal system of measurement, analogous to those which we have already seen to belong to the tetragonal system: to distinguish them from those formerly treated of we shall call them *trigonal muarif angles*, while the others might have been styled *tetragonal muarif angles*.

61. If two angles be both muarif of the same system, their sum and their difference are so; a right angle being regarded as muarif in the tetragonal and an angle of 60° as muarif in the trigonal system.

This has already been shown to be true for the tetragonal system; the proof for angles of the trigonal system is of the same nature.

62. If a trigon be constructed with two of its angles muarif of the trigonal system, its sides are commensurable with each other, and its area with the equilateral trigons constructed on the sides.

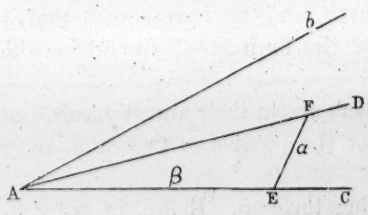
The proof of this is analogous to that already given for the other system.

63. If at the two extremities of a line taken as a base, any number of trigonal muarif angles be made, their sides, continued indefinitely, cut each other into segments commensurable with the base, and intercept areas which are commensurable with the equilateral trigon described on the base.

This is merely an extension of the preceding proposition.

64 If at any of the points of intersection of the preceding figure, other trigonal muarif angles be made; the distances intercepted by these new lines are commensurable with the base and the intercepted surfaces with the equilateral trigon on the base.

65. If at the point of contact of a straight line with a circle, trigonal muarif angles be made; if the extremities of the chords so formed be joined, and if tangents be applied at the extremities of those chords, all the intercepted distances are commensurable with the side of the inscribed equilateral trigon, and all the areas with the area of that trigon.



66. To construct a trigonal muarif angle which may approximate to any given angle.

Let it be proposed to compute, in integers, the sides of a trigon which shall have one of its angles equal to a given angle BAC, and another either 120° or 60°.

If the given angle BAC exceed 60°, it is impossible to have either of the remaining angles 120°; we shall therefore deduct 60°, or if need be, 120° from BAC, thus leaving bAC less than 60°. Having bisected bAC by the line AD, draw EF, making AEF = 120°; and deter-

mine, by the method of continued fractions, or by any other process, two numbers, α and β, which may represent with sufficient nearness the ratio FE : EA; and then compute a, b, c, the sides of a rational trigon having one angle 120° by the formulæ,—

$$c=(\alpha+\beta)^2-\alpha\beta\,;\ b=(\alpha+\beta)^2-\beta^2\,;\ a=\beta^2-\alpha^2,$$

then the angle of this trigon opposite to a is double of not the angle DAC, but an angle sufficiently near to DAC, wherefore that angle is nearly equal to bAC.

This is the general process, analogous to that given for the tetragonal system: but in special cases we may use peculiar methods.

Example 1.

67. To construct a rational trigon having one angle 120°, and of which the containing sides may differ by unit. That is, in other words, to construct a trigonal muarif angle approximating to 30°.

If c, b, a, be the sides of such a trigon, and if $b=a+1$, we have $c^2=3a^2+3a+1$, that is to say, $c^2=3\ (a+\frac{1}{2})^2+\frac{1}{4}$; or multiplying by 4.

$$(2c)^2=3\,(2a+1)^2+1,$$

and therefore $2c$ and $2a+1$ must be the numerator and denominator of a fraction converging to $\sqrt{3}$. Now, we have for $\sqrt{3}$,—

$$\begin{array}{cccccccccccc} & 1\,; & 1, & 2\,; & 1, & 2\,; & 1, & 2\,; & 1, & 2\,; & \\ 0 & 1 & 1 & 2 & 5 & 7 & 19 & 26 & 71 & 97 & \\ \overline{1} & \overline{0} & \overline{1} & \overline{1} & \overline{3} & \overline{4} & \overline{15} & \overline{15} & \overline{41} & \overline{56}' & \&c. \end{array}$$

of which the alternate terms,—

$$\begin{array}{cccccccccc} & -4 & -4 & -4 & -4 & -4 & -4 & -4 & & \\ 1 & 2 & 7 & 26 & 97 & 362 & 1351 & 5042 & & \\ \overline{0}' & \overline{1}' & \overline{4}' & \overline{15}' & \overline{56}' & \overline{209}' & \overline{780}' & \overline{2911}' & \&c. \end{array}$$

satisfy the equation $p^2=3q^2+1$. These may be formed by means of the multiplier -4. Of these again, the alternate terms have their numerators even, viz.:—

$$\begin{array}{ccccccc} & -14 & -14 & -14 & -14 & & \\ 2 & 26 & 362 & 5042 & 70226 & & \\ \overline{-1}' & \overline{15}' & \overline{209}' & \overline{2911}' & \overline{20545}' & \&c. \end{array}$$

and of these, if we put the numerator equal to $2c$, and the denominator to $2b-1$, or to $2a+1$, we find the cases,—

$$\begin{array}{lllll} c=1, & 13, & 181, & 2521, & \&c. \\ b=1, & 8, & 105, & 1456, & \&c. \\ a=0, & 7, & 104, & 1455, & \&c. \end{array}$$

and it may be remarked, that this progression of fractions may be continued by help of the multiplier -14.

The intermediate fractions which have their numerators odd, give trigons of which the sides differ by 2, thus,—

$c=7,\ 97,\ 1351,\ 18817$, &c.
$b=5,\ 57,\ 781,\ 10865$, &c.
$a=3,\ 55,\ 779,\ 10864$, &c.

Example 2.

68. To construct in integers a trigon having one angle 120° and another nearly 45°.

In this case the angle FAE is $21\frac{1}{2}°$, while AFE is $37\frac{1}{2}°$, wherefore the ratio of β to a is identic with that of sin 37° 30′ to sin 22° 30′, that is, of ·6087614 to ·3826834. On approximating to this ratio by the method of continued fractions, we obtain the quotients 1, 1, 1, 2, 3, 1, 14, &c., whence the values,—

$$\begin{array}{cccccccc} & 1 & 1 & 1 & 2 & 3 & 1 & \\ 0 & 1 & 1 & 2 & 3 & 8 & 27 & 35 \\ 1 & 0 & 1 & 1 & 2 & 5 & 17 & 22 \end{array}, \text{ &c.}$$

and thence the trigons,—

$c=7,\ 19,\ 43,\ 1477,\ 2479$, &c.
$b=5,\ 16,\ 35,\ 1207,\ 2024$, &c.
$a=3,\ 5,\ 13,\ 440,\ 741$, &c.

69. In a given circle to inscribe a polygon of which all the sides and diagonals may be commensurable with the side of the inscribed equilateral trigon, while the polygon may approximate to a given inscribed polygon.

The solution of this problem is analogous to that of the corresponding problem of the tetragonal system.

SECTION 5.—*On Muarif Angles in General.*

70. In the first branch of our inquiry we treated of trigons having one angle right, in the second branch we extended our researches to trigons having angles of 60° or 120°, and in either case we arrived at general theorems analogous to each other. I proceed now to consider whether there may be other classes of muarif angles possessing the same generic properties.

If in the figure of article 48 we suppose that the exterior angle EFG′ is described by θ, we have $AB^2=a^2+2ab\cos\theta+b^2=P$, and similarly, $A\beta^2=a^2+2a\beta\cos\theta+\beta^2=Q$, and all the equalities given on page 739 hold good, with the exception of HI, which, instead of being represented by aa, is now represented by $2aa\cos\theta$, and H′I′, which becomes $2ba\cos\theta$, and we obtain

$$\begin{aligned} AD^2=P.Q&=(b^2+2ab\cos\theta+a^2)\,(\beta^2+2a\beta\cos\theta+a^2) \\ DG&=a\beta+ba+2aa\cos\theta \\ AG&=b\beta-aa. \end{aligned}$$

If in this we suppose $a=a$, $b=\beta$, we obtain

$$\begin{aligned} \mathrm{AD=C}&=\beta^2+2a\beta\cos\theta+a^2 \\ \mathrm{AG=B}&=\beta^2\qquad\qquad -a^2 \\ \mathrm{GD=A}&=2a\,(\beta+a\cos\theta), \end{aligned}$$

Wherefore, if in a trigon $A\gamma\beta$, the two sides β, a, be denoted by integers, while the cosine of the angle $A\gamma\beta$ is rational, the three sides of the trigon AGD having an angle GAD double of $\gamma A\beta$, will be commensurable.

The subject may be presented in another light, thus—

Let the cosine of the angle θ be denoted by the rational fraction $\frac{s}{t}$, then c, b, a, being the sides of any trigon having $180°-\theta$ for the angle opposite c, we have

$$\begin{aligned} c^2&=b^2+\frac{2s}{t}ab+a^2 \\ &=b^2+\frac{2s}{t}ab+\frac{s^2}{t^2}a^2+\frac{t^2-s^2}{t^2}a^2 \\ &=(b+\frac{s}{t}a)^2+\frac{t^2-s^2}{t}a^2 \end{aligned}$$

or,

$$(tc)^2=(tb+sa)^2+(t^2-s^2)a^2,$$

whence

$$(tc+tb+sa)\,(tc-tb-sa)=(t+s)\,(t-s)\,x^2y^2,$$

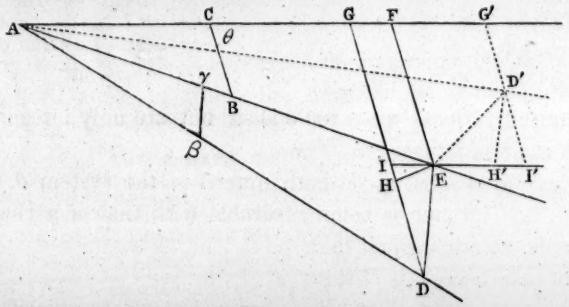
if we put xy (either of which may be unit) for a. Decomposing the latter member into factors prime to each other, we may put

$$\begin{aligned} tc+tb+sa&=(t+s)x^2 \\ tc-tb-sa&=(t-s)y^2, \end{aligned}$$

which give

$$\begin{aligned} \mathrm{C}=2tc&=(t+s)x^2 & &+(t-s)y^2\,; \\ \mathrm{B}=2tb&=(t+s)x^2 & -2sxy\quad &-(t-s)y^2\,; \\ \mathrm{A}=2ta&= & 2txy\,; & \end{aligned}$$

in which x and y may be any two numbers whatever; it is needless, however, to assume them with a common divisor.

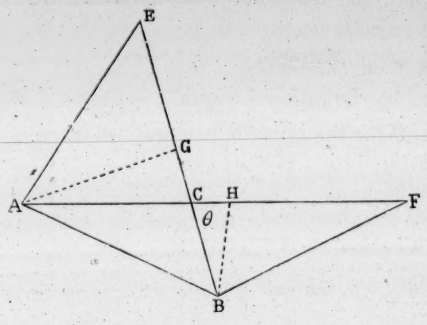


When the three sides BC, CA, AB, of a trigon, having the angle ACB equal

to the supplement of θ, have been determined in integers, the angles BAC, CBA may be said to be *muarif of the system* θ, and for the present we may call θ the determining angle of the system.

71. Every rational sided obtuse-angled trigon is accompanied by two acute-angled trigons having their sides also rational and their areas commensurable.

For let ABC be an obtuse angle, and let the three lines BC, CA, AB, be expressed by three integer numbers, a, b, c. From A to the extension of BC, and from B to the extension of AC, inflect AE, BF, each equal to AB: then since $BA^2 - AC^2 = BC \cdot CE$, CE is represented by the quotient $\frac{c^2-b^2}{a}$, while CF is represented by $\frac{c^2-a^2}{b}$, both of which are rational. If EG be cut off equal to BC, and FE equal to AC, it is evident that the angles CAG, HBC, are each double of the complement of FCB or θ; wherefore the sides of ACE are $a+2b\cos\theta$, b, c, while those of BCF are a, $b+2a\cos\theta$, C.



The areas ACB, ACE, BCF, are proportional to the rectangles AC, CB, AC, CE, BC, CF, and must be commensurable since the containing sides are so.

72. If each of two angles be muarif of the same system θ, their sum and difference are also muarif of that system.

Let (figure, page 747) CAB, γAβ, be two angles muarif of the system $\theta = 180 - ACB = 180° - A\gamma\beta$, and let a, b, c; α, β, γ, be the integer numbers which represent the sides of the two trigons; make AD equal to Cγ units, and complete the construction as in article 48, only observing to draw EH making EHG equal to θ, and D′H′ making D′H′I′ equal to θ, then we easily obtain the values DE = D′E′ = Ca, AE = Cβ, EF = $a\beta$, AF = $b\beta$, EH = $a\alpha$, DH = $b\alpha$, D′H′ = $b\alpha$ as before, while HI $= 2a\alpha\cos\theta$, H′I′ $= 2b\alpha\cos\theta$, whence

AD $= c\gamma$	AD′ $= c\gamma$
AG $= b\beta - a\alpha$	AG′ $= b\beta + a\alpha + 2b\alpha\cos\theta$
DG $= a\beta + b\alpha + 2a\alpha\cos\theta$	D′G′ $= a\beta - b\alpha$,

which values are all rational when $\cos\theta$ is so, but are only integers when $\cos\theta = 0$, or when $2\cos\theta$ is integer.

73. If two angles of a trigon be both muarif of the system θ, its sides are commensurable, and its area is commensurable with that of a rhombus on the linear unit, having an angle equal to θ.

The truth of this theorem is obvious.

74. If at the extremities of any line assumed as a base muarif angles of the system θ be made, and the sides of these be indefinitely produced, all the inter-

cepted distances are commensurable with the base, and all the areas with the rhombus constructed on the base with the angle θ.

75. If at any of the intersections of the preceding article muarif angles of the same system be made, the segment and areas so intercepted are all commensurable.

76. If at the point of contact of a straight line with a circle any number of muarif angles of the system θ be made, all the lines joining the extremities of the chords so formed, and all the tangents there applied continued indefinitely, intercept segments and areas which are commensurable with each other.

77. If one muarif angle of the system θ be equal to an angle of the system ϕ, the two systems are identic.

Hence it follows that the determination of a system by the angle θ is not quite appropriate; that is to say, the angle θ, no more than the angle ϕ, can be regarded as the modulus or *mastar* of the system.

78. The numerical expressions for the sines of all muarif angles of the same system involve the same irreducible surd. Or,

The numerical expressions for the areas of all muarif figures of the same system, when expressed in squares of the linear unit, involve the same irreducible surd. Let $\frac{s}{t}$ and $\frac{S}{T}$ be the expressions for the cosines of two angles θ and Θ, then have we

$$\sin\theta = \frac{\sqrt{(t^2-s^2)}}{t}, \quad \sin\Theta = \frac{\sqrt{(T^2-S^2)}}{t},$$

and

$$\cos(\theta+\Theta) = \frac{sS}{tT} - \frac{\sqrt{(t^2-s^2)}\,.\,\sqrt{(T^2-S^2)}}{tT};$$

now if the two angles θ and Θ be both muarif of one system, their sum $\theta+\Theta$ must also be so; that is to say, the cosine of $\theta+\Theta$ must be rational, and for this it is necessary that the product of

$$\sqrt{(t^2-s^2)} \quad \text{by} \quad \sqrt{(T^2-S^2)}$$

be also rational, and this can only be when t^2-s^2 and T^2-S^2 involve the same unsquare factors.

Since the area of any trigon expressed in squares of the linear unit is half the product of the two containing sides multiplied by the sine of the included angle, the same irreducible surd must enter into the expression for the area.

79. All muarif angles of which the sines involve the same irreducible surd, belong to the same system.

Let θ and ϕ be two muarif angles, of which the sines are respectively $\sin\theta = \frac{e}{f}\sqrt{m}$ and $\sin\phi = \frac{g}{h}\sqrt{m}$, m being a number which has no square divisor; then, if the angles be muarif of any systems, their cosines must be rational. Now

$$\begin{aligned}\cos(\theta+\phi) &= \cos\theta\,.\,\cos\phi - \sin\theta\,.\,\sin\phi \\ &= \cos\theta\,.\,\cos\phi - \frac{eg}{fh}m\end{aligned}$$

which is rational, wherefore the sum of the two angles is muarif, and thus they must belong to the same system.

80. The irreducible surd which enters into the expression for the sines of the angles of one system, may be called the *modulus* or *mastar* of that system; we shall generally denote it by the symbol $\sqrt{m}$.

81. Having given the modulus of a system, to find general expressions for the sines and cosines of all the angles belonging to it.

Let $\frac{e}{f}\sqrt{m}$ be the sine of any angle ϕ, then the expression for the cosine of that angle is,—

$$\cos\theta=\frac{\sqrt{(f^2-me^2)}}{f},$$

and in order that θ may be a muarif angle, it is necessary that its cosine be rational, or that f^2-me^2 be a square number. We may then put $f^2-me^2=p^2$, or $f^2-p^2=me^2$. Now, whether e be prime or composite, we may always put $e=xy$, when we do not exclude unit from the values of x and y; and similarly we may put $m=\mu\nu$ with the same generalisation; our equation of condition then becomes,

$$(f+p)(f-p)=\mu\nu x^2y^2,$$

wherefore in general,

$$f+p=\mu x^2,\ f-p=\nu y^2,$$

and

$$f=\tfrac{1}{2}(\mu x^2+\nu y^2);\quad p=\tfrac{1}{2}(\mu x^2-\nu y^2)$$

so that

$$\frac{2xy\sqrt{\mu\nu}}{\mu x^2+\nu y^2}=\sin\theta;\quad \frac{\mu x^2-\nu y^2}{\mu x^2+\nu y^2}=\cos\theta,$$

and also,

$$\frac{2xy\sqrt{\mu\nu}}{\mu\nu x^2+y^2}=\sin\theta;\quad \frac{\mu\nu x^2-y^2}{\mu\nu x^2+y^2}=\cos\theta,$$

are the expressions for the sines and cosines of all muarif angles of the system $\sqrt{\mu\nu}$, x and y being susceptible of all integer values including unit.

It is obvious, that in assuming values for x and y, we may reject those couples which have a common divisor.

82. The sines and cosines of the sum and difference of two muarif angles, determined by the above formulæ, belong to the same forms.

Let

$$\cos\theta=\frac{\mu x^2-\nu y^2}{\mu x^2+\nu y^2},\quad \cos\Theta=\frac{\mu x'^2-\nu\gamma^2}{\mu x'^2+\nu\gamma^2},$$

$$\sin\theta=\frac{2xy\sqrt{\mu\nu}}{\mu x^2+\nu y^2},\quad \sin\Theta=\frac{2x'\gamma\sqrt{\mu\nu}}{\mu x'^2+\nu\gamma^2},$$

then

$$\cos(\theta+\Theta)=\frac{(\mu xx'-\nu y\gamma)^2-\mu\nu(x\gamma+yx')^2}{(\mu xx'-\nu y\gamma)^2+\mu\nu(x\gamma+yx')}$$

$$\sin(\theta+\Theta)=\frac{2(\mu xx'-\nu y\gamma)(x\gamma+yx')}{(\mu xx'-\nu y\gamma)^2+\mu\nu(x\gamma+yx')^2}\sqrt{\mu\nu}$$

which can be brought into the preceding forms by multiplying each member of the fractions by μ or by ν.

83. The tetragonal system has for its modulus the symbol $\sqrt{1}$.

84. The modulus of the trigonal system is $\sqrt{3}$.

85. If a trigon be constructed, having one angle muarif of one system, and another muarif of another system, its sides are incommensurable.

For if ϕ and θ be two muarif angles, the one involving the surd $\sqrt{\mu}$ and the other the surd $\sqrt{\nu}$, the sine of $\phi+\theta$ would involve the product $\sqrt{\mu\nu}$, but then the cosine would involve the two surds $\sqrt{\mu}$, $\sqrt{\nu}$ separately, wherefore the side intermediate between ϕ and θ would be incommensurable with either of the other two.

86. Having given the three integers which represent the sides of a rational sided trigon, to find the modulus of the muarif system to which its angles belong.

Let a, b, c, be the three sides, then, the area of the trigon being s, we have,

$$4\,S=\sqrt{\{(a+b+c)\,(-a+b+c)\,(a-b+c)\,(a+b-c)\}}$$

wherefore, if we decompose each of the four numbers $a+b+c$, $-a+b+c$, $a-b+c$, $a+b-c$, into its prime factors, and reject all those factors which occur twice, the square root of the product of the remaining factors is the required modulus.

Thus we have the following cases:—

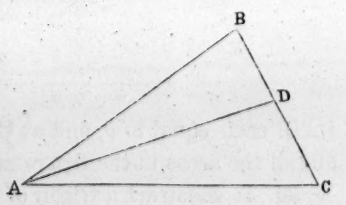
a	b	c	Modulus.	a	b	c	Modulus.
2	3	4	$\sqrt{15}$	9	10	11	$\sqrt{2}$
3	4	5	$\sqrt{1}$				
4	5	6	$\sqrt{7}$	3	5	7	$\sqrt{3}$
5	6	7	$\sqrt{6}$	5	7	9	$\sqrt{11}$
6	7	8	$\sqrt{15}$	7	9	11	$\sqrt{195}$
7	8	9	$\sqrt{5}$	9	11	13	$\sqrt{35}$, &c.

87. If m be prime, for every pair of values of x and y, we have two angles, according as we combine m with x^2 or with y^2; but if m be composite, then for every way in which m can be represented as a product $\mu\nu$, we have two angles.

Section 6.—*Miscellaneous Propositions.*

88. To construct a trigon, such that the three sides and the line bisecting one of the angles may be all commensurable.

Let the angle BAC of the trigon BAC be bisected by the line AD; it is required to determine the dimensions, so that all the lines be represented by integer numbers.



From the principles of geometry we know that BA : AC : : BD : DC, while the square of AD is the difference between the two rectangles BA.AC and BD.DC. If we denote the ratio of BA to AC by $p:q$, p and q being prime to each other, we may put $BA=xp$, $AC=xq$; $BD=yp$, and $DC=yq$, whence BA.AC

$=x^2pq$ and $BD.DC=y^2pq$, so that $AD^2=(x^2-y^2)\ pq$; thus it seems that the product $(x+y)\ (x-y)\ pq$ must be a square; now p and q are prime to each other, wherefore they must be found as factors in the product $(x+y)\ (x-y)$, the remaining factors being in couples. Hence the following cases are possible,—

1*st*.	$x+y=a^2cpq$;	$x-y=b^2c$
2*d*.	$x+y=a^2c$;	$x-y=b^2qc$
3*d*.	$x+y=a^2pc$;	$x-y=b^2qc$
4*th*.	$x+y=a^2qc$;	$x-y=b^2pc$

in all of which the effect of the c is only to augment the numbers.

Hence the solutions,—

1.	$2x=a^2pq+b^2$;	$2y=a^2pq-b^2$;
2.	$2x=a^2+b^2pq$;	$2y=a^2-b^2pq$;
3.	$2x=a^2p+b^2q$;	$2y=a^2p-b^2q$;
4.	$2x=a^2q+b^2p$;	$2y=a^2q-b^2p$:

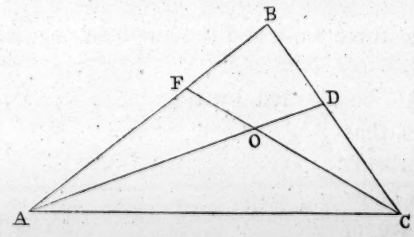
which give

$BC=(a^2pq-b^2)\ (p+q)$;	$CA=(a^2pq+b^2)q$;	$AB=(a^2pq+b^2)p$;
$BC=(a^2-b^2pq)\ (p+q)$;	$CA=(a^2+b^2pq)q$;	$AB=(a^2+b^2pq)p$;
$BC=(a^2p-b^2q)\ (p+q)$;	$CA=(a^2p+q^2q)q$;	$AB=(a^2p+b^2q)p$;
$BC=(a^2q-b^2p)\ (p+q)$;	$CA=(a^2q+b^2p)q$;	$AB=(a^2q+b^2p)q$:

in which p, q, a, b, may be assumed at will, subject only to the restriction, that the values of BC be not negative. These four solutions are complementary in pairs.

A much more elegant solution of the problem is obtained from the properties of muarif angles. Since the trigons BAD, DAC have their sides commensurable, and their angles BAD, DAC alike, they must belong to the same muarif system. Wherefore, in any one system assume $BAD=DAC=\theta$ and $ADB=\phi$, then we have sin $ABD=\sin(\phi+\theta)$, sin $ACD=\sin(\phi-\theta)$ so that all the ratios are rational, since all the sines involve the same irreducible surd.

89. To construct a trigon, such that the three sides and the lines bisecting two of the angles may be all commensurable.

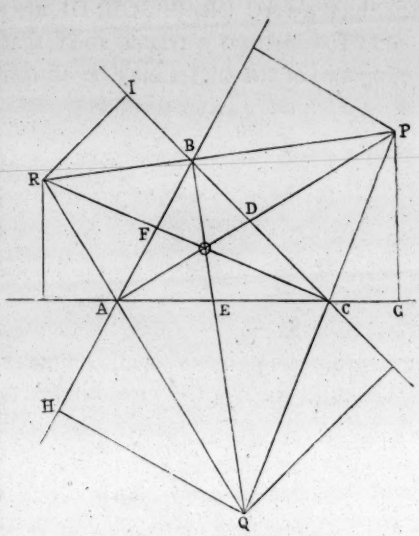


To construct a trigon ABC, such that its three sides and the lines AD, CF bisecting two of its angles may be all commensurable.

In any system of muarif angles, assume two, θ and ϕ, of which the sum may be less than a right angle; then having taken any base AC, make at A, CAD, DAB, each equal to θ, and at C, ACF, FCB, each equal to ϕ, then all the lines and all the areas in the figure are commensurable.

90. To construct a trigon of which the three sides and the three lines bisecting the three angles may be all commensurable.

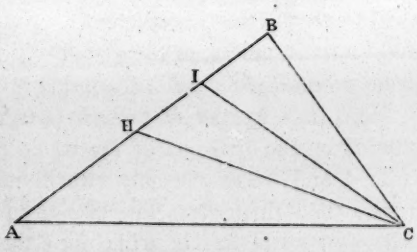
The halves of the three angles of any trigon make together one right angle, wherefore they can only be all muarif in the system to which the right angle belongs; and if two of them be muarif of that system, the third must also be so. Hence, if the two angles θ and ϕ be taken from the tetragonal system, and the trigon ABC be constructed, having $BAC=2\theta$, $ACB=2\phi$, ABE, the half of the third angle ABC, is also muarif, and therefore the segments intercepted on and by the six lines AB, BC, CA, AD, BE, CF, are all commensurable.



Moreover, if we draw QAR, RBP, PCQ, bisecting the supplemental angles, and then draw perpendiculars from O, P, Q, R, to the three sides, all the distances intercepted on those lines are also commensurable; hence, of such a trigon, the three sides, the three altitudes, the radius of the circumscribed circle, the radii of the four circles of contact, the distances of the centres of those four circles from each other, and from the corners of the trigon, as well as all the segments of the lines bisecting the angles internally and externally, are commensurable, and may therefore be expressed in integer numbers.

Example.

If we take $DAC=\theta=36^\circ\ 52'$, $ACF=\phi=22^\circ\ 37'$, we have $CBE=30^\circ\ 31'=\psi$, and in $\theta=\frac{3}{5}$, $\cos\theta=\frac{4}{5}$, $\sin\phi=\frac{5}{13}$, $\cos\phi=\frac{12}{13}$; whence $\sin\psi=\cos(\theta+\phi)=\frac{33}{65}$, $\cos\psi=\sin(\phi+\theta)=\frac{56}{65}$; from which we obtain the sines of the whole angles, viz., $\sin A=\frac{24}{25}$, $\sin C=\frac{120}{169}$, $\sin B=\frac{3696}{4225}$. The three sides must be proportional to these sines, so that, when the common divisors are taken out, we obtain AC = 154, BA = 125, CB = 169.

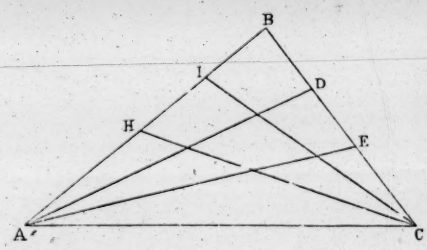


91. To construct a trigon such that the three sides and the two lines trisecting one of the angles may be all commensurable.

Having assumed an angle ϕ muarif of any system and less than 60°, make ACB equal to 3ϕ, and then at A make any

angle θ muarif of the same system, and less than the supplement of 3ϕ, and then the lines AC, CB, BA, CH, CI, are all commensurable.

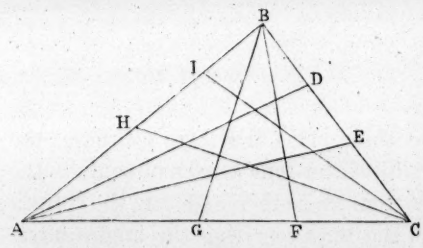
92. To construct a trigon such that the three sides and the four lines trisecting two of the angles may be all commensurable.



Assume two angles θ and ϕ, the sum of which must be less than 60°, and make CAB equal to 3θ, ACB equal to 3ϕ, then all the lines AB, BC, CA, AD, AE, CH, CI, and all the segments of those lines, are commensurable.

93. To construct a trigon of which the three sides and the six lines trisecting the three angles may be all commensurable.

The third parts of the three angles A, B, C, make together 60°, wherefore if these third parts be muarif of any system, 60° must belong to that system. Such a figure, then, can only belong to the trigonal system of which the modulus is $\sqrt{3}$.



Assume then from the trigonal system two angles, θ and ϕ, of which the sum is less than 60°, then the defect of their sum from 60° is also muarif of that system. Make BAC equal to 3θ, ACB equal to 3ϕ, and trisect the angles; these angles are all muarif, and consequently all the lines and all their segments are commensurable, while all the areas are commensurable with equilateral trigons constructed on any of the lines or parts.

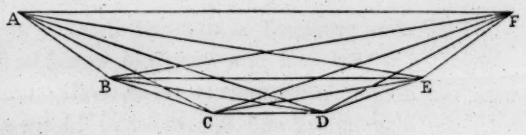
94. It is impossible to construct a trigon such that its sides and also the lines dividing its angles into more than three equal parts may be all commensurable.

If it were proposed to construct a trigon such that its sides and the lines dividing each of two of its angles into n equal parts may be all commensurable, we should only have to assume θ and ϕ muarif angles of any system, but such that $n(\theta+\phi)$ may be less than 180°; and then to make the angles at A and C equal to $n\theta$ and to $n\phi$ respectively. But the nth part of the remaining angle would not be muarif, unless, in the system $\sqrt{1}$, n were 2, or in the system $\sqrt{3}$, n were 3; for no submultiple of 180°, excepting 90° and 60°, can be muarif of any system.

Here it is worthy of remark that the tetragonal and trigonal systems are founded on the divisions of the whole revolution into 4 and into 6 equal parts; while 4 and 6 are the only two divisors which separate the prime numbers greater than themselves into two groups; those groups being, for 4, of the two forms $4n+1$ and $4n-1$; while for 6 they are classed as of the forms $6n+1$ and $6n-1$.

95. If a series of equal straight lines be drawn, making equal angles with each other, the angle being the supplement of the double of a muarif angle of any system, the distances of the extremities of the lines from each other are commensurable with the lines.

Let the equal lines AB, BC, CD, DE, EF, &c., make equal angles ABC, BCD, BDE, DEF, &c., such that the supplement of ABC is double of a muarif angle of any system; then all the diagonals AC, AD, AE, AF; BD, BE, BF, &c., are commensurable with AB.

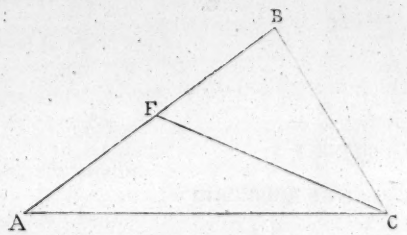


For the angle BAC being half the supplement of ABC is muarif; so is CDA its double, DEA its triple, and so on; wherefore, all the angles of the figure being muarif, all the sides of the trigons are commensurable.

If the angles belong to the tetragonal system, the sides are also commensurable with the radii of the circles described, one through the points A, B, C, D, E, F, and the other to touch the lines AB, BC, CD, &c.; and also the co-ordinates of the points referred to any system of rectangular axes making muarif angles with any of the lines may be obtained rational.

96. To construct a trigon such that its three sides and the line joining the middle of one side with the opposite corner may be all commensurable.

Having bisected the side AB of the trigon ABC in F, and joined CF, we have $AC^2+CB^2=2AF^2+2FC^2$, so that the problem becomes this: to find two square numbers whose sum is double of the sum of other two square numbers. Put $BC=a$, $CA=b$, $AF=k$, $FC=l$, then



$$a^2+b^2=2k^2+2l^2=(k+l)^2+(k-l)^2$$

so that the sum of the squares of a and b is also the sum of the squares of $k+l$ and $k-l$; now we have seen that every number which can be divided into two squares in more than one way is the product of two or more prime numbers of the form $4n+1$. Hence we have only to take any two or more prime numbers $\alpha, \beta, \gamma, \delta$, of the form $4n+1$, and decompose their product into two squares in two different ways, so as to obtain

$$\alpha\beta\gamma\delta=a^2+b^2=p^2+q^2,$$

and then we have $k=\frac{1}{2}(p+q)$, $l=\frac{1}{2}(p-q)$, or doubling all in order to avoid fractions,

$$BC=2A,\ CA=2b,\ AF=p+q;\ FC=p-q.$$

It is evident that we may introduce powers of the primes α, β, γ, and decompose such a product as $\alpha^2\beta^3\gamma\delta^4$ into two squares in two different ways.

For example, if we take $\alpha=5$, $\beta=13$, we obtain $65=7^2+4^2=8^2+1^2$, whence

$$BC=8,\ CA=14,\ AF=FB=9,\ FC=7.$$

If CF were produced to an equal distance, and the extremity of the produced part joined with A and B, a rhomboid would be formed having its sides 8 and 14 with the diagonals 18 and 14 respectively; or, halving, the sides are 4 and 7, with the diagonals 7 and 9. As 5 and 13 are the smallest primes of the class $4n+1$, we may infer that the above are the smallest dimensions of a rhomboid having its sides and diagonals all integers.

The subject may be viewed in another light thus: let the cosine of the angle at F be denoted by the fraction $\frac{st}{uv}$, in which one or both of the factors of the numerator may be 1 or zero; while one of the factors of the denominator may be unit; then denoting AF by k and FC by l as before, we have

$$a^2=k^2+2\frac{st}{uv}\,kl+l^2=u^2x^2+2stx\lambda+v^2\lambda^2$$

$$b^2=k^2-2\frac{st}{uv}\,kl+l^2=u^2x^2-2stx\lambda+v^2\lambda^2$$

if we put

$$k=ux,\ l=v\lambda.$$

Hence

$$a^2-b^2=(a+b)\,(a-b)=4stx\lambda=4sx\,.\,t\lambda,$$

which is satisfied on making

$$a=2sx+\tfrac{1}{2}\,t\lambda;\ b=2sx-\tfrac{1}{2}\,t\lambda.$$

which gives

$$a^2=4s^2x^2+2stx\lambda+\tfrac{1}{4}\,t^2\lambda^2,$$

so that we must have

$$u^2x^2+v^2\lambda^2=4s^2x^2+\tfrac{1}{4}\,t^2\lambda^2;$$

or

$$(4s^2-u^2)\,x^2=(v^2-\tfrac{1}{4}\,t^2)\,\lambda^2;$$

that is

$$(2s+u)\,.\,(2s-u)\,x^2=v^2\lambda^2-\tfrac{1}{4}\,t^2\lambda^2,$$

whence

$$(2s+u)+(2s-u)\,x^2=2v\lambda=2l;\ 2ux=2k$$
$$(2s+u)-(2s-u)x^2=t\lambda;$$

so that ultimately, multiplying by 2 to remove fractions,

$$2a=A=4sx+(2s+u)-(2s-u)\,x^2$$
$$2b=B=4sx-(2s+u)+(2s-u)\,x^2$$
$$2k=K=2ux$$
$$2l=L\qquad(2s+u)+(2s-u)\,x^2$$

in which any values, positive or negative, may be assigned to s, u, and x.

Right-angled Trigons.

a	β	c	b	a	A.			a	β	c	b	a	A.		
					°	′	″						°	′	″
1	2	5	4	3	36	52	11·64			·289	240	161	33	51	18·11
2	3	13	12	5	22	37	11·51	2	17	293	285	68	13	25	10·81
1	4	17	15	8	28	04	20·94			·305	273	136	26	28	51·75
		·25	24	7	16	15	36·74			·305	224	207	42	44	28·48
2	5	29	21	20	43	33	10·15	12	13	313	312	25	4	34	52·43
1	6	37	35	12	18	55	28·71	11	14	317	308	75	13	41	07·98
4	5	41	40	9	12	40	49·38			·325	323	36	6	21	34·78
2	7	53	45	28	31	53	26·84			·325	253	204	38	52	48·25
5	6	61	60	11	10	23	19·89	9	16	337	288	175	31	17	04·17
		·65	63	16	14	15	00·12	5	18	349	299	180	31	02	53·59
		·65	56	33	30	30	36·85	8	17	353	272	225	39	35	51·90
3	8	73	55	48	41	06	43·51			·365	364	27	4	14	31·90
		·85	84	13	8	47	50·69			·365	357	76	12	01	04·84
		·85	77	36	25	03	27·42	7	18	373	275	252	42	30	03·65
5	8	89	80	39	25	59	21·23			377	352	135	20	58	58·63
4	9	97	72	65	42	04	30·11			377	345	152	23	46	38·34
1	10	101	99	20	11	25	16·27	10	17	389	340	189	29	04	08·08
3	10	109	91	60	33	23	54·57	6	19	397	325	228	35	03	04 07
7	8	113	112	15	7	37	41·34	1	20	401	399	40	5	43	29·32
		·125	117	44	20	36	34·89	3	20	409	391	120	17	03	41·50
4	11	137	105	88	39	57	58·36	14	15	421	420	29	3	56	59·52
		·145	144	17	6	43	58·52			·425	416	87	11	48	44·22
		·145	143	24	9	31	38·22			·425	304	297	44	19	57·68
7	10	149	140	51	20	00	57·46	12	17	433	408	145	19	33	53·33
6	11	157	132	85	32	46	44·69			·445	437	84	10	52	50·40
		·169	120	119	44	45	37·00			·445	396	203	27	08	27·12
2	13	173	165	52	17	29	32·38	7	20	449	351	280	38	34	48·32
9	10	181	180	19	6	01	32·07	4	21	457	425	168	21	34	06·95
		·185	176	57	17	56	42·92	10	19	461	380	261	34	28	58·50
		·185	153	104	34	12	19·64			481	480	31	3	41	42·80
7	12	193	168	95	29	29	13·65			481	360	319	41	32	40·24
1	14	197	195	28	8	10	16·44			·485	483	44	5	12	18·45
		·205	187	84	24	11	22·25			·485	476	93	11	03	18·28
		·205	156	133	40	27	58·98			493	475	132	15	31	49·20
		·221	220	21	5	27	09·44			493	468	155	18	19	28·91
		·221	171	140	39	18	27·54			·505	456	217	25	26	55·36
2	15	229	221	60	15	11	21·44			·505	377	336	41	42	32·10
8	13	233	208	105	26	47	06·00	5	22	509	459	220	25	36	30·71
4	15	241	209	120	29	51	46·19	11	20	521	440	279	32	22	42·29
1	16	257	255	32	7	09	09·60			533	525	92	9	56	22·13
		·265	264	23	4	58	44·78			533	435	308	35	18	00·89
		·265	247	96	21	14	21·51	10	21	541	420	341	39	04	23·91
10	13	269	260	69	14	51	46·15			·545	544	33	3	28	17·07
9	14	277	252	115	24	31	46·36			·545	513	184	19	43	53·80
5	16	281	231	160	34	42	28·99	14	19	557	532	165	17	13	52·67

Right-angled Trigons—continued.

α	β	c	b	a	A.		
					°	′	″
		565	493	276	29	14	30·30
		565	403	396	44	29	53·01
13	20	569	520	231	23	57	08·15
1	24	577	575	48	4	46	18·80
8	23	593	465	368	38	21	28·83
5	24	601	551	240	23	32	11·67
17	18	613	612	35	3	16	23·35
16	19	617	608	105	9	47	53·46
		625	527	336	32	31	13·48
		629	621	100	9	08	52·23
		629	460	429	43	00	10·33
4	25	641	609	200	18	10	50·00
13	22	653	572	315	28	50	29·58
6	25	661	589	300	26	59	29·29
12	23	673	552	385	34	53	39·74
1	26	677	675	52	4	24	18·71
		685	684	37	3	05	46·73
		685	667	156	13	09	50·00
		689	680	111	9	16	15·34
		689	561	400	35	29	21·63
		697	672	185	15	23	31·57
		697	528	455	40	45	10·32
5	26	701	651	260	21	46	15·79
15	22	709	660	259	21	25	34·50
		725	644	333	27	20	33·40
		725	627	364	30	08	13·11
2	27	733	725	108	8	28	22·05
		745	713	216	16	51	14·19
		745	624	407	33	06	50·91
9	26	757	595	468	38	11	13·14
19	20	761	760	39	2	56	15·37
12	25	769	600	481	38	43	04·76
17	22	773	748	195	14	36	41 47
		785	783	56	4	05	26·94
		785	736	273	20	21	03·66
		793	775	168	12	13	51·63
		793	665	432	33	00	31·40
11	26	797	572	555	44	08	08·88
5	28	809	759	280	20	14	57·63
14	25	821	700	429	31	30	08·45
10	27	829	629	540	40	38	46·60
		841	840	41	2	47	39·70
		845	837	116	7	53	25·34
		845	836	123	8	22	11·40
18	23	853	828	205	13	54	21·30
4	29	857	825	232	15	42	23·86
		865	816	287	19	22	39·26
		865	703	504	35	38	16·00
6	29	877	805	348	23	22	43·45
16	25	881	800	369	24	45	41·46
		901	899	60	3	49	05·90
		901	780	451	30	02	12·19
		905	777	464	30	50	39·56
		905	663	616	42	53	43·70
		925	924	43	2	39	51·98
		925	756	533	35	11	05·45
20	23	929	920	129	7	58	54·57
19	24	937	912	215	13	15	54·12
10	29	941	741	580	38	03	04·37
		949	900	301	18	29	32·01
		949	851	420	26	16	04·97
13	28	953	728	615	40	11	25·62
		965	957	124	7	22	57·98
		965	884	387	23	38	34·72
4	31	977	945	248	14	42	17·13
		985	864	473	28	41	55·19
		985	697	696	44	57	31·90
6	31	997	925	372	21	54	29·24
15	28	1009	840	559	33	38	34·14
22	23	1013	1012	45	3	32	45·81
11	30	1021	779	660	40	16	21·37
		1025	1023	64	3	34	47·36
		1025	897	496	28	56	26·12
3	32	1033	1015	192	10	42	41·94
		1037	988	315	17	41	01·07
		1037	812	645	38	27	40·84
5	32	1049	999	320	17	45	40·74
10	31	1061	861	620	35	45	26·60
13	30	1069	780	731	43	08	33·42
		1073	975	448	24	40	41·44
		1073	952	495	27	28	21·14
2	33	1093	1085	132	6	56	11·25
16	29	1097	928	585	32	13	36·62
		1105	1104	47	2	26	15·90
		1105	1073	264	13	49	20·83

Trigons of 120°.

a	β	c	b	a	A.			a	β	c	b	a	A.		
1	2	7	5	3	21°	47′	12″·5	8	13	337	272	105	15°	39′	14″·9
1	3	13	8	7	27	47	44·7	1	18	343	323	37	5	21	37·3
2	3	19	16	5	13	10	24·7	3	17	349	280	111	15	59	17·8
1	5	31	24	11	17	53	47·6	5	16	361	231	185	26	20	49·6
3	4	37	33	7	9	25	48·0	9	13	367	315	88	11	59	06·3
1	6	43	35	13	15	10	41·4	4	17	373	273	152	20	39	55·7
3	5	49	39	16	16	25	35·1	7	15	379	259	176	23	42	48·9
4	5	61	56	9	7	20	27·6	11	12	397	385	23	2	52	33·2
2	7	67	45	32	24	25	57·7	2	19	403	357	80	9	53	57·2
1	8	73	63	17	11	38	06·2	9	14	403	333	115	14	18	27·6
3	7	79	51	40	26	00	28·2	8	15	409	304	161	19	55	55·2
1	9	91	80	19	10	25	02·8	1	20	421	399	41	4	50	17·0
5	6	91	85	11	6	00	32·3	3	19	427	352	123	14	26	44·9
3	8	97	57	55	29	24	33·5	6	17	427	253	240	29	07	40·0
2	9	103	77	40	19	39	10·3	11	13	433	407	48	5	30	32·5
5	7	109	95	24	10	59	33·8	5	18	439	299	205	23	51	14·6
6	7	127	120	13	5	05	09·1	7	17	457	287	240	27	03	08·7
1	11	133	120	23	8	36	47·7	1	21	463	440	43	4	36	47·8
4	9	133	88	65	25	02	22·8	3	20	469	391	129	13	46	49·8
3	10	139	91	69	25	27	39·8	12	13	469	456	25	2	38	45·3
5	9	151	115	56	18	44	02·4	5	19	481	336	215	22	46	27·2
1	12	157	143	25	7	55	35·3	9	16	481	369	175	18	21	56·7
3	11	163	112	75	23	28	59·0	2	21	487	437	88	9	00	11·4
7	8	169	161	15	4	24	30·4	7	18	499	301	275	28	30	25·8
4	11	181	105	104	29	50	30·2	6	19	511	325	264	26	34	41·4
7	9	193	175	32	8	15	20·2	11	15	511	451	104	10	09	06·2
2	13	199	165	56	14	06	19·3	9	17	523	387	208	20	08	47·7
1	14	211	195	29	6	50	09·5	4	21	541	425	184	17	07	48·8
3	13	217	160	87	20	19	00·0	13	14	547	533	27	2	26	59·9
8	9	217	208	17	3	53	24·8	1	23	553	528	47	4	13	15·7
6	11	223	168	85	19	16	29·5	11	16	553	473	135	12	12	19·5
5	12	229	145	119	26	44	44·3	3	22	559	475	141	12	37	03·3
1	15	241	224	31	6	23	45·2	10	17	559	440	189	17	01	33·8
3	14	247	187	93	19	01	50·4	5	21	571	416	235	20	52	49·9
7	11	247	203	72	14	37	20·0	8	19	577	368	297	26	28	21·5
2	15	259	221	64	12	21	24·4	7	20	589	351	329	28	55	47·6
5	13	259	155	144	28	46	59·5	13	15	589	559	56	4	43	22·8
9	10	271	261	19	3	28	51·6	1	24	601	575	49	4	02	56·0
7	12	277	217	95	17	16	41·7	3	23	607	520	147	12	06	23·4
6	13	283	192	133	24	00	59·8	9	19	613	423	280	23	18	06·5
4	15	301	209	136	23	02	06·1	5	22	619	459	245	20	02	45·2
9	11	301	279	40	6	36	31·0	14	15	631	616	29	2	16	51·8
1	17	307	288	35	5	39	58·3	4	23	637	513	200	15	46	40·1
3	16	313	247	105	16	53	20·7	12	17	637	552	145	11	22	09·7
10	11	331	320	21	3	08	58·8	11	18	643	517	203	15	52	02·4

Trigons of 120°—continued.

a	β	c	b	a	A.			a	β	c	b	a	A.		
9	20	661	441	319	24°	42′	17″·8	13	20	829	689	231	13°	57′	51″·4
8	21	673	400	377	29	01	15·2	4	27	853	713	232	13	37	25·0
2	25	679	621	104	7	37	21·1	10	23	859	560	429	25	37	36·7
13	17	679	611	120	8	48	14·1	1	29	871	840	59	3	21	47·0
11	19	691	539	240	17	30	18·3	15	19	871	795	136	7	46	17·5
1	26	703	675	53	3	44	36·7	3	28	877	775	177	10	03	58·1
6	23	703	493	312	22	36	12·8	13	21	883	715	272	15	28	20·0
3	25	709	616	159	11	11	55·8	5	27	889	704	295	16	42	03·4
5	24	721	551	265	18	33	37·3	8	25	889	561	464	26	52	21·5
15	16	721	705	31	2	08	02·4	7	26	907	627	413	23	13	29·9
13	18	727	637	155	10	38	24·7	17	18	919	901	35	1	53	24·4
12	19	733	600	217	14	51	19·6	1	30	931	899	61	3	15	10·3
7	23	739	480	371	25	46	14·5	9	25	931	544	531	29	35	59·9
10	21	751	520	341	23	09	20·0	3	29	937	832	183	9	44	15·8
1	27	757	728	55	3	37	27·0	5	28	949	759	305	16	09	38·6
3	26	763	667	165	10	47	38·6	12	23	949	696	385	20	34	09·0
9	22	763	477	403	27	13	13·7	11	24	961	649	455	24	12	24·8
15	17	769	735	64	4	07	59·4	7	27	967	680	427	22	28	59·5
2	27	787	725	112	7	04	46·2	4	29	973	825	248	12	45	07·8
7	24	793	527	385	24	51	47·6	17	19	973	935	72	3	40	27·4
11	21	793	583	320	20	27	17·2	9	26	991	595	549	28	40	12·4
6	25	811	589	336	21	01	34·8	13	23	997	767	360	18	13	20·8
9	23	817	495	448	28	21	06·2								
16	17	817	800	33	2	00	16·7								
14	19	823	728	165	9	59	55·3								

XLVI.—*On the Relations, Structure, and Function, of the Valves of the Vascular System in Vertebrata.*

By James Bell Pettigrew, M.D. Edin., Assistant in the Museum of the Royal College of Surgeons of England. Communicated by William Turner, M.B. (Plates XXVIII., XXIX.)

(Read 21st March 1864.)

Introductory Remarks.

The rapid advances made of late in the diagnosis of cardiac and other diseases affecting the organs of circulation, render the present inquiry into the normal or healthy condition of the valves of the vascular system, not more important anatomically, than medically. As the nature and composition of the parts in which valves are found in some instances materially influence their action, I have deemed it necessary to advert briefly to the properties and structure of the veins and arteries, when describing the venous and arterial or semilunar valves; and to the arrangement of the muscular fibres in the ventricles, when pointing out the peculiarities of the auriculo-ventricular ones. As, moreover, much information is to be obtained by comparing analogous structures, I have, in the present instance, not confined my observations to any particular form of valve, but have examined in succession the entire valvular arrangements of the fish, the reptile, the bird, and the mammal; my object being to arrive, if possible, at a correct knowledge of the more elaborate varieties as they exist in man, and in the higher mammalia.

In order to simplify the numerous relations and complicated structure of the several valves met with, as well as to obviate the necessity for entering largely into anatomical details, the present paper has been fully illustrated by photographs and drawings from dissections, and from casts representing the valves in action. The photographs, thirty-four in number, were taken for the most part from the specimens while in the fresh or recent state, by Mr Ayling and myself. Of the drawings, twenty-three in number, that marked 33, Plate XXVIII. and the last six of Plate XXIX., are by my friend Dr Henry Season Wilson. The remaining fourteen are by myself. For permission to examine and figure several of the specimens illustrating the peculiarities of the valvular arrangements of the fish and reptile, I am indebted to the kindness of the Council of the Royal College of Surgeons of England. The dissections and casts, which are numerous, were made with a special view to this inquiry, and are preserved in the Museum of the Royal College of Surgeons of England, where they may be consulted.

Structure of the Veins and Venous Valves.

Regarding the composition of the veins, there is, as the reader is aware, some difference of opinion, authorities not being agreed either as to the number or nature of the coats. This may in part be explained by the variation in the thickness of the coats themselves, these, according to John Hunter,* becoming thinner and thinner in proportion to the size of the vein, the nearer they approach to the heart. In moderate-sized veins an external, a middle, and an internal coat are usually described; the first consisting of cellular, fibrous, and elastic tissue, interlacing in all directions; the second, of waved filaments of areolar tissue, with a certain admixture of non-striped muscular fibres, which run circularly, obliquely, or even longitudinally;† the third, consisting of one or more strata of very fine elastic tissue, minutely reticulated in a longitudinal direction, the innermost stratum (when several are present) being lined by epithelium. Of these layers, the second and third, from the fact of their contributing to the formation of the venous valves, are the most important. The coats of the veins, as has been long known, are tough, elastic, and possessed of considerable vital contractility. Of these qualities, the toughness prevents undue dilatation of the vessel when distended with blood; the elasticity and vital contractility assisting the onward flow of that fluid, and *tending to approximate the segments of the valves*, by contracting in the direction of the axis of the vessel. As the valves of the veins are very ample and very flexible, they readily accommodate themselves to the varying conditions in which they are placed by the elasticity and contractility of the vessel, and by the reflux of the blood.

The valves of the veins vary as regards the number of the segments composing them, and also slightly as regards structure. In the smallest veins, and where small veins enter larger ones (Plate XXVIII. fig. 9 *b*), one segment only is present. In middle-sized veins, as they occur in the extremities, two segments (Plate XXVIII. figs. 3, 4, and 7 *a b*), are usually met with;‡ while in the larger veins, as in the internal jugular of the horse, three, and even four segments (Plate XXVIII. figs. 1 and 2, *a b c, f g h*), are by no means uncommon.§

The segments, whatever their number, are semilunar in shape‖ (Plate XXVIII.

* Hunter on the Blood, pp. 180, 181.

† Dr Chevers says, that in the deep as well as in some of the superficial veins of the trunk and neck, the middle coat is composed of several layers of circular fibres, with only here and there a few which take a longitudinal course; while the middle coat of the superficial and deep veins of the limbs consists of a circular layer, and immediately within this of a strong layer of longitudinal fibres.—*Med. Gazette*, 1845, p. 638.

‡ In the heart of the frog-fish, sun-fish, sturgeon, American devil-fish, python, and crocodile, a semilunar valve, consisting of two segments, guards the orifice of communication between the sinus venosus and the right auricle.

§ When four segments are present, two are usually more or less rudimentary (Plate XXVIII. fig. 2 *f g*).

‖ John Hunter in speaking of the form of the venous valves, says, their free edges are *cut off straight*, and are not curved as in the arteries. This, however, is not the case; as may be seen by

figs. 3 and 4 *a b*); the convex border being attached to the wall of the vessel obliquely (Plate XXVIII. fig. 5 *a b*), the crescentic or concave margin, which is free (Plate XXVIII. fig. 4 *c*), and directed towards the heart, projecting into the vessel. When one segment constitutes the valve, and it occurs in the course of a vein, it is placed obliquely in the vessel (Plate XXVIII. fig. 9), its attached convex border (*a*) occupying rather more than a half of the interior. When the segment occurs at the junction of a smaller with a larger vein, its convex border is attached to a half or more of the orifice of the smaller one where it joins the larger, its free margin running transversely to the larger trunk. In such cases the segment *acts as a moveable partition or septum, common alike to both vessels*, but its position and relations are such, that while it readily permits the blood from the smaller vein to enter the larger one, it effectually prevents its return.

When the valve consists of two segments, they are semilunar in shape, and very ample, the vertical measurement of each, being not unfrequently nearly twice that of the diameter of the vessel itself (Plate XXVIII. figs. 3 and 4 *a b*). In such cases both segments are usually of the same size, so that they divide the vessel into two equal parts (*e*). They are placed obliquely with regard to each other (Plate XXVIII. fig. 5 *a b*), their convex borders, which are attached to the interior of the vessel, starting from a common point above (Plate XXVIII. fig. 2 *i*), and gradually diverging (*e*) to curve round and reunite on the opposite side of the vessel (*d*); their concave and free margins inclining towards each other (Plate XXVIII. fig. 12 *e*), and being directed, as in the more simple valve, towards the heart. The free margins of the two segments, like the attached ones, start from a common point (Plate XXVIII. fig. 4 *r*), but such is the shape of the segments, and such the angle at which they are placed with regard to each other, that they do not diverge to the same extent, *but run more or less parallel.** This relation of the segments to each other above, is in part accounted for by the presence of *a fibrous structure* (Plate XXVIII. fig. 4 *r*), *which extends from the wall of the vessel into the interior*, and supports them at a certain distance from the sides of the vessel. The fibrous structure referred to is well seen in the semilunar valves of the pulmonary artery and aorta (Plate XXVIII. fig. 36 *n n'*), and seems to have escaped observation. In a line corresponding to the attached border of each of the segments (Plate XXVIII. fig. 4 *a b*), *the middle and internal coats of the vein are thickened*, as may be ascertained by a vertical section, or by

reference to photographs 1, 2, 3, and 4, Plate XXVIII. The edges referred to are least curved when the valve is distended or in action (Plate XXVIII. fig. 13 *e*), but the curve is never altogether absent.—*Treatise on the Blood*, pp. 181, 182.

* I was much struck, on injecting the external saphenous vein of the human subject from the dorsum of the foot, to find, on dissection, that the free margins of some of the segments were in contact throughout; clearly showing, that when the segments are allowed to float in a fluid, they are so projected against each other, that even the slightest reflux will instantly close them.

introducing coloured plaster of Paris* into the vessel. I particularly direct attention to this circumstance, as the thickenings referred to *form fibrous zones* (Plate XXVIII. fig. 6 *h*), *which extend for a short distance into the substance of the segments*, and afford them a considerable degree of support. They further assist in preserving the shape of the segments, and in enabling them to maintain the proper angle of inclination—the said angle inclining the segments towards each other in the mesial plane of the vessel (Plate XXVIII. figs. 3 and 4 *e*). When a valve, consisting of two segments, is situated at the junction of a smaller with a larger vein, one of the segments is usually placed between the two vessels at the point of juncture (Plate XXVIII. fig. 11 *b*), the other on the wall of the smaller vein (*a*). The position of the segments in such instances varies, their long diameter sometimes running parallel with the larger vessel, sometimes obliquely, but more commonly transversely. When the valve consists of three segments (Plate XXVIII. figs. 1 and 8, *a b c*, *r s t*), the segments, as a rule, are unequal in size, one of them being generally a little larger (*t*) than either of the other two (*r s*). They are semilunar in shape, as in the smaller and middle-sized veins, and differ from the latter in being less capacious. The tri-semilunar valves in the veins, may therefore be regarded as intermediate between the fully developed bi-semilunar valves found in the veins of the extremities, and the fully developed tri-semilunar valves which occur at the origin of the pulmonary artery and aorta. The existence of valves in the veins is indicated externally by a dilatation or enlargement of the vessel; the dilatation consisting of one, two (Plate XXVIII. figs. 3, 5, and 12 *h g*), or three (Plate XXVIII. fig. 8 *g a b*) swellings, according as the valve is composed of one two, or three segments. These dilatations or swellings are analogous to the sinuses of VALSALVA in the arteries (Plate XXVIII. figs. 17 and 18 *d*), their direction in the veins of the extremities being from below upwards and from within outwards. They form, with the segment to which they belong, open sinuses or pouches which look towards the heart, and as they extend nearly as far in an outward direction as the segments project inwardly, they give a very good idea of the size and shape of the segments themselves. The only point regarding the dilatations deserving of special attention, *is the gradual thinning in a direction from above downwards, of those portions of the coats of the vessel which enter into their formation.* The thinning referred to, is well seen when vertical sections of the vessel are made, or when the vein is distended with coloured plaster of Paris, as recommended. The swellings present a deeper colour the nearer we approach to the attached border of the segments, the attached borders, on account of their greater thickness, appearing as dense fibrous zones (Plate XXVIII. fig. 5 *a b*, fig. 6 *h*). The object of the swellings is evidently

* I have derived much information from the employment of this material; its use having enabled me to determine with something like accuracy, the relation of the segments of the valves to each other when in action, and other points connected with the physiology of the heart.

twofold,—*firstly, to cause the blood to act on the segments of the valve from above downwards, and from without inwards, in the direction of the mesial plane, or of the axis of the vessel,* according as there are two or three segments present; and, *secondly, to increase the area over which the pressure exerted by the reflux of the blood extends.* When the vein is opened between the segments, when two are present, each of the segments is seen to form two curves,—one curve giving its concave or free margin (Plate XXVIII. fig. 4 *c*), the other its convex or attached one (*a*); but when the section is carried through the wall of the vein, and through the centre of one of the segments, one curve only is obtained (Plate XXVIII. fig. 3 *g*); and it is useful to remember this, as it shows with what facility the structures entering into the formation of the segments, viz., the lining membrane of the vessel and certain parts of the middle and internal coats, are given off. It also shows how the lower portion of the dilatation (*g*), while it supports a certain quantity of the refluent blood, guides by far the greater quantity on to the valves (*b*), rendering their closure not a matter of accident, but of necessity.

The segments of the venous valves are exceedingly flexible, and so delicate as to be semi-transparent. They possess great strength and a considerable degree of elasticity.* Usually they are described as consisting of a reduplication of the fine membrane lining the vessel, strengthened by some included fibro-cellular tissue, the whole being covered with epithelium. This description, however, is much too general to convey any very accurate impression of their real structure, and the following, drawn up from the examination of a large number of specimens taken from man, the horse, ox, sheep, and other animals, may prove useful.

When one of the segments of a well-formed bi-semilunar valve removed from the human femoral vein, is stained with carmine, fixed between two glasses, and examined with the microscope or pocket lens, the subjoined phenomena are witnessed:—

1*st*, The lining membrane of the vessel covered with epithelium is seen to form the investing sheath of the segment, no breach of continuity being anywhere perceptible.

2*d*, Large quantities of white fibrous tissue, mixed up with areolar and yellow elastic tissue, from the middle and internal coats of the vessel, are observed to extend into the segment. The fibres composing these tissues pursue a definite arrangement.

Thus running along the concave or free margin of the segment (Plate XXVIII. fig. 14 *a*), as likewise on the body, especially where the segments join each other (*b*), are a series of very delicate fibres, consisting principally of yellow elastic tissue. These fibres proceed in the direction of the long diameter

* Hunter denies the elasticity of the segments, on the ground that the valvular membrane is not formed of a reduplication of the lining membrane of the vessel, an opinion at variance with recent investigation.—*Treatise on the Blood*, pp. 181, 182.

of the segment, but transversely to the course of the vessel, and may be denominated *the horizontal fibres.*

Running in a precisely opposite direction, and confining themselves principally to the body of the segment, are a series of equally delicate fibres (*c*), having a like composition, and which, for the sake of distinction, may be described as *the vertical series.* These two sets of fibres are superficial, and to be seen properly, a power magnifying from 200 to 250 diameters is required.

Radiating from the centre of the segment (Plate XXVIII. fig. 15 *e*) *towards its attached border* (*i i'*), *and seen through the more delicate horizontal and vertical ones, is a series of stronger and deeper fibres,* composed of white fibrous and yellow elastic tissue, the former predominating. Still stronger and deeper than either of the fibres yet described, and proceeding from the attached border of the segment (Plate XXVIII. fig. 16 *s t*), *is a series of oblique fibres,* continuous in very many instances with corresponding fibres in the middle coat of the vessel. These fibres cross each other with great regularity, and form the principal portion of the segments. They are most strongly marked at the margin of the convex border of the segment, where they form a fibrous zone or ring, which, as has been explained, supports the segment, and carries it away from the sides of the vessel into the interior. I have also detected, in the vicinity of the attached border of the segment, some non-striped muscular fibres. The segment of a venous valve is therefore a highly symmetrical and complex structure, the fibrous tissues composing it, being arranged in at least three well-marked directions; viz., horizontally, vertically, and obliquely. The great strength which such an arrangement is calculated to impart to the segment is readily understood.

In conclusion, the segment is thinnest at its free margin, and thickest towards its attached border; the body being a little thicker than the free margins, and where the extremities or narrow portions of the segments join each other, but not so thick as the attached border.

The Venous Valves in Action.

The manner in which the venous valves act, is well seen when the vein is suspended perpendicularly overhead, and water, oil, glycerine, or liquid plaster of Paris, poured into it by an assistant from above; the vein beneath the valve being cut away, the better to expose the segments to the view of the spectator. When the valve consists of one segment only, the fluid is observed *to force it obliquely across the vessel,* and to apply its free crescentic margin to the interior or convex surface with such accuracy as to prevent even the slightest reflux. When two segments occur in the course of a vein, *they are forced by the fluid simultaneously towards each other in the mesial plane of the vessel* (Plate XXVIII. figs. 3 and 12 *e*), the sinuses (*gh*) behind the segments becoming distended, and directing the

current and regulating to a certain extent the amount of pressure. The closure in this instance is almost instantaneous, and so perfect that not a single drop escapes. It is effected by *the free margins of the segments, and a large proportion of the sides*, coming into accurate contact, the amount of contact increasing in the inverse of the pressure applied. If liquid plaster of Paris be used for distending the vein, and the specimen is examined after the plaster has set, one is struck with the great precision with which the segments act (Plate XXVIII. figs. 6 and 11 *a b*), these coming together so symmetrically, that they form by their union *a perpendicular wall or septum* (Plate XXVIII. figs. 6, 11, and 12, *e*) *of a beautifully crescentic shape** (Plate XXVIII. fig. 13 *e*). This fact is significant, as it clearly proves that the concave or free margins of the segments, and a considerable proportion of the sides, run parallel to each other when the valve is in action, a circumstance difficult of comprehension, when it is remembered that the convex borders of the segments are attached obliquely to the walls of the vessel, and that the segments, when not in action, incline towards each other at a considerable angle. The very accurate apposition of the segments, when the valve is closed, is to be traced :—

1*st*, *To the direction and shape of the venous sinuses*, which conduct the fluid employed, on to the segments in almost equal quantities.

2*d*, *To the disposition of the free margins of the segments*, which, as was explained, run side by side, and are supported by fibrous structures which carry them away from the sides of the vessel for some distance; and,

3*d*, *To the amplitude of the segments themselves*, which allows them to come together without difficulty, and when the pressure is applied, to flatten themselves against each other to form the perpendicular crescentic wall adverted to.

In the event of two segments occurring at the entrance of a smaller into a larger vein, one of them being situated at the junction of the smaller vessel with the main trunk (Plate XXVIII. fig. 11 *b*), the other on the wall of the tributary branch (*a*); the former, *i.e.*, the common or septal segment, is forced by the fluid in *a slightly outward direction*, the latter *in an opposite or inward direction*, the free margins and sides of the segments being by this arrangement accurately applied to each other to form an impervious wall or septum (*e*) as already described. When the entrance of a smaller into a larger vessel is guarded by two segments situated on the tributary branch at its orifice, their action is precisely the same as when they are placed in the course of a vein. When three segments are present, as happens in the larger trunks, the closure is effected in a manner greatly resembling that by which the semilunar valves of the pulmonary artery and aorta

* In order to see the perpendicular wall formed by the flattening of the sides of the segments against each other when the valve is in action, the vein and the plaster should be cut across immediately above the valve, and the segments forcibly separated by introducing a thin knife between them. In fig. 13, Plate XXVIII., one of the segments has been quite removed.

are closed; the fluid employed, in virtue of the direction given to it by the venous sinuses, *causing each of the segments* (Plate XXVIII. fig. 8 *r s t*) *to bend or double upon itself at an angle* (*i*) *of something like* 60°;* the three lines formed by the doubling and union of the three segments dividing the circle corresponding to the wall of the vessel, into three nearly equal parts. In the doubling of the segments upon themselves, *each segment regulates the amount of bending which takes place in that next to it*, and as the free margins of the segments so bent advance synchronously towards the axis of the vessel, *they mutually act upon and support each other*. As the three segments are attached obliquely to the wall of the vessel, while the free margins, after the folding has taken place, are inclined towards and run parallel to each other (*a b*), they form an inverted dome consisting of three nearly equal parts, the margins of the segments, and a certain portion of the sides, when the pressure is applied, flattening themselves against each other to form three crescentic partitions or septa† which run from the axis of the vessel towards the circumference.

The tri-semilunar valve, as will be seen from the foregoing explanation, is closed in a very different manner from the bi-semilunar one. The occlusion of the vessel, however, is not the less complete; the segments, when three are present, *being wedged into each other in a direction from above downwards, and from without inwards;* the first of these movements, by tending to flatten the segments, pressing their margins and sides together: the second, by urging the segments towards the axis of the vessel, impacting them more and more tightly, especially towards their apices or points. As the apices or points formed by the doubling of the segments whilst in action, are composed principally of the flexible and free crescentic margins, and are at liberty to move until the wedging process is completed, a careful examination has satisfied me, *that they rotate to a greater or less extent before the valve is finally closed.* This spiral movement, which is simply indicated in the venous valves, is more strongly marked in the semilunar ones of the pulmonary artery and aorta (Plate XXVIII. figs. 26, 27, and 28, *v*, *w*, *x*,) and attains, as will be shown subsequently, a maximum in the auriculo-ventricular valves of the mammal (Plate XXIX. figs. 53 and 54, *m i n, s r*).

By whatever power the blood in the veins advances—whether impelled by the heart alone, or by muscular contractions occurring in different parts of the body, or by rythmic movements which take place in the vessels themselves, or by efforts of inspiration, or by all or combinations of these; there can, I think, be little doubt that this fluid, in its backward or retrograde movement, acts to a great extent mechanically on the valves as described. It ought, however, to be borne in mind that the veins and the valves are vital structures, and that

* The angle is never precisely 60°, from the fact of the segments varying slightly as regards size.

† The crescentic partitions, as they occur in the semilunar valves of the pulmonary artery and aorta, are shown at *b b'*, of fig. 25, Plate XXVIII.

although a perfect closure may be effected by purely mechanical means in the dead vein, it is more than probable that in the living one, the contraction of the coats of the vessel exercises a regulating influence.

Structure of the Arteries and Arterial Valves.

The coats of the arteries, as is well known, are thicker than those of the veins, while the layers composing them are more numerous. The external coat, according to Henle, consists of an outer layer of areolar tissue in which the fibres run obliquely or diagonally round the vessel, and an internal stratum of elastic tissue; the middle coat in the largest arteries, according to Raüschel, being divisible into upwards of forty layers. The layers of the middle coat consist of pale, soft, flattened fibres, with an admixture of elastic tissue, the fibres and elastic tissue being disposed circularly round the vessel. The internal coat is composed of one or more layers of fibres, so delicate that they constitute a transparent film, the film being perforated at intervals, and lined with epithelium. The arteries, as might be expected from their structure, and as was proved by the admirable experiments of John Hunter, whose beautiful preparations I have had an opportunity of examining, possess a high degree of elasticity and vital contractility, and are extensible and retractile both in their length and breadth; the power of recovery, according to that author, being greater in proportion as the vessel is nearer the heart. From this it follows that the pulmonary artery and aorta are most liable to change in dimensions. As, however, any material alteration in the size of the pulmonary artery and aorta might interfere with the proper function of the semilunar valves situated at their orifices, it is curious to note that the great vessels arise from strong and comparatively unyielding fibrous rings. These rings (particularly the aortic one) are so dense as to be almost cartilaginous in consistence, and Professor Donders* has lately discovered, that they contain stellate corpuscles similar in many respects to those stellate and spicate corpuscles, found in many forms of cartilaginous tumours. They have been more or less minutely described by Valsalva,† Gerdy,‡ Dr John Reid,§ and Mr W. S. Savory,‖ and merit attention because of their important relations to the segments of the semilunar valves. The following description of the aortic and pulmonic fibrous rings, has been drawn up chiefly from the examination of a large number of human hearts. Each ring, as will be seen by a reference to

* "Onderzockingen betrekkeligh den bouw van het menchelijke hart," in "Nederlandsch Lancet" for March and April 1852.

† Opera Valsalvæ, tom. i. p 129.

‡ Journal Complimentaire, tom. x.

§ Cyc. Anat. and Phy. article "Heart," pp. 588, 589. London, 1839.

‖ Paper read before the Royal Society in December 1851.

Plate XXVIII. figure 30, taken from a photograph of a boiled heart, consists, as was shown by REID, of three convex portions (*r s t*). Each convex portion is directed from above downwards, and from without inwards, and as it unites above with that next to it, the two when taken together form a conical-shaped prominence (*x*), which is adapted to one of the three triangular-shaped interspaces occurring between the segments of the valve (Plate XXVIII. fig. 17 *h*). The arterial rings are therefore placed obliquely, the under surface, which gives attachment to many of the fibres of the ventricles anteriorly (Plate XXVIII. fig. 30 *d d'*), resting on the rounded oblique border of the ventricular walls (Plate XXVIII. fig. 17 *e*). The ring surrounding the pulmonary artery, as was pointed out by REID, is broader, but not quite so thick as that surrounding the aorta, and both are admirably adapted for the reception of the large vessels which, as was shown by that author, originate in three festooned borders. These borders, I am inclined to think, consist of two parts,—*an outer* (Plate XXVIII. fig. 17 *r r'*), composed of the outer, and a small portion of the central layer of either the aorta or pulmonary artery; and *an inner* (*t*), composed principally of the central and inner layers. The outer border (Plate XXVIII. fig. 18 *r*), which is the thinner of the two, is attached *to the superior and outer margin* of one or other of the fibrous rings (*g*), chiefly by the serous membranes; the inner (Plate XXVIII. fig. 17 *t*), which projects further in a direction from above downwards, and corresponds to the thickened convex border of the segments (*s*), to the formation of which it contributes, being attached *to the inferior and inner margin* (*g'*). These points are well seen, when a vertical section is made of the aorta or pulmonary artery between the segments composing the semilunar valves. In such a section (Plate XXVIII. fig. 17), the vessels are observed to be thickened in a direction from above downwards, the thickening beginning at the point where the segments meet above (*b*), and gradually increasing until the vessels bifurcate (*r' t*). The reverse of this holds true of those portions of the vessels which enter into the formation of the sinuses of VALSALVA (Plate XXVIII. fig. 18 *i*), these being unusually thin, particularly where attached (*r*).* As the thickened portions of the vessels correspond to the fixed margins of the segments (Plate XXVIII. figs. 17 and 35 *b t n*), and extend between them in an arched direction above (*c*), they give the precise boundaries of the sinuses of VALSALVA (Plate XXVIII. figs. 17 and 18 *d*), and furnish the segments with *three fibrous frameworks* analogous, in some respects, to the thickenings which occur in similar situations in the veins. *These frameworks extend for a short distance into the segments* (Plate XXVIII. fig. 17 *b*, and fig. 36 *n*), and assist not only in affording the segments

* The several points adverted to are seen to advantage in the whale (*Physalus antiquorum*, Gray), the aorta of which I had an opportunity of dissecting for the Museum of the Royal College of Surgeons of England.

the requisite degree of support, but in carrying them away from the sides of the vessel, and in inclining them towards each other at such an angle as insures that their free margins, especially where they unite above (Plate XXVIII. fig. 36 *b c*), shall be more or less parallel when the valve is in action. That the frameworks afford the support here indicated, is proved by the fact, that when liquid plaster of Paris is introduced into the ventricles, and forced through the pulmonary artery and aorta, in the direction of the circulation, the attached borders of the segments do not fall back into the sinuses of VALSALVA (Plate XXIX. figs. 50 and 51 *v w*) to the same extent as the sides and free margins, but project so as to furnish the casts thus obtained, with corresponding depressions (*r s*). The sinuses of VALSALVA are formed above by the dilatations or expansions of the great vessels, and the one occupies a higher position than either of the other two. They are further unequal in size (Plate XXVIII. fig. 26 *w x v*), the highest and smallest occurring anteriorly, that which is intermediate in size being placed posteriorly, while the lowest and largest is directed towards the septum. They correspond in situation and dimensions to the segments behind which they are found, and differ from the venous sinuses in being more capacious, a section of the sinus and its segment (which is likewise very ample) giving a sweep of nearly half a circle (Plate XXVIII. fig. 18 *b i s*). As a result of this amplitude, those portions of the segments which project into the vessel are, during the action of the valve, closely applied to each other throughout a considerable part of their extent (Plate XXVIII. fig. 26 *a b c*); the great size of the sinuses furnishing an increased quantity of blood for pressing the segments from above downwards, and from without inwards, or in the direction of the axis of the vessel. *The sinuses of* VALSALVA *curve towards each other in a spiral direction;* and this ought to be attended to in speaking of the action of the semilunar valves, as the sinuses direct the blood spirally on to the mesial line of each segment (Plate XXVIII. fig. 26 *v w x*), and cause the segments to twist and wedge into each other, as represented at *v w x* of figs. 26, 27, and 28, Plate XXVIII. In order to determine this point, I procured a fresh pulmonary artery and aorta, and after putting the valves into position with water, caused an assistant to drop liquid plaster of Paris into the vessels. The greater density of the plaster gradually displaced the water, and I was in this way furnished with accurate casts of the sinuses and of the valves. The segments of the semilunar valves, unlike the venous ones, are almost invariably three in number.* They differ in size and in position, and in this respect resemble the sinuses of VALSALVA, to the inside of which they are found. Thus the segment which is smallest is situated anteriorly, and occupies a higher position than either of the others; that which is second in

* Dr JOHN HUGHES BENNETT speaks of a case in which four were present, but whether the additional segment was congenital, or the result of disease, is not easy to determine. "Principles and Practice of Medicine," 1858, p. 550.

size being placed posteriorly, and a little lower than the anterior segment. The remaining segment, which is the lowest, is directed towards the septum. They are flexible, more or less opaque, very strong, and somewhat crescentic in shape (Plate XXVIII. figs. 19 and 20). In structure, the semilunar valves are intermediate between the venous and auriculo-ventricular ones. They consist of a reduplication of the fine membrane lining the vessel, strengthened by certain tendinous bands, and, as was first satisfactorily demonstrated by Mr W. S. SAVORY, of a considerable quantity of yellow elastic tissue.* Some of the older anatomists, among whom may be mentioned LANCISCI,† SENAC,‡ MORGAGNI,§ WINSLOW,‖ and COOPER,¶ believed that they had detected the presence of carneous or muscular fibres; but HALLER,** and many since his time, have gravely doubted the accuracy of their observations. Very recently, Mr MOORE†† has figured two sets of muscular fibres, which he has termed according to their supposed action, dilators and retractors; and Dr MONNERET‡‡ has described two similar sets, which, for like reasons, he has named elevators and depressors. I have sought in vain for the muscular fibres in question, and am inclined to think that when found, they have been mistaken for the tendinous bands accidentally stained with blood. The tendinous bands have hitherto been regarded as following three principal directions,—one band being said to occupy the free margin, and to be divided into two equal parts by the nodulus or Corpus Arantii, otherwise called Corpusculum Morgagni, and Corpus sesamoideum; a second band, proceeding from points a little above the middle of the segment, and curving in an upward direction towards the Corpus Arantii; the third band, which is the thickest, surrounding the attached border of the segment. A careful examination of a large number of mammalian hearts, particularly those of man, has induced me to assign to the semilunar valves a more intricate structure. In a healthy human semilunar valve§§ taken from the pulmonary artery, the following seems to be the arrangement. Proceeding from the attached extremities of the segment above (Plate XXVIII. fig. 19 *b*, fig. 20 *x*, and fig. 29 *a*), and running along its free margin, is a delicate tendinous band, which gives off still more delicate

* PURKINGE and RAEUSCHEL had detected elastic tissue in the Corpora Arantii, but knew nothing of its existence throughout the other portions of the valves. Of its presence I have frequently satisfied myself.

† De motu Cordis.

‡ Traité de la Structure de Cœur, livre i.

§ Adversaria Anatomica Omnia.

‖ Exposition Anat. de la Structure du Corps Humain, p. 592

¶ Myotomia Reformata.

** Elementa Physiologiæ. Liber iv. sect. 10.

†† Med. Gazette, March 8, 1850.

‡‡ Lancet, Dec. 29, 1850.

§§ It is very difficult to get a perfectly healthy human semilunar valve, especially if the patient is at all advanced in years. Out of twenty adult hearts examined by me, nearly a half of that number had the valves abnormally thickened.

slips (Plate XXVIII. figs. 20 and 29 *r*), to radiate in a downward and inward direction, *i.e.* in the direction of the mesial line (Plate XXVIII. fig. 20 *c*) and body of the segment. These fine slips interdigitate in the mesial line, and are attached below to the uppermost of a series of very strong fibrous bands which occupy the body of the segment (Plate XXVIII. figs. 20 and 29 *s*). In the interspaces between the slips, the valve is so thin as to be almost transparent. Those portions of the segments included within the delicate fibrous band, running along the free margin and the uppermost of the stronger bands occupying the body, and which are situated to the right and left of the mesial line, are somewhat crescentic in shape (Plate XXVIII. fig. 20 *r*), and have, from this circumstance, been termed lunulæ. They do not form the perfect crescents usually represented in books, the horns of the crescents directed towards the mesial line of the segment (Plate XXVIII. fig. 20 *c*), being much broader than those directed towards the extremities, or where the segments unite above (Plate XXVIII. fig. 20 *b*). The object of this arrangement is obvious. The crescentic portions referred to, are those which, when the segment is folded upon itself during the action of the valve, are accurately applied to corresponding and similar portions of the two remaining segments (Plate XXVIII. fig. 25 *b b'*). If, however, the lunulæ had been symmetrical, in other words, if they had terminated in well-defined horns towards the mesial line, or where the segments fold upon themselves, then the union between the segments in the axis of the vessel (Plate XXVIII. fig. 25 *x*), where great strength is required, *would have been very partial*, and consequently very imperfect.

Proceeding from the attached extremities of the segments at points a little below the origins of the marginal band, and curving in a downward and inward direction, is the first of the stronger bands (Plate XXVIII. fig. 20 *b*). The band referred to essentially consists of two portions, these splitting up into brush-like expansions as they approach the mesial line (*c*), where they interdigitate and become strongly embraced. Other and similar bands, to the extent of three (*s*) or four, usually the latter number, are met with (Plate XXVIII. fig. 29 *v*), and as they all curve in a downward and inward direction, and have finer bands running between them in a nearly vertical direction, they suspend the body of the segment; so that when water is poured upon it, the various parts of which it is composed, radiate from the attached or convex border like a fan; each band dragging upon that above it; the whole deriving support from the thickened convex border. The bands, which are thus six in number, are best seen on that aspect of the segments which is directed towards the sinuses of Valsalva.* They are thickest at their attached extremities, where they interlace slightly, and are mixed up to a greater or less extent with the pale, soft, flattened fibres, and elastic tissue of the central layer

* The surfaces of the segments directed towards the axis are perfectly smooth, and so facilitate the onward flow of the blood.

of the vessel itself. They in this manner form a fibrous zone (Plate XXVIII. fig. 17 *b n*), which corresponds to the attached convex border of the segment, and may be regarded as an expansion of the inner of the two divisions (Plate XXVIII. fig. 17 *t r*) into which, as I formerly pointed out, the pulmonary artery and aorta resolve themselves.

As the bands under consideration are exceedingly strong when compared with those occurring in other portions of the segment; and project in an inward direction, or towards the axis of the vessel, when the preparation is sunk in water; their function, as ascertained from numerous experiments on the semilunar valves of a whale (*Physalus antiquorum*, Gray), seems to be the following:—

1*st*, *They carry the body of the segment away from the sides of the vessel, and incline the free margins towards each other, at such an angle, as necessitates the free margins of neighbouring segments, being always more or less in apposition.* In this they are assisted by the thickened portion of the pulmonary artery (Plate XXVIII. figs. 17 and 36 *b n*) which projects between the segments (Plate XXVIII. fig. 36 *n n'*) where they unite above, and by the fibrous zones which correspond to the convex border of each segment.

2*d*, *The stronger fibres suspend the body of the segment from above, and permit the reflux of blood to act more immediately upon the mesial line of each segment where thinnest* (Plate XXVIII. fig. 20 *c*), *and where least supported;* to occasion that characteristic folding of the segment upon itself, when the valve is in action.

Other bands, intermediate in thickness between those occupying the free margin and the body of the segment, are found towards its lower portion (Plate XXVIII. figs. 20 and 29 *o*). These bands cross and interdigitate to a greater or less extent, and as their prevailing direction is from below upwards, are instrumental in keeping the *lower portions* of the segment, away from the sides of the vessel. Each segment may therefore be described as consisting of three portions—a superior and thinner portion, an inferior and thicker one, and a central portion, which is the thickest of all. It ought also to be remarked that the three portions of the segment corresponding to its mesial line, where the folding occurs when it is in action, are comparatively thinner than the parts to either side of the line in question. The varying thickness of the segments is well seen in the semilunar valve of the whale (Plate XXVIII. fig. 29 *a v n*).

While the foregoing may be considered a literal description of a healthy human semilunar valve, there are modifications to which it is necessary to direct attention. Thus, in some instances, the band occupying the margin of the segment splits up near its origin, as represented at Plate XXVIII. fig. 19 *b*, and maps out a triangular portion (*c*). This portion is very thin, and contains delicate tendinous fibres, which terminate in brush-shaped expansions towards the mesial line. The remaining and stronger bands are similar to those already described, but the difference in the thickness of the several portions of the valve, is not so marked.

In other cases (Plate XXVIII. fig. 21), the marginal band not only splits up and contains a fully developed Corpus Arantii (*d*), but gives off two or more well-marked tendinous slips (*a*) which connect the free margin with the stronger or central portion of the segment (*i*). Radiating from the Corpus Arantii as a centre (Plate XXVIII. fig. 24 *d*), and proceeding along the free margin, I have sometimes detected a series of hair-like fibres (*e*), which are apparently of use in strengthening this the weakest portion of the segment. This is the more probable, since other and similarly delicate fibres proceed from the attached extremities in the direction of the Corpus Arantii. On other occasions, the tendinous bands proceeding from the marginal one (Plate XXVIII. fig. 22 *s*) are abnormally thickened (*t*), and terminate in brush-shaped expansions in the body of the segment (*v*); the body under such circumstances, projecting in an upward direction towards the Corpus Arantii (*d*). In such cases, those portions of the valve ($\overline{r\,r'}$) which occur between the thickened bands proceeding from the marginal one, are exceedingly thin, and in some diseased conditions altogether awanting, so that the segment very much resembles one of the segments of the mitral or tricuspid valve, with its chordæ tendineæ. That there is an analogy between the semilunar and the mitral and tricuspid valves, and that the chordæ tendineæ is a further development, seems probable from the fact, that in the bulbus arteriosus of certain fishes, as in the grey and basking sharks (Plate XXIX. figs. 41 and 48), Lepidosteus, &c. (Plate XXIX. fig. 40 *b*), the semilunar valves are furnished with what may be regarded as rudimentary chordæ tendineæ, (Plate XXIX., fig. 48. *a*), while in the auriculo-ventricular valves of fishes, which have hitherto been regarded as semilunar, but which exhibit some of the peculiarities of the mitral valve of the mammal, chordæ tendineæ in various stages of development occur.

A scheme of the arrangement of the tendinous bands in the semilunar valves has been given at Plate XXVIII. fig. 23, and shows the segments to be not only bilaterally symmetrical, but to be constructed on a plan which secures the greatest amount of strength with the least possible material; the bands mutually acting upon and supporting each other. Thus the bands marked *a* and *d*, which represent the central portion of the segment, split up into brush-shaped expansions, one portion of each curving in an upward direction (*b e*), and representing the tendinous slips proceeding from the marginal one (*r*); the remaining portions curving in a downward direction (*f c*), and giving the inferior set of fibres which curve from below, towards the body of the segment (*a d*). The Corpus Arantii is rarely present in a perfectly healthy semilunar segment; nor will its absence occasion surprise, when it is remembered that its presence materially interferes with the folding of the segments upon themselves, when the valve is in action. That its existence is not necessary to the perfect closure of the valve, is proved by its complete absence in a great number of cases. In the semilunar valve of the

whale, where one would have naturally expected it in perfection, I could not detect even a trace of it.

What has been said of the semilunar valves of the pulmonary artery, may with equal propriety be said of those of the aorta; the only difference being that the segments are stronger and more opaque, to harmonise with the greater strength of the left ventricle.

The Arterial or Semilunar Valves in Action.

As the manner in which the semilunar valves are closed, does not seem to be well understood, the following experiments conducted with various fluids and liquid plaster of Paris, may prove interesting:—

When the aorta is cut across two inches or so above the aortic semilunar valve, and water introduced, the segments, if watched from beneath, are seen to act with great alacrity, the smallest segment (Plate XXVIII. figs. 26, 27, and 28, *w*), which is situated highest, *descending with a spiral swoop*, and first falling into position; the middle-sized segment (*x*), which is placed a little lower, *descending in like manner, and fixing the first segment by one of its lunulæ or crescentic surfaces* (Plate XXVIII. fig. 26 *a*); the third and largest segment (*v*), which occupies a lower position than either of the others, *descending spirally upon the crescentic margins* (*b c*) *of the other two, and wedging and screwing them more and more tightly into each other*. The spiral movement, as has been already explained, is occasioned by the direction of the sinuses of VALSALVA, which curve towards each other, and direct the blood in spiral waves upon the mesial line of each segment (*w x v*).

It is well seen when liquid plaster of Paris is used, as the plaster, on setting, enables the experimenter to examine the relations of the segments to each other at leisure. Figures 27 and 28, Plate XXVIII., have been taken from specimens so prepared. On removing one of the segments in such specimens, it is found to be folded upon itself (Plate XXVIII., fig. 27 *w*), and to present two semilunar surfaces, each of which is accurately applied to a corresponding and similar surface of that segment of the valve which is next to it (Plate XXVIII. fig. 25 *b b'*). The union, therefore, between any two of the segments of a semilunar valve, is analogous in many respects, to that occurring in a venous valve consisting of two segments. There is, however, this difference; in a venous valve, *the segments, simply flatten themselves against each other in the mesial plane of the vessel*, to form a perpendicular crescentic wall (Plate XXVIII. figs. 11 and 13 *e*); whereas in thes emilunar valves, the segments in addition *curve into each other*, and so form three perpendicular crescentic walls, each of which radiates from the axis of the vessel (Plate XXVIII. fig. 26 *r s o*). In the venous valve, moreover, those portions of the segments which come into apposition *form systemetrical crescents* (Plate XXVIII. fig. 13 *e*);

whereas in the semilunar one, the surfaces referred to, are *non-symmetrical;* in other words, the horns of the crescents forming the lunulæ, are broader towards the mesial line of the segments (Plate XXVIII. fig. 20 *c*) than where they meet above (*b*). As a result of this want of symmetry in the lunulæ or opposing surfaces of the semilunar valves, the apices or central portions of the segments come together in the axis of the vessel *throughout a considerable space* (Plate XXVIII. fig. 25 *x*), and form a union of the most perfect description. The extent of the union increases in the inverse of the pressure applied (compare dotted lines, Plate XXVIII. fig. 25 *m m'* with plain ones *b b'*), and is rendered very secure from the segments being wedged into each other in a direction from above downwards and from without inwards (Plate XXVIII. fig. 26 *v w x*). Retzius,* who figures the manner of closure of the semilunar valves, does not seem to have been aware of this fact, for he represents the segments as coming together in the axis of the vessel at three points, an arrangement which could scarcely fail to occasion a certain amount of regurgitation. If the closure of the semilunar valves be watched from above, other phenomena are observed. When, for example, the aorta and semilunar valve of the whale were sunk in water and permitted to remain undisturbed, the thicker portions of each segment were seen to project in an upward and inward direction, the free margins being by this arrangement brought more or less closely into contact, and supported on a level corresponding to the top of the sinuses of Valsalva. When, however, the preparation was raised in the vessel, so that the water acted from above on the central and more unsupported portions of the segments; the free margins, together with the more moveable parts of the bodies, descended to the extent of fully an inch and a half. In so doing, the free margins of the segments were projected against and accurately applied to each other, clearly showing that the fluid, because of its weight and the spiral downward and inward direction communicated to it by the sinuses of Valsalva, is sufficient to effect the closure. When the closure was taking place, the segments fell into position in rotation, but at so nearly the same interval of time, that they mutually regulated the amount of downward and inward movement; and so prevented each other from protruding too far into the interior of the vessel. When the hand was introduced into the aorta, which the great size of the specimen† readily permitted, and one of the segments was pushed in an outward direction, it was found to apply itself to the sinus of Valsalva behind it, with more or less accuracy; the extremities of the segments, where they unite above, projecting to form three ridges, which are *spirally inclined with reference to each other*, and are no doubt useful in directing the blood into the aorta proper. From the foregoing description of the venous and arterial semilunar valves in mam-

* Om Mekanismen af Semilunar Valvlernes tillolutning.

† In this case, the aorta had a girt of 27 inches; the average size of the segments being 9 inches by 7.

malia, it will be evident that there is nothing, either in their structure or relations, to betoken any great degree of activity on their part. That these structures are, on the contrary, principally passive, seems certain from the fact, that a stream of water or other fluid directed upon them from above as recommended, at once closes the orifices which they guard.

Structure of the Bulbus arteriosus of the Fish, and of the Ventricle of the Fish and Reptile; Semilunar and other Valves found therein.

The semilunar valves in the bulbus arteriosus of the fish, and the auriculo-ventricular valves in the fish and reptile, differ from the venous and arterial ones, in being, for the most part, connected either directly or indirectly, or exposed in some way to the influence of muscular contractions. In order the better to understand the position which these valves occupy in the gradually ascending scale of valvular arrangements, a brief description of the bulbus arteriosus, of the fish, and of the ventricle of the fish and reptile, is necessary. In the ventricle of the fish, the fibres, as I have pointed out elsewhere,* consist of three layers;—an external layer, in which they proceed from base to apex, and occasionally interdigitate and become strongly embraced; an internal layer, in which they are aggregated into fascicular bundles, and have a more or less vertical reticulated arrangement; and a central layer, in which they run transversely, or at right angles to the fibres of the external and internal layers. These layers are connected to each other by certain fibrous bands, which run in a direction from without inwards. Rising from the base of the ventricle anteriorly is a muscular structure of a more or less bulbous form, the so-called bulbus arteriosus (Plate XXIX. figs. 38, 39, 40, 41, and 48), the arrangement of the fibres in which, resembles that in the ventricle itself. The ventricle of the fish, and the bulbus arteriosus contract in every direction, and in this respect they are analogous to the veins and arteries, which, as John Hunter showed, are extensible and retractile, both in their length and breadth. One point to be noted in the ventricle of the fish, is the absence of musculi papillares; the auriculo-ventricular valves being so placed, that certain of the fasciculi constituting the internal layer, run parallel to them, and extend, in not a few instances, into their substance. The effect of this arrangement is to modify the action of the valves in question; and I direct attention to the circumstance, because of the purely mechanical views entertained by some with regard to them; views which to me appear inconsistent with the nature of the textures involved. The arrangement of the fibres in the ventricle of the reptile is nearly the same as that in the fish. There is, however, this difference, and it is worthy of mention as bearing directly upon the structure and function of the auriculo-ventricular valves in this

* On the Arrangement of the Muscular Fibres, in the Ventricles of the Vertebrate Heart, with Physiological Remarks.—*Phil. Trans.*, vol. 154, pp. 445–47.

class of animals. The fibres of the external and internal layers pursue a slightly spiral course. The spiral direction of the fibres here indicated is so marked in the ventricles of the bird and mammal, as to influence not only the position of the musculi papillares and carneæ columnæ, but also the shape of the ventricular cavities, and the closure of the mitral and tricuspid valves. In the bulbus arteriosus of the fish, the valves as a rule, may be said to be fairly within the range of muscular influence, and it is interesting to note that in this structure, the segments vary both as regards number, size, and shape. Thus, in the frog-fish (*Lophius piscatorius*), the origin of the bulbus arteriosus is guarded by a semilunar valve, consisting of two ample and very delicate segments (Plate XXIX. fig. 47 *a*), resembling those found in the middle-sized veins (Plate XXVIII. fig. 3; compare with *a b*); while in the sun-fish (*Orthagoriscus mola*, Schneider), the same aperture is guarded by a semilunar valve, consisting of three segments (Plate XXIX. fig. 43 *a b c*); the segments being analogous in every respect to those found in the largest veins (Plate XXVIII. fig. 1; compare with *a b c*). As the valve in these cases is situated between the bulbus arteriosus and the ventricle, and surrounded by a fibrous ring similar to that occurring at the origin of the pulmonary artery and aorta, it is not affected by the structures between which it is situated to any great extent. The semilunar valves in the frog-fish and sun-fish, may therefore be regarded as connecting links between the venous and arterial ones in the bird and mammal; and that more complex system of analogous valves, which is found in the bulbus arteriosus of the fish generally. In the bulbus arteriosus of the skate (*Raia batis*), the segments occupy the whole of the interior of the bulb, and are arranged in three pyramidal rows of five each (Plate XXIX. fig. 38 *a b c*). As the segments in this instance are very small, and altogether inadequate to the obliteration of the bulbus cavity, they must be looked upon as being useful only in supporting the column of blood in its onward progress; it being reserved for the segments at the termination of the bulb, which are larger and more fully developed, to effect the closure. The action of the segments in the bulbus arteriosus of the skate, is rendered more perfect by the pressure from without, caused by the contraction of the bulb itself (*d*). In the bulbus arteriosus of the sturgeon (*Accipenser sturio*), the segments are arranged in three rows of eight each (Plate XXIX. fig. 39 *a*). They are more delicate, and less perfectly formed than in the skate. In the bulbus arteriosus of the American devil-fish (*Cephalopterus giorna*), they increase to thirty-six, are more imperfect than in any of the others, and are supported by three longitudinal angular muscular columns. As these segment-bearing columns, from their shape, project into the cavity, so as almost to obliterate it during the contraction of the bulb, they in this way bring the free margins of the segments together. The orifices of the bulbus arteriosus, however, are not closed by the imperfect segments referred to; these being guarded by two well-formed and fully developed tri-semilunar

valves, the one of which is situated at the beginning, the other at the termination of the bulb. In the bulbus arteriosus of the grey shark (*Galeus communis*), we have a slightly different arrangement, the two rows of segments of which the valve is composed being connected to each other by means of tendinous bands, resembling chordæ tendineæ (Plate XXIX. fig. 48 *a*). In the bulbus arteriosus of the Lepidosteus (Plate XXIX. fig. 40 *b*), and that of the basking shark (*Selachi maxima*, Cuv.), (Plate XXIX. fig. 41 *b*), the same arrangement prevails; the segments being stronger and less mobile, and the tendinous bands which bind the one segment to the other, more strongly marked than in the grey shark. As the tendinous bands referred to are not in contact with the wall of the bulbus arteriosus, but simply run between the segments, and are in some instances, as in the basking shark, very powerful (Plate XXIX. fig. 41 *b*), they must be regarded in the light of sustaining or supporting structures; their function being probably to prevent eversion of the segments. Other examples might be cited, but sufficient have been adduced to show, that the nature, as well as the number and arrangement of the segments, is adapted to the peculiar wants of the structure in which they are situated; and it ought not to be overlooked, that when a multiplicity of segments are met with in an actively contracting organ, the two act together or in unison.

If we now direct our attention to the auriculo-ventricular valves of the fish and reptile, similar modifications as regards the number of the segments, and the presence or absence of chordæ tendineæ and analogous structures, present themselves. Thus in the heart of the serpent (*Python tigris*), the two crescentic apertures by which the blood enters the posterior or aortic division of the ventricle, are each provided *with a single semilunar valve*. The same may be said of the aperture of communication, between the left auricle and ventricle of the crocodile (*Crocodilus acutus*) and of the sturgeon (*Accipenser sturio*, Linn.) In the heart of the Indian tortoise (*Testudo Indica*, Vosmaer), the left auriculo-ventricular orifice is guarded *by a single membranous fold*, the right orifice having in addition *a slightly projecting semilunar ridge*, which extends from the right ventricular wall, and may be regarded as the rudiment of the fleshy valve which guards the same aperture in birds (Plate XXIX. fig. 45 *g h*). In the heart of the bulinus, frog-fish, American devil-fish, grey shark, and crocodile, the auriculo-ventricular orifice is guarded by a semilunar valve consisting *of two cusps or segments;* while in the sturgeon, sun-fish, and others, it is guarded *by four, two larger and two smaller*.

So much for the number of the segments constituting the auriculo-ventricular valves in fishes and reptiles; but there are other modifications which are not less interesting physiologically. In the bulinus, frog-fish, and crocodile, the segments of the valves are attached to the auriculo-ventricular tendinous ring, and to the sides of the ventricle, and *have no chordæ tendineæ*. In the sun-fish

(Plate XXIX. fig. 43 *f*), the valve is destitute of chordæ tendineæ likewise; but in this instance the muscular fibres are arranged *in the direction of the free margin of the segments of the valve*, and no doubt exercise an influence upon them. In the grey shark the membranous folds forming the segments, *are elongated at the parts where they are attached to the ventricular walls*, these elongated attachments being more or less split up, so as to resemble *chordæ tendineæ*.

In the American devil-fish the semilunar valve consists of two strong well-developed membranous folds, which, like the preceding, are attached by elongated processes to the interior of the ventricular wall; *these processes consisting of distinct tendinous slips*, which are attached *to rudimentary musculi papillares*.

In the sturgeon (Plate XXIX. fig. 37), *three tendinous chords* (*b*) *from rudimentary musculi papillares*, are seen to extend into the half of each of the segments; while in the left ventricle of the dugong, *six chords, proceeding from tolerably well-formed musculi papillares, are distributed to the back, and six to the margins of each of the segments*. It is, however, in the bird and mammal, particularly the latter, that the musculi papillares are most fully developed, and the chordæ tendineæ most numerous—the number of tendinous chords, inserted into each of the segments, amounting to eighteen or more (Plate XXVIII. fig. 33 *r r'*, *s s'*). As the auriculo-ventricular valves are attached either to the interior of the ventricle, or to the musculi papillares or carneæ columnæ, it is plain that the contraction of the ventricle must influence them to a greater or less extent. That, however, the presence of muscular substance in no way interferes with the efficiency of the valves, is proved by the fact, that some valves are partly muscular and partly tendinous, a few being altogether muscular. Thus, in the heart of the cassowary, the right auriculo-ventricular orifice is occluded by a valve, which is partly muscular and partly tendinous; the muscular part, which is a continuation of two tolerably well-formed musculi papillares, extending into the tendinous substance of the valve, where it gradually loses itself. In the right ventricle of the crocodile (Plate XXIX. fig. 42 *r*), a muscular valve, resembling that found in the right ventricle of birds, exists.

In birds the muscular valve (Plate XXIX. figs. 45 and 46 *g h i*) is usually described as consisting of two parts, from the fact of its dependent or free margin (*g*) being divided into two portions by a spindle-shaped muscular band (*h*), which connects it with the right ventricular wall (*j*). As, however, the wall consists of one continuous fold towards the base (*i*), and the two portions of the margin are applied during the systole not to each other but to the septum (*e' e'' e'''*), it is more correct to say that the valve is single; the spindle-shaped muscular band representing the musculus papillaris of the right wall of the ventricle with its attached chordæ tendineæ.* In the serpent, the opening between the right

* For the relations, structure, and function, of the muscular valve in birds, see paper already referred to. Phil. Trans. vol. 154, pp. 470–1–2.

and left ventricle, occurs as a spiral slit in the septum (Plate XXIX. fig. 44 *r*), and is guarded by two projecting muscular surfaces, which are rounded off for this purpose. The opening into the left ventricle also occurs as a muscular slit (*s*); and the orifices of many of the venous sinuses are closed by purely muscular adaptations; the fibres in such instances running parallel to the slit-like opening (Plate XXVIII. fig. 10), and being continuous with two or more bundles of fibres (*b c*), which supply the place of musculi papillares. From the great variety in the shape and structure of the auriculo-ventricular valves, and from the existence in almost all of tendinous chords, which connect them with actively contracting textures, there can, I think, be little doubt, that they possess an adaptive power peculiar in a great measure to themselves; this power being traceable to the contractile properties residing in muscle.

As it would greatly exceed the limits of the present paper, to give a detailed account of the structure of the numerous auriculo-ventricular valves, to which allusion has been made, I have selected for description the auriculo-ventricular valves of the mammal, and those of man more particularly. Before, however, entering upon this the most difficult part of the present investigation, a brief account of the arrangement of the muscular fibres in the ventricles seems indispensable; these, as has been explained, modifying the action of the valves to a very considerable extent.

Arrangement of the Muscular Fibres in the Ventricles of the Mammal—Shape of the Ventricular Cavities, &c.

The fibres of the ventricles in the mammal, as I have ascertained from numerous dissections,* are arranged in seven layers; three external, a fourth or central, and three internal. The fibres constituting these layers in the left ventricle, to which these remarks more particularly apply, pursue a spiral direction; the external fibres becoming more and more oblique, in a direction from left to right downwards, as the central layer is approached—the internal fibres becoming more and more vertical, in a direction from right to left upwards, as it is receded from. The fibres, therefore, of corresponding external and internal layers, cross each other. The fibres of the several layers are further arranged in two sets; the two sets, forming each of the external layers, being continuous at the apex and at the base, with two similar sets belonging to a corresponding internal layer. This arrangement of the fibres renders the ventricles bilaterally symmetrical, and in part accounts for the great precision with which the heart acts, and for its rolling movements. Its bearing on the action of the organ is obvious, for as muscular fibres contract in the direction of their length, the more vertical external and internal fibres, diminish the ventricular cavities from above downwards, and from

* Of these upwards of an hundred are preserved in the University of Edinburgh Anatomical Museum, where they may be examined. For a detailed description of the specimens, and for accurate representations thereof, see Phil. Trans. vol. 154, pp. 445–500, Plates 12 to 16.

below upwards; the downward movement preceding the upward by an almost inappreciable interval of time. In that brief space, however, which elapses between the downward and upward movements, the ventricles, owing to the contraction of the more circular fibres, are visibly diminished from without inwards; and it is important to note this circumstance, as the auriculo-ventricular orifices are, at this instant, reduced in size, and the mitral and tricuspid valves, consequently liable to a certain amount of displacement. The ventricular wall of the left ventricle, as was known to GERDY and other investigators, is thickest at the upper part of its middle third (Plate XXVIII. fig. 35 *j*), and tapers towards the apex (*w*) and base (*v'*) respectively; and it is interesting to observe that the thickest part of the ventricular wall, corresponds with the widest portion of the ventricular cavity, whence the blood is projected into the aorta; a fact of some significance, since the contractions at this point are necessarily more intense than at any other. As the two sets of fibres composing the first external layer are continuous at the left apex with the two sets of fibres forming the carneæ columnæ and musculi papillares, and these structures, especially the latter, bear an important relation to the segments of the bicuspid valve, with which they are connected by the chordæ tendineæ, a more minute description than that given of the other layers, is requisite for clearness. On looking at the left auriculo-ventricular opening (Plate XXVIII. fig. 30 *b*), the fibres of the first layer are seen to arise from the fibrous ring surrounding the aorta (*a*), and from the auriculo-ventricular tendinous ring (*n*) in two divisions; the one division (*d*) proceeding *from the anterior portions of the rings*, and winding in a spiral nearly vertical direction, from before backwards, to converge and enter the apex *posteriorly;* the other set (Plate XXVIII. fig 30 *f*) proceeding *from the posterior portions of the rings*, and winding in a spiral direction from behind forwards, to converge and enter the apex *anteriorly*. Having entered the apex, the two sets of external fibres are collected together, and form the musculi papillares and carneæ columnæ; the one set, viz., that which proceeded from the auriculo-ventricular orifice anteriorly and entered the apex posteriorly, curving round in a spiral direction from right to left upwards, and forming *the anterior musculus papillaris* (Plate XXIX. fig. 50 *y*), *and the carneæ columnæ next to it;* the other set, which proceeded from the auriculo-ventricular orifice posteriorly, and entered the apex anteriorly, curving round in a corresponding spiral direction, and forming *the posterior musculus papillaris* (Plate XXIX. figs. 50 and 51 *x*), *and adjoining carneæ columnæ*. As the external fibres converged on nearing the apex, so the internal continuations of these fibres radiate towards the base; and hence the conical shape of the musculi papillares. I am particular in directing attention to the course and position of the musculi papillares, as they have hitherto, though erroneously, been regarded as simply vertical columns, instead of more or less vertical *spiral* columns.* The necessity for in-

* Plate XXIX. fig. 49, *x y*, gives the spiral track of the musculi papillares.

sisting upon this distinction will appear more evident when I come to speak of the influence exerted by these structures on the segments of the bicuspid valve. It is worthy of remark, that while the left apex is closed by two sets of fibres, the left auriculo-ventricular orifice is occluded during the systole by the two flaps or segments constituting the bicuspid valve (Plates XXVIII. and XXIX. figs. 28 and 51 *m n*). The bilateral arrangement, therefore, which obtains in all parts of the ventricle and in the musculi papillares, extends also to the segments of the valve in question. What has been said of the arrangement of the fibres in the left ventricle, applies with slight modifications to the fibres of the right one; and many are of opinion (and I also incline to the belief) that the tricuspid valve, is in reality bicuspid in its nature (Plate XXVIII. fig. 34 *m n;* and Plate XXIX. fig. 51 *g h*). The shape of the ventricular cavities of the heart of the mammal greatly influences the movements of the mitral and tricuspid valves, by moulding the blood into certain forms, and causing it to act in certain directions. It is seen to advantage when the ventricles are filled with wax or plaster of Paris, and the ventricular parietes removed to expose the casts or moulds thus obtained (Plate XXIX. figs. 50 and 51).

The form of the left ventricular cavity, which I regard as typical, is that of a cone twisted upon itself (Plate XXIX. fig. 49); the twist or spiral running from left to right of the spectator, and being especially well marked towards the apex.* The cone tapers slightly towards its base (*b*), and the direction of its spiral corresponds with the direction of the fibres of the carneæ columnæ and musculi papillares (Plate XXIX. fig. 50 *x y*). As the two spiral musculi papillares project into the ventricular cavity, it follows that between them, two conical-shaped spiral depressions or grooves, are found (Plate XXIX. fig. 49 *q j*). These grooves, which are especially distinct, are unequal in size; the smaller one (Plate XXIX. figs. 49 and 50 *j*) beginning *at the right side of the apex*, and winding in an upward spiral direction, to terminate *at the base of the external or left and smaller segment of the bicuspid valve* (Plate XXIX. figs. 50 and 51 *n*); the larger groove (Plate XXIX. fig. 49 *q*) beginning *at the left side of the apex*, and pursuing a similar direction, to terminate *at the base of the internal or right and larger segment* (Plate XXIX. fig. 51 *m*).

Running between the grooves in question, and corresponding to the septal aspect of the ventricular cavity, is yet another groove, larger than either of the others (Plate XXIX. fig. 51 *q*). The third or remaining groove winds from *the interior of the apex posteriorly*, and conducts *to the aorta* (*a*), which, as the reader is aware, *is situated anteriorly*. The importance of these grooves physiologically cannot be over-estimated, for I find that in them the blood is arranged or moulded *into three spiral columns*, and that towards the end of the diastole and the beginning of the systole, the blood in the two lesser ones is forced in two spiral streams upon the segments of the bicuspid valve, which are in this

* In this description the heart is supposed to be placed on its apex.

way progressively elevated towards the base, and twisted and wedged into each other, until regurgitation is rendered impossible (Plate XXIX. fig. 51 *m n*). When the bicuspid valve is fairly closed, the blood is directed towards the third and largest groove, which, as has been stated, communicates with the aorta. The spiral action of the mitral valve, and the spiral motion communicated to the blood when projected from the heart, is due to the spiral arrangement of the musculi papillares and fibres composing the ventricle, as well as to the spiral shape of the left ventricular cavity. These points are determined in the following manner:— When a cast of the interior of the left ventricle is made, by introducing liquid plaster of Paris into the left ventricular cavity, by means of a tube inserted into the aorta and reaching to the left apex, it is found, on cutting away the parietes of the ventricle, that the segments of the mitral valve are borne up on the plaster,* and wedged into each other on a level with the ventricular orifice. It is further found, that the two spiral streams of plaster (now spiral columns) which closed the segments of the mitral valve, merge towards the base, into the third column, communicating with the aorta. That portion, therefore, of the left ventricular cavity (Plate XXIX. figs. 50 and 51 *o*), which corresponds to the conus arteriosus or infundibulum of the right one (Plate XXIX. fig. 50 *i*), is conical in form. It is moreover furnished with three conical-shaped spiral depressions, which in the cast appear as conical-shaped *spiral prominences* (*p o*), and are continuous with the three spiral columns of plaster proceeding from the apex of the ventricle. As the apices of the three conical-shaped infundibuliform prominences referred to are directed between the three segments of the aortic semilunar valve, the blood from this arrangement must on its onward progress throw the semilunar segments hastily apart, by causing them to fall back upon the spirally disposed sinuses of Valsalva (*k w v*). The spiral channel, which is thus provided for the blood, is not confined to the heart, but extends for a short distance into the great vessels. As the semilunar valves are closed by a reverse movement to that by which they are opened, it is not difficult to perceive how the spiral action of the segments constituting them is induced.

What has been said of the left ventricular cavity and aorta, applies, with slight alterations, to the right ventricular one and the pulmonary artery (Plate XXIX. figs. 50 and 51 *c i*), the cone formed by the right cavity being flattened out and applied to or round the left.

Intricate Structure of the Mitral and Tricuspid Valves in Mammalia; Relations of the Cordæ tendineæ to the Segments and to the Musculi papillares.

The auriculo-ventricular valves are composed of segments, which differ in size, and are more or less triangular in shape. They are much stronger than

* In order to see the spiral movement of the segments to advantage, the plaster ought to be made very thin. Should any difficulty occur, the experimenter is recommended to use water until he is familiar with the phenomena to be observed.

the segments of the semilunar valves, which in some respects they resemble in structure as well as function. They are very dense, and quite opaque, unless at the margins and apices, where they are frequently remarkably thin. They unite at the base where thickest, to form a ring, which is attached to one or other of the fibrous rings surrounding the auriculo-ventricular orifices (Plate XXVIII. fig. 30 *n n'*). The auriculo-ventricular rings, which are consequently intimately related to the segments, have been variously described, the majority of investigators regarding them as highly developed structures, which afford attachment, not only to the valves, but to all the fibres of the auricles and ventricles. A careful examination of the rings in question in boiled hearts, has led me to a different conclusion. They afford attachment to the fibres of the auricles (Plate XXVIII. fig. 35 *y d*), and to the valves (*m n*), *but to almost none of the fibres of the ventricles* (*v*). They are most fully developed anteriorly, and on the septum, where they form a dense fibrous investment. The left ring, like everything else pertaining to the left ventricle, is more fully developed than the right; but neither the one nor the other can compare in breadth, or thickness, with either of the arterial rings (*a k*). The influence, therefore, which they exert on the dilatation and contraction of the auriculo-ventricular orifices (Plate XXVIII. fig. 30 *b l*) must be immaterial. The position of the segments in the auriculo-ventricular orifices, and their relation to the musculi papillares, is deserving of attention. On looking into the auriculo-ventricular orifice of the left or typical ventricle, when the clots which usually fill the ventricular cavity have been removed, it is found to be partially obliterated, by two principal segments; the one of which is larger than the other (Plate XXVIII. fig. 28 *m*). The larger segment, which is obliquely suspended between the auriculo-ventricular and aortic openings, occupies a somewhat *internal and anterior position;* while the smaller one (*n*), which runs parallel to it, occupies a more or less *external and posterior position.* Between the principal segments, two smaller accessory segments, are usually found. In the right ventricle, the principal segments are three in number, and are of different sizes,—the smallest running parallel to the septum; the largest being placed anteriorly and inclined to the right side; the one which is intermediate in size occupying a more posterior position. These, also, have smaller accessory segments placed between them. The segments, whatever their size, are attached by their bases to the auriculo-ventricular tendinous rings (Plate XXVIII. fig. 30 *n n'*), and, by their margins and apices, to the spiral musculi papillares (Plate XXVIII. fig. 28 *a b c d*), by means of the chordæ tendineæ.

The segments of the mitral valve, to which the following description, drawn from an extensive examination of mammalian hearts,* more particularly applies,

* Of the hearts examined may be mentioned those of man, the elephant, camel, whale (*Physalus antiquorum*, Gray), mysticetus, horse, ox, ass, deer, sheep, seal, hog, porpoise, monkey, rabbit, and hedgehog.

consist of a reduplication of the endocardium, or lining membrane of the heart, containing within its fold, large quantities of white fibrous tissue, and, as was pointed out by Mr W. S. Savory, and after him, by Professor Donders,* of a moderate amount of yellow elastic tissue. The white fibrous tissue greatly preponderates, and is derived principally from the chordæ tendineæ, which split up into a vast number of brush-shaped expansions, prior to being inserted into the segments. The fibrous or tendinous expansions, which assume the form of bands, may consequently be regarded as prolongations of the chordæ tendineæ. They are analogous, in many respects, to similar bands in the semilunar valves; the only difference being, that in the semilunar valves, the bands referred to, instead of being free, as in the present instance, are involved in the valvular substance. (Compare fibrous bands marked in Plate XXVIII. fig. 20, with chordæ tendineæ and brush-shaped expansions, marked *s* in Plate XXVIII. fig. 33). As each of the segments composing the bicuspid or mitral valve, like the left ventricle itself, is bilaterally symmetrical, it will be convenient, when speaking of these structures, to describe, in the first instance, only the half of one of the segments; and in order to do this the more effectually, it will be necessary to consider each musculus papillaris, as essentially consisting of two portions;† *a superior and external portion* (Plate XXVIII. figs. 28 and 33 *a*), which gives off two, usually three chordæ tendineæ (*s*) to *that half of the anterior segment* of the mitral valve (*m*) which is next to it; and *an inferior and internal portion* (*b*), which also gives off three tendinous chords; these being inserted into the adjacent *half of the posterior segment* (*n*). The three tendinous chords which proceed from the superior external portion (Plate XXVIII. fig. 31 *r*), subdivide, and are inserted, by the brush-shaped expansions spoken of, into the half of the anterior segment posteriorly, in nine different places.‡ Of these, three are inserted into the mesial line of the segment (Plate XXVIII. figs. 31, 32, and 33, *r*), viz., into the base (*g*), central portion (*f*), and apex (*e*); three into the basal, central, and apicial portion of the free margin (Plate XXVIII. fig. 33 *s'*); and three into intermediate points between the mesial line and the margin (*r'*). On some occasions, as in the mitral valve of the whale, a slightly different arrangement prevails; three chordæ tendineæ being inserted into the mesial line of the segment at the base, at the centre, and at the apex; an additional chord *going to the free margin near the apex;* three into intermediate points between the mesial line and the margin; a second additional chord *going to the central*

* Professor Donders describes the yellow elastic tissue as being most abundant in the upper surface of the segments.

† The musculi papillares in the human and other hearts (Plate XXVIII. figs. 28, 31, and 33, *ab*, *cd*) either bifurcate, or show a disposition to bifurcate at their free extremities, so that the division of the chordæ tendineæ into two sets is by no means an arbitrary one.

‡ The number of insertions vary in particular instances. In typical hearts, however, it is remarkably uniform.

portion of the margin. A third and independent chord, *goes to the base of the margin.* In the whale, as will be observed, the arrangement is virtually the same as that first given; the insertions being nine in number,—three into the mesial line, three into the free margin, and three into intermediate points. The tendinous chords pursue different directions prior to insertion; the three which are inserted into the mesial line of the segment, and are the longest and strongest (Plate XXVIII. figs. 31, 32, and 33 *r*), being less vertical than those which are inserted nearer the margin (*r'*); these in turn being less vertical than the ones inserted into the margin, which are the shortest and most delicate. The basal chord of each set, on the contrary, is more vertical than that beneath it, or nearer the apex (*x*); the apicial chords being more or less horizontal. As there is a disposition on the part of the higher and more central chordæ tendineæ (Plate XXVIII. fig. 31 *g*) to overlap, by their terminal brush-shaped expansions, those below and to the outside of them, the segment is found to diminish in thickness from the base (*z*) towards the apex (*x*) and from the mesial line towards the periphery or margin; the basal and central portions of the segment being comparatively very thick, the apicial and marginal portions very thin (Plate XXVIII. fig. 32 *v x*); so thin, indeed, that in some hearts, particularly in the right ventricle, they present a cobwebbed appearance. As the marginal portions form the counterparts of the lunulæ in the semilunar valves, and are those parts of the segments which come into accurate apposition when the valve is in action, they are, from this circumstance, entitled to consideration. When a perfectly healthy mitral valve from an adult, or, still better, from a fœtus at the full time or soon after birth, is examined, the portions referred to are found to be of a more or less crescentic shape (*vide* that part of the mitral valve to which the chordæ tendineæ marked *r'*, fig. 33, Plate XXVIII., are distributed), and so extremely thin, that the slightest current in the fluid in which they are examined causes them to move like ciliæ. The physiological value of this delicacy of structure, and consequent mobility, is very great; as the most trifling impulse causes the marginal parts of the segments, which are naturally in juxtaposition, to approach towards or recede from each other. The half of one of the segments of the mitral valve may be regarded as consisting of a reduplication of the endocardium or lining membrane of the heart, supported or strengthened in all directions by nine or more tendinous brush-shaped expansions; these expansions being arranged in three vertical rows of three each (Plate XXVIII. fig. 33), with much precision, and according to a principle which is seldom deviated from. In addition to the reduplication of the lining membrane and the tendinous expansions referred to, LANCISI,* SENAC,† and KÜRSCHNER‡ have ascertained that there is a

* De Motu Cordis. † Traité de la Structure du Cœur, livre i. p. 76.
‡ WAGNER's Handwörterbuch, art. "Herzthäjtigkeit."

slight admixture of true muscular fibres.* As the tendinous expansions of the half of the segment described, bifurcate or split up, and run into similar expansions from the other or remaining half of the same segment, to become strongly embraced in the mesial line; a complete segment may be described as consisting of a reduplication of the endocardium or lining membrane, enclosing in its fold certain muscular fibres, and eighteen or more tendinous expansions; the chordæ tendineæ, on which these expansions are situated, proceeding from the anterior and posterior musculi papillares equally (Plate XXVIII. fig. 28 *a c*), and pursuing different directions, to meet in the mesial line and form angles (Plate XXVIII. fig. 32) which become more and more obtuse in a direction from above downwards. When, therefore, a segment is examined by being held against the light, or by the aid of a dissecting lens or microscope, it is found to consist of tendinous striæ running transversely, obliquely, and more or less vertically; the striæ of opposite sides being so disposed that they mutually act upon and support each other; an arrangement productive of great strength, and one which secures that the segments shall be at once tightened or loosened, by the slightest contraction or relaxation of the musculi papillares. The intermediate accessory segments of the mitral valve, resemble the principal ones, in structure and general configuration. They are, however, comparatively speaking, very thin; and the chordæ tendineæ inserted into them differ from those inserted into the principal segments, in having a more vertical direction, and in being longer and more feeble. The description given of the bicuspid valve, applies, with trifling alterations in particular instances, to the tricuspid, if allowance be made for an additional large segment, and three or more accessory segments. With regard to the smallest of the three large segments forming the tricuspid, I have to observe, that in all probability, it is simply an over-developed accessory segment; the so-called tricuspid valve being in reality a bicuspid one (Plate XXVIII. fig. 34 *m n*). Nor is this to be wondered at, when it is stated† that the right ventricle, is a segmented portion of the left, and partakes of its bilateral symmetry even in matters of detail. The opinion here advanced is by no means new, but it appears to me that the point has not been sufficiently investigated, and we are in want of statistics regarding it. In ten human hearts which I examined for this purpose, no less than four had well-marked bicuspid valves in both ventricles; and on looking over a large collection of miscellaneous hearts in the Museum of the Royal College of Surgeons of England, I found that nearly a third of them had the peculiarity adverted to; if indeed that can be called peculiar which seems to me to be typical. When two principal segments, with two or more accessory segments, occlude the right auriculo-ventricular orifice, as happens in Plate

* According to Mr Savory's observations, the muscular fibre is found more particularly at the upper or attached border of the valves.

† Phil. Trans., vol. cliv., pp. 464–67.

XXVIII. fig. 34 *m n*, the distribution of the chordæ tendineæ (*o p*) in the segments is the same as that given when describing the bicuspid of the left ventricle; but when there are three principal segments, with as many or more accessory segments, the distribution is varied to meet the exigencies of the case; there being a tendency in each of the chordæ tendineæ to divide into three; one of the chords so divided being inserted by one of its slips into the mesial line of the segment at the base posteriorly; by another into the margin of the segment, likewise at the base; and by the remaining slip into a point intermediate between the mesial line and the margin. Other chords, similarly divided, are inserted at intervals in a direction from above downwards, or from base to apex (Plate XXVIII. fig. 34 *o*), the insertions in each case proceeding from the mesial line towards the margin. In some instances, though more rarely, a mixed arrangement prevails; the insertions of the three tendinous slips running from the mesial line of the segment towards the margin, and from the base to the apex indiscriminately; but whatever the arrangement on the one side of a segment, it is, as a rule, the same on the other, so that one of the segments of a true tricuspid valve is as symmetrical in its way as a segment of a bicuspid one. The musculi papillares in the right ventricle of a typical heart, are two in number, as in the left. When, therefore, the right auriculo-ventricular opening is closed by a tricuspid valve, an additional origin is required for the chordæ tendineæ; and in such a case these chords spring from the two musculi papillares, and from the right side of the septum behind the fleshy pons, either from a rudimentary papillary muscle, or from carneæ columnæ, or from the smooth wall of the septum. The number of papillary muscles in the right ventricle (and the same remark applies to the left, although not to the same extent) vary somewhat; the two typical ones being frequently seen to bifurcate at their free extremities, and others to spring up in their vicinity. On these occasions the origins of the chordæ tendineæ are increased, but this does not materially affect their insertion, which is remarkably uniform. The tricuspid valve, differs from the bicuspid as regards actual strength, the segments being comparatively thinner. This delicacy of structure, extends to the chordæ tendineæ and to the musculi papillares, in fact, to everything pertaining to the right ventricle; the walls of which, as is well known, are only half the thickness of those of the left ventricle. The comparative feebleness of the tricuspid valve, is no doubt traceable to the smaller amount of force it is called upon to withstand, the pulmonic circulation being less vigorous than the systemic. From the foregoing description, it will be seen, that the chordæ tendineæ are inserted into every portion of the bicuspid (Plate XXVIII. figs. 31, 32, and 33) and tricuspid valves; and as they freely decussate with each other in all directions, by means of their terminal brush-shaped expansions, and are of infinite variety as regards length and strength; those at the base and posterior aspect of each segment being long and exceeding strong; while those

at the margins and towards the apices are short, and in some instances as delicate as hairs, it follows that every part of the valves in question, bears a graduated relation to, and is under the control and domination of, the conical-shaped spiral musculi papillares, whose power to contract is now well established.* It is therefore my impression, and my belief is shared by others, that the chordæ tendineæ ought to be regarded as the satellites of the actively contracting musculi papillares, under whose guidance they have to perform, not only a very important, but a very delicate function, and one which could not by any possibility, be accomplished by a simply mechanical arrangement.

The Mitral and Tricuspid Valves of the Mammal in Action.

The theories which have long divided the attention of physiologists with regard to the action of the mitral and tricuspid valves, are two in number; one sect maintaining, that the valves *are acted upon mechanically by the blood*, as if they were composed of inanimate matter; the other believing, that *they form part of a living system*, their movements being traceable to their connection with the musculi papillares, which are actively contracting structures.

In the mechanical theory, the segments of the valves are supposed to be *passively* floated up by the blood, which acts upon them from beneath during the systole, and brings their edges or free margins into such accurate apposition as enables the segments completely to occlude the auriculo-ventricular orifices. In these movements *the musculi papillares and carneæ columnæ* are said *to take no part;* the chordæ tendineæ acting *mechanically*, like so many stays, to prevent eversion of the segments in the direction of the auricles.

In the vital theory, on the other hand, the segments of the valves are supposed to be from the first *under the control of the musculi papillares;* these structures, by contracting, drawing the lips or free margins of the segments closely together in the axis of the auriculo-ventricular openings, to form two impervious cones, the apices of which project downwards into the cavities of the ventricles.

In these movements, it is maintained, *the blood takes no part*, the chordæ tendineæ, which are regarded as the proper tendons of the musculi papillares, acting as adjusters or adapters of the segments;† a function which their varying length and strength readily enables them to perform.

I need scarcely add, that these theories are diametrically opposed to each

* Dr John Reid states from experiment, that the carneæ columnæ act simultaneously with the other muscular fibres of the heart, and that the *musculi papillares* are proportionally more shortened during their contraction than the heart itself taken as a whole. He attributes this to the more vertical direction of the musculi papillares, and to their being free towards the base and in the direction of the ventricular cavities.—*Cyc. of Anat. and Phy.*, art. "Heart," p. 601. London, 1839.

† In one specimen which I dissected, the chordæ tendineæ contained a large amount of muscular fibre, and were so thickened as to resemble rudimentary musculi papillares.

other. The vital theory which was espoused by MAYO and BOUILLAUD,* has been defended with great ability by Dr JOHN REID,† who says, that if the lips of the valves were merely floated up to the orifice, a greater quantity of the blood would regurgitate into the auricles during the systole of the ventricle than if the lips were assisted or brought together by an active force. This author alludes to, and very properly attaches considerable importance to the fact, that the musculi papillares to which the valves are attached by the chordæ tendineæ, contract with the other portions of the ventricular walls. He also points out the uniform position and course of the musculi papillares and chordæ tendineæ, and shows, as bearing directly upon the question, how the chordæ tendineæ in the left ventricle pass from each musculus papillaris to both lips of the mitral valve. One statement, however, made by him requires to be noticed. When speaking of the mitral valve he says, that the lip of the posterior or smaller segment though it may be drawn inwards so as to meet that of the larger and more moveable one, *is so bound down as to be scarcely capable, in most cases, of being floated up on a level with the orifice.* I have examined a large number of hearts, young and old, human and otherwise, with a view to determine this point, and in no instance do I remember an example where the peculiarity adverted to was observable. The anterior or larger segment, from its attachments and shape, naturally rises higher, just as the anterior and septal segments of the tricuspid valve for similar reasons, rise higher than the posterior segment; but in every case, the posterior or smaller segment rises sufficiently high, not only fairly to meet the anterior and larger one, but if a sufficiency of pressure is exercised, to protrude in an upward direction, so as to form a convexity which encroaches upon the left auricular space.

In the valvular controversy, as in most others, a certain amount of truth is to be found on either side; and I have to express my conviction, that both theories (conflicting though they appear) are virtually correct as far as they go, but that neither the one nor the other is sufficient of itself to explain the gradual, and to a certain extent self-regulating process, by which the auriculo-ventricular valves are closed and kept closed. On the contrary, I believe that the closure is effected partly by *mechanical* and partly by *vital* means. In other words, that the blood towards the end of the diastole and the beginning of the systole forces the segments in an upward direction, and causes their margins and apices to be so accurately applied to each other as to prevent even the slightest regurgitation; whereas, during the systole, and towards the termination of that act, the valves are by the contraction of the musculi papillares, dragged down by

* These investigators proposed to call the musculi papillares the tensor, elevator, or adductor muscles of the valves.

† *Op. cit.* pp. 361, 362.

the chordæ tendineæ into the ventricular cavities to form two dependent cones; this downward movement of the segments permitting the blood in the auricles to descend into the ventricles, so as to relieve the congestion of the former.

Granting that the foregoing hypothesis is correct, there is yet another point *as to the manner of the closure*, to which I am particularly anxious to direct attention, as it is of primary importance, and appears to me, by some unaccountable means, to have hitherto escaped observation. I refer to the spiral form assumed by the blood in the ventricular cavities, which, as has been already partially explained, causes it, towards the end of the diastole and the beginning of the systole, to act in *spiral waves* mechanically (Plate XXIX. fig. 49 *j q*) on the segments, with the effect of *twisting and wedging them into each other in a spiral upward direction* (Plate XXIX. fig 51 *m n*, and figs. 53 and 54 *m i n*, *r s*). I allude, also, to the spiral course pursued by the musculi papillares (Plate XXIX. figs. 49 and 50 *x y*); these structures, as the systole advances, contracting in such a manner as occasions the spiral descent of the segments into the ventricular cavities (Plate XXIX. figs. 52 and 55 *m n*, *r s*), to form *two spiral dependent cones*, the apices of which are directed towards the apices of the ventricles. As the decrease of the blood in the ventricles is followed, as has been stated, by a corresponding increase in the auricles; the blood in the latter assists in keeping the free margins and apices of the segments from being everted by the uniform pressure exercised on them by the blood in the former, during the systole. From this account of the closure of the auriculo-ventricular valves, it will be perceived that the valvular segments form two moveable partitions or septa, which rise and fall during the action of the heart, in the same way that the diaphragm rises and falls during the respiratory efforts. The advantages arising from such an arrangement are very great. *When the ventricles are full of blood*, and the auricles empty or comparatively so, the valvular septa are convex towards the base of the heart, and protrude into the auricular cavities. When, however, *the auricles are full of blood*, and the ventricles all but drained of it, the valvular septa descend so as to protrude in a downward or opposite direction. Certain portions, therefore, of the auriculo-ventricular cavities are common alike to the auricles and to the ventricles; and it is important to note this fact, as the valvular septa by their rising and falling, at one time increase the size of the ventricular cavities while they diminish the auricular ones, and *vice versa*. The principal object gained by the descent of the segments into the ventricles is the diminution of the ventricular cavities towards the base; the dependent cones formed by the valves fitting accurately into the conical-shaped interspaces situated between the slanting heads of the musculi papillares and the auriculo-ventricular tendinous rings. As the musculi papillares, on the contraction of the ventricles, mutually embrace and twine round each other, the obliteration of the ventricular cavities is by this arrangement rendered very complete. "That the ventricles empty themselves during the systole, is rendered probable

from analogy, for on watching the hearts of cold-blooded animals, they are found towards the end of the contractile act to become quite pale, not, as Harvey supposed, from the blood being pressed out of the parietes, but from the blood in their cavities seen through their transparent sides being almost entirely expelled." An important inference to be deduced from the spiral nature of the ventricular fibres and ventricular cavities and *the undoubted spiral action of the auriculo-ventricular valves* is the effect produced on the blood as it leaves the ventricles, that fluid being unquestionably projected by a wringing or twisting movement, which communicates to it *a gliding spiral motion.* This view is favoured by the spiral inclination of the sinuses of VALSALVA to each other, these structures gradually introducing the blood so projected into the vessels. How far the rotatory movement referred to, extends into the arteries, is difficult to determine; but when the smooth cylindrical nature of the vessels, and the great velocity and force with which the blood travels, is taken into account, there is every reason to suppose that the distance is considerable.

The Mechanical and Vital Theories of the Action of the Mitral and Tricuspid Valves considered.

That the theory which attributes the closure of the auriculo-ventricular valves to the mechanical floating up of the segments from beneath by the blood, forced by the auricles* into the ventricles, distending equally in all directions,† is of itself inadequate to explain all the phenomena, is, I think, probable from analogy and the nature of things; for if a merely mechanical arrangement of parts was sufficient for the closure of the gradually contracting auriculo-ventricular orifices, then, it may be asked, why were these apertures in birds and mammals not furnished with sigmoid or semilunar valves similar in all respects to those met with in the veins and arteries? The answer to this question is no doubt to be found in the nature of the structures in which the valves are situated, as well as in the circulation itself. In the veins, as is well known, the movements of the blood are sluggish—the contraction of the vessels being feeble, and not consequently calculated to interfere to any great extent with the closure of the valves. In the arteries, where the circulation is more vigorous, and the contractions of the vessels more decided, the valves are surrounded by dense fibrous rings, which protect them alike from the contractions of the ventricles, and the contractility

* According to HARVEY, LOWER, SENAC, HALLER, and others, the auricles contract with a very considerable degree of energy.

† "In a quantity of fluid submitted to compression, the whole mass is equally affected and similarly in all directions."—*Hydrostatic Law.* Dr GEORGE BRITTON HALFORD attributes the closure of the auriculo-ventricular valves entirely to the pressure exercised by the auricles on the blood forced by them into the ventricles. That, however, this is not the sole cause, will be shown further on.

and elasticity of the vessels. In the bulbus arteriosus of fishes, where the area of activity of the valves is not thus circumscribed, and where they are exposed to the influence of muscular contraction, *the segments are not only increased in number, but chordæ tendineæ*, in the shape of tendinous bands, *begin to make their appearance.* In the auriculo-ventricular valves of fishes and reptiles, *chordæ tendineæ in various stages of development occur*, these being attached to the interior of the ventricle *to more or less fully developed musculi papillares;* the musculi papillares, which occur only in the reptilia, being in no instance so well marked *as in the ventricles of the aves and in the mammalia.* As we thus rise in the scale of being, and the requirements of the circulation become greater, it will be observed *that the relation of the segments to actively contracting structures,* becomes more and more defined. In the ventricle of the fish, as I pointed out, the fibres proceed in wavy lines from base to apex, and from apex to base, from without inwards and circularly; so that the organ contracts and dilates very much *as one would shut and open the hand.* In the reptilia, the external and internal fibres pursue a slightly spiral direction—*the ventricles rotating more or less when in action.* In the cold-blooded animals, moreover, as every one is aware, the circulation is languid or slow, so that an arrangement of valves similar in some respects, though more complex than that which exists in the veins and venous sinuses and in the arteries, amply suffices. In the hearts, however, of the warm-blooded animals, where the ventricles are composed entirely of spiral fibres, and where the circulation, on account *of the sudden twisting and untwisting of the fibres is very rapid*, a system of valves, which will act with greater rapidity and precision, is absolutely necessary. But functional precision implies structural excellence; and hence that exquisite arrangement of parts in the auriculo-ventricular valves of mammals, whereby every portion of every segment (by reason of the ever varying length and strength of the chordæ tendineæ) bears a graduated relation to the musculi papillares and carneæ columnæ. Although the partial closing of the valves during the diastole may be, and is occasioned by the uniform expansion of the blood owing to the force exercised upon it by the contraction of the auricles, still it must be evident to all who reflect, that this cause is not of itself adequate to the complete closure, and for a very obvious reason. The blood, which is the expanding force, derives its power solely from the contraction of the auricles, and enters the ventricular cavities by the auriculo-ventricular orifices. Once in the ventricles, however, the blood has no inherent expansive power, by which it can of its own accord entirely shut off or close, the apertures by which it entered. This act, as I shall show presently, requires for its consummation, the force exercised by the contraction of the ventricles at the commencement of the systole. Admitting, however, that the expansion of the blood was adequate to the closure of the auriculo-ventricular valves at one period,—say at the end of the ventricular diastole,

when the ventricles are full of blood and the auriculo-ventricular orifices widest,—it is scarcely possible that it could keep them closed towards the end of the systole, when the auriculo-ventricular orifices are greatly diminished in size and the blood itself all but ejected. A regulating and motor power, therefore, in addition to the blood, for adapting the different portions of the segments of the valves to the varying conditions of the auriculo-ventricular orifices and cavities during the systole, seems requisite. Such a power, in my opinion, resides in the conical-shaped spiral musculi papillares with their proper tendons—the chordæ tendineæ.

That the theory which ascribes the closing of the auriculo-ventricular valves, entirely "to the contraction of the musculi papillares," is likewise of itself insufficient, appears for the following reasons:—

1*st*, If the valves which, at the commencement of the ventricular diastole, hang free in the ventricular cavities, and are undoubtedly floated mechanically upwards, so as to have their edges approximated by the blood towards the termination of that act, were dragged upon at the instant of contraction from above downwards, or in an opposite direction to that in which the force by which they were brought together acts, the segments of the valves, instead of being further approximated, would inevitably be drawn asunder, and regurgitation to a fatal extent supervene.

2*d*, By such an arrangement as Dr HALFORD has satisfactorily shown, the cavities of the ventricles would not only be materially diminished at a very inconvenient time,* but a certain amount of the force required for the expulsion of the blood from the ventricular cavities, would be expended in closing the valves. While, therefore, it seems essential for the approximation of the auriculo-ventricular valves, that they should first ascend with the ascending columns of blood occasioned by the injection of the ventricles by the auricles, so as to have their margins gradually and accurately approximated, in order that when the contraction of the ventricles takes place, they may be instantly closed,† thereby effectually preventing regurgitation; so, for their continued closure, it seems necessary,

* Dr HALFORD states his belief, "that the segments of the valves are forced even beyond the level of the auriculo-ventricular orifices, and in this way become convex towards the auricles, and deeply concave towards the ventricles." In his zeal for the enlarged accommodation of the ventricles, he forgets that the auricles are equally entitled to consideration, and that it is unfair to give to the one and take from the other; for if, as he argues, the segments of the valves form a convex partition, whose convexity throughout the entire systole of the ventricle points in the direction of the auricles, the space beyond the level of the orifices is appropriated from the auricles without compensation. As, however, such an arrangement could not fail materially to inconvenience the auricles, when *they are fullest* of blood, we naturally turn to the ventricles for redress. The additional space required is, as I have already shown, supplied by the descent of the segments of the bicuspid and tricuspid valves towards the end of the systole when the ventricles are almost drained of blood.—*On the Time and Manner of Closure of the Auriculo-ventricular Valves.* Churchill, London, 1861.

† The margins of the segments of the valves at the end of the ventricular diastole are so close as to be nearly in contact. The slightest amount of pressure, therefore, suffices for the instantaneous closure. As, moreover, regurgitation is prevented in proportion to the rapidity with which the closure is effected, the efficiency of this arrangement is at once apparent.

in the second place, that they should descend with the rapidly decreasing columns of blood occasioned by the uniform and continued contraction of the ventricles and musculi papillares, in order that they may adapt themselves to the reduced size of the auriculo-ventricular orifices, and in order that the ventricular cavities may be diminished towards the base, as well as in every other direction. My belief consequently is, that the valves, like the ventricles themselves, have a passive and an active state; and another which, while it is neither strictly passive nor active, may, for the sake of distinction, be regarded as the neutral state. The passive state, corresponds to the diastole of the ventricles; the active state, to the systole; and the neutral or intermediate state, to that brief period which embraces the termination of the diastole and the commencement of the systole.* As, however, the action of the valves, is, to a certain extent, dependent upon, and induced by the action of the ventricles, the following slight differences as regards time, are to be noted. The passive state of the valves corresponds to that period in which *their segments are floated mechanically upwards, and their margins partially approximated* (Plate XXIX. figs. 52 and 55 *mn*, *rs*) by the blood forced by the auricles into the ventricles, during the dilatation of the latter; the neutral state, to that almost inappreciable interval which succeeds the sudden contraction of the ventricles, in which the blood set in motion is arranged in spiral columns, and acts in such a way as not only instantly closes the valves (Plate XXIX. figs. 53 and 56 *mn*, *rs*), but *screws and wedges the segments thereof, into each other*,† *in an upward spiral direction* (Plate XXIX. figs. 54 and 57 *mn*, *rs*). The active state, corresponds to the period occupied by the progressive contraction of the ventricles. During this period, *the valves are dragged forcibly downwards by the contraction of the musculi papillares, in an opposite direction to that by which they ascended; and are twisted into or round each other, to form spiral dependent cones.* In the active stage, as in the neutral, the blood acts from beneath, and keeps the delicate margins and apices of the segments of the valves, in accurate contact. That the foregoing is the true explanation of the gradual approximation and continued closure of the auriculo-ventricular valves, there can, I think, be little doubt, both from the disposition and structure of the parts, and from experiment. If, *e.g.*, the coagula be carefully removed from perfectly fresh ventricles, and two tubes of appropriate calibre be cautiously introduced past the semilunar valves, and securely fixed in the aorta and pulmonary artery,‡ and the preparation be

* In speaking of the closure of the valves, it is of great importance to remember, that the action, although very rapid, is a strictly progressive one, and necessarily consists of stages. In this, however, as in many other vital acts, it is often very difficult (if not indeed impossible) to say precisely where the one stage terminates and the other begins.

† This act takes place just before the blood finds its way into the aorta and pulmonary artery, the amount of pressure required for shutting and screwing home the auriculo-ventricular valves being less than that required for raising the semilunar ones.

‡ Strictly speaking, the tubes should be introduced into the auriculo-ventricular orifices, as it is through these apertures that the blood passes during the dilatation of the ventricles. As, however,

sunk in water until the ventricular cavities fill, it will be found, when one of the tubes,* say that fixed in the aorta, is carefully blown into, that the segments of the bicuspid valve roll up from beneath in a spiral direction (Plate XXIX. figs. 52 and 55 *rs*), in a progressive and gradual manner; each of the two larger or major segments, by folding upon itself, more or less completely, in a direction from within outwards, forming itself into a provisional or temporary cone, the apex of which is directed towards the apex of the left ventricle (*first stage, in which the crescentic margins and apices of the segments are slowly approximated by the uniform expansion of the blood forced into the ventricle by the auricle*). As the pressure exerted by the air is gradually increased, and the action of the valve is further evolved, the segments, folded upon themselves as described, *are gradually elevated, until they are on the same plane with the auriculo-ventricular fibrous ring;* where they are found to be wedged and screwed into each other, and present a level surface above (Plate XXIX. figs. 53 and 56 *rs*).

At this, the second stage of the closure, the crescentic margins of the segments are observed to be accurately applied to each other, *to form two perpendicular crescentic walls*, which accord in a wonderful manner with similar walls formed by the union of the semilunar valves; in fact, the manner of closure is, to a certain extent, the same in both; the segments in either case, being folded upon themselves by the blood, and presenting delicate crescentic margins, which are flattened against each other, in proportion to the amount of pressure employed. When the crescentic margins of the segments, are so accurately applied to each other as to become perfectly unyielding, and the distending process is carried beyond a certain point, the bodies or central portions of the segments bulge in an upward or downward direction as happens; the segments of the mitral valve, protruding into the auricle (Plate XXIX. figs. 54 and 57 *rs*); those of the semilunar one, into the ventricle (Plate XXVIII. figs. 27 and 28 *v wx*).

This completes the first and second stages of the process, by which the mitral or bicuspid valve is closed; but the more important, as being the more active and difficult, is yet to come. *This consists in adapting the segments of the valve, to the gradually diminishing auriculo-ventricular orifice; and in dragging them down into the left ventricular cavity*, to diminish the ventricle towards the base. By this act, the segments, as has been shown, are made to form a spiral dependent cone, an arrangement which renders the obliteration of the ventricular cavity towards the base, a matter of certainty.

The third stage of the closure of the mitral valve, entirely differs from the

the insertion of tubes, however small, into the auriculo-ventricular openings would necessarily prevent the complete closure of the valves, there is no good reason why the plan recommended in the text should not be adopted.

* Thin metallic tubes with unyielding parietes are best adapted for this purpose, as they can be readily fixed in the vessels, and the amount of pressure exercised by the breath on the valves asily ascertained.

first and second stages; inasmuch as the chordæ tendineæ, on the contraction of the musculi papillares, drag the segments in a downward direction, to adapt them to the altered conditions of the auriculo-ventricular orifice, and ventricular cavity. That this downward movement, actually takes place, is proved as follows. If a portion of the fluid be withdrawn by applying the mouth to the tube in the aorta, so as to create a certain amount of suction, the segments of the bicuspid valve, are found gradually to descend in a spiral direction (Plate XXIX. figs. 52 and 55 *r s*); forming, as they do so, a spiral cone, whose apex becomes more and more defined in proportion as the suction is increased; the water in the interior keeping the margins of the segments, accurately in apposition, and thereby maintaining the symmetry. If, again, the musculi papillares, be cut out of the ventricular walls and made to act in the direction of their fibres, *i. e.*, in a spiral direction from left to right downwards; they will be found, in virtue of being connected by the chordæ tendineæ more or less diagonally to either segment of the bicuspid valve, to act simultaneously on that side of the segment which is next to them; the musculus papillaris marked *a b* in fig. 31, Plate XXVIII., acting spirally on the margin and apex (*x*) of the larger or anterior segment, in the direction of the arrow near it; that marked *c d*, acting spirally on the margin and apex (*v*) of the smaller or posterior segment, in a precisely opposite direction. (Compare direction of arrows marked *a* and *d*.) The effect of these apparently incongruous movements, on the segments, is very striking.

The space which naturally exists between the segments (Plate XXVIII. fig. 31 *v x*; fig. 28 *m n*), and which corresponds to the distance between the points of origin of the two sets of chordæ tendineæ (*a* and *b*), is gradually but surely diminished, and the segments twisted into or round each other.

This arrangement, I may observe, while it facilitates the spiral movement adverted to, absolutely forbids any other.

It is rendered perfect by the pressure exercised on the delicate margins of the segments by the spiral columns of blood as already explained.

One complete closure of the mitral or bicuspid valve, may therefore be briefly stated, as follows:—

The segments, are first floated gently and gradually upwards, by the uniform expansion of the blood forced into the left ventricle, during the diastole, by the contraction of the left auricle. *This is a purely mechanical act, and during its performance the valve, and chordæ tendineæ, are entirely passive.* When, however, the ventricle suddenly contracts, the margins of the valve, which were in apposition, although not in actual contact, are rapidly approximated (the left auriculo-ventricular opening being instantly closed*); and the two spiral columns

* Regurgitation (as has been already stated) is prevented in the inverse of the rapidity with which the closure takes place.

of blood set suddenly in motion by the ventricular systole, *force the segments of the valve, in an upward spiral direction, rendering them more and more tense until they reach the level of the ventricular orifice; at which point, they are twisted and wedged into each other;* the chordæ tendineæ limiting the amount of upward motion, to prevent retroversion and regurgitation. As, however, the blood finds its way through the aorta, which it does the instant the segments of the valve are screwed home,* *the segments gradually but rapidly descend in an opposite direction to that by which they ascended, their descent being occasioned, regulated, and minutely graduated by the contraction of the musculi papillares aided by the chordæ tendineæ and by the ascending spiral columns of blood;* an arrangement which insures that the delicate margins of the segments, are always closely and accurately applied to each other; for the chordæ tendineæ and the blood in the auricles acting from above, while the spiral columns of blood in the ventricles act from beneath, the delicate margins in question, are effectually prevented from falling towards the ventricular walls.† This downward action of the valve, musculi papillares, and chordæ tendineæ, which is of essential importance in adapting the former to the diminishing condition of the left auriculo-ventricular orifice and ventricular cavity, continues until the blood is completely ejected, and the segments of the valve are twisted or plaited into each other to form a dependent spiral cone, whose apex, is directed towards the apex, of the ventricle. By the time this happens, *i. e.*, by the time the blood is ejected from the ventricle, and the cone in question fairly formed, the left auricle is distended; and due advantage being taken of the extra space afforded by the descent of the valve, the blood assumes a spiral and conical or wedge-shaped form, which is the best possible for pushing aside the segments, already in the most favourable position for falling away from the ventricular axis, towards the ventricular walls. The same phenomena are repeated with unerring regularity, with each succeeding action of the heart. What has been said of the manner of closure of the mitral or bicuspid valve, applies, I need scarcely add, with slight modifications to the tricuspid.

Resume.

The points which have been more particularly dwelt upon in the present investigation, and on which the writer has endeavoured to throw additional light, are these:—

* When the segments of the mitral valve are screwed home, the whole force of the ventricular contraction is expended in raising the aortic semilunar valve, and until the screwing home has taken place, the latter action is impossible, as the ventricle up till this point is compressible.

† The serious results which might arise from the segments of the valve falling towards the ventricular walls, or away from the axis of the cavity, is especially prevented by the attachments of the chordæ tendineæ; the principal and more internal chordæ tendineæ (Plate XXVIII. fig. 33 *r s*) being, as I have shown, attached to the backs or more external surfaces of the segments, an arrangement which makes their rapid approximation towards the ventricular axis inevitable.

First, An attempt has been made to point out the intimate structural relation, existing between the veins, and venous valves; how the segments of the venous valves are composed principally of white fibrous and yellow elastic tissue, arranged in at least three well-marked directions; and how the segments are so disposed, that their free margins, unless when the blood is actually passing between them, are always more or less in apposition. An attempt has also been made, to demonstrate the nature of the apposition, by the employment of plaster of Paris, injected while in the fluid state into the distal and proximal extremities of the vessels. In the veins, as was pointed out, *the closure is to a great extent mechanical*; the segments, when two are present, being forced together in the mesial plane of the vessel, by the contraction of its walls, but principally by the weight of the refluent blood.

Secondly, The structure and relations of the arterial or semilunar valves, particularly in man, have been examined afresh; a more precise description than that hitherto given of the segments in systematic treatises on anatomy having been essayed. The great vessels have further been shown to bifurcate at their origins, and to be greatly thickened between the segments, which they support and incline towards each other—an arrangement calculated to bring the free margins of the segments more or less closely together, unless when pushed aside by the advancing column of blood during the systole. The sinuses of Valsalva have, in addition, been shown to vary in size; the one curving towards the other in a spiral direction, and causing the blood to act in spiral waves upon the segments of the semilunar valves, which by this means are twisted and wedged into each other, when the reflux occurs. In the arteries, the action of the semilunar valves, as has been explained, *is for the most part mechanical*, the strong fibrous rings situated at the aortic and pulmonic orifices, tending to counteract the inconvenience, which might be supposed to result from an excess of vital contractility in the vessels, and the ventricles. The arterial semilunar valves, may be said to differ from the bi-semilunar venous ones, in having their segments wedged together by a spiral movement, which in the venous valves, is little more than indicated.

Thirdly, The bulbus arteriosus of fishes, has been shown to be a contractile organ, and to contain in its interior a system of valves, the segments of which, are, as a rule, more numerous than in either the veins or arteries. They have, in some instances, tendinous bands, resembling chordæ tendineæ, running between them; and are for the most part, arranged in tiers; so that the blood which is not caught by the one set, falls into, and is supported by the next. The action of these valves, as will readily be inferred, *is partly mechanical, and partly vital*; for the contraction of the bulb must be regarded as contributing to the closure. They are, therefore, an advance upon the valves of the veins and arteries, both as regards their number, and the manner of their closure.

Fourthly, In the reptiles, as has been demonstrated, *the valves are partly tendinous, and partly muscular;* while in the right ventricle of the bird, *they are altogether muscular.* Here, then, may be witnessed the first trace of a self-regulating power—*actively contracting muscular fibre*, taking the place of *non-contractile fibrous tissue.*

In the auriculo-ventricular valves, there is immense variety; these including most of the forms referred to, and others exhibiting a still higher degree of differentiation. They are, for the most part, characterised by the presence of chordæ tendineæ, which connect them with the interior of the ventricles, or the structures arising therefrom; viz. the carneæ columnæ and musculi papillares. The auriculo-ventricular valves, therefore, differ from the semilunar valves proper. In some instances, only one semilunar flap is present; and this may be either *altogether fibrous*, or *partly fibrous and partly muscular, or altogether muscular.* In a second, there are two flaps or segments, so arranged *that their long diameters correspond to the direction of the muscular fibres lining the ventricular cavity; the segments being continuous with the muscular fibres referred to.* In a third, the two segments are attached to the interior of the ventricle *by rudimentary chordæ tendineæ.* In a fourth, two accessory or smaller segments, are added to the two principal ones; *the whole being attached by well developed chordæ tendineæ to rudimentary musculi papillares.* In a fifth, which is the most perfect form of valve, as it exists in man and in the higher mammalia, the segments are from four to six in number, *most exquisitely and symmetrically formed, and attached by minutely graduated chordæ tendineæ to highly developed carneæ columnæ and spiral musculi papillares.*

The action of the auriculo-ventricular valves, owing to the want of uniformity in the number, structure, and relations of their segments, is varied. It is, however, on all occasions, carefully adapted to the wants of the circulation, and to the configuration of the ventricles and ventricular cavities; these cavities, as has been pointed out, adapting or moulding the blood, and causing it to act in given directions. Thus in the fish, where the circulation is slow, and where the ventricle is conical in shape, and composed of fibres interlacing in all directions, the segments, where two are present, are forced towards each other by the uniform expansion of the blood, and by the contraction of the ventricles, in a manner analogous to that by which the segments of the bi-semilunar venous valves are approximated by the retrogressive movements of the slowly advancing venous blood, assisted to a slight extent by the vital contractility of the vessels.

In the reptile, where the circulation is also languid or slow, the shape of the ventricle, owing to the fibres pursuing a more or less spiral direction, is that of a cone slightly twisted upon itself. As the spiral arrangement extends also to the valves, their action may be aptly compared to that which obtains in the valves of the largest veins, and in the arteries. It is, however, in the

auriculo-ventricular valves of the bird and mammal, that the spiral action of the segments becomes most conspicuous; the nature of the action being unavoidably determined, by the unmistakably spiral arrangement of the muscular fibres composing the ventricles, and by the spiral nature of the musculi papillares and ventricular cavities. As, however, the action of these valves, has been already explained at great length, further allusion to them at this stage is unnecessary.

The valves of the vascular system of the vertebrata, as will be perceived from this summing up, form a progressive and gradually ascending series; the valves in the veins exhibiting a lower type than those in the arteries; the valves in the arteries, being less fully developed than the valves occurring in the bulbus arteriosus and in the auriculo-ventricular orifice of the fish; the valves in the fish, being less highly differentiated than the valves in the reptile and bird; these again falling short both in complexity and adaptive power to those met with in the mammal. In the mammal the valvular arrangements may be said to culminate.

Description of the Plates.

Plate XXVIII.

Figs. 1 and 2. External Jugular Veins of Horse inverted. Show valves, consisting of two (*d e*), three (*a b c*), and four (*f g h*) segments. (See pp. 763, 764.)

Fig. 3. Section of External Jugular Vein of Horse. Shows valve, consisting of two segments (*a b*), with dilatations (*g*), corresponding to the sinuses of Valsalva, in the arteries. (See pp. 763, 764, 767.)

Fig. 4. External Jugular Vein of Horse opened. To show the relations of the segments (*a b*) above (*r e*). (See p. 763.)

Fig. 5. Portion of Femoral Vein distended with plaster of Paris. Shows dilatations (*h g*) in the course of the vessel corresponding to the position of the valve. (See p. 765.)

Fig. 6. Shows Venous Valve, consisting of two segments (*a b*), in action. (See p. 767.)

Fig. 7. The same, not in action. (See p. 763.)

Fig. 8. Venous Valve from External Jugular of Horse, consisting of three segments. (See p. 764.)

Fig. 9. Venous Valve, consisting of one segment, situated at the entrance of a smaller into a larger vein. (See p. 763.)

Fig. 10. Venous Sinus from Auricle of Heart of Sturgeon. (See p. 782.)

Fig. 11. Femoral Vein distended with plaster of Paris. Shows venous valves in action, where a smaller vessel enters the larger one (*a b*), and in the main trunk (*a' b'*). (See pp. 767, 768.)

Fig. 12. Vertical Section of Vein distended with plaster of Paris. Shows the nature of the union between the segments (*e*). (See p. 767.)

Fig. 13. The same, the section being carried between (*e*) instead of across or through the segments. (See p. 767.)

Figs. 14, 15, and 16. Show the Structure of the Venous Valves. (See p. 766.)

Fig. 17. Section carried through Pulmonary Artery and Right Ventricle of Human Heart, between the segments of the semilunar valves (*s*). Shows the variation in the thickness of the vessel (*a b*), and how it bifurcates (*r r'*) at its origin. (See p. 770.)

Fig. 18. A similar section, carried through the middle of one of the segments (*s*). Shows how the Pulmonary Artery (*a b*) behind the segments diminishes in thickness in a direction from above downwards (*i*). See p. 770.)

Figs. 19, 20, 21, 22, 23, and 24, Show the Structure of the Semilunar Valves in the Human Pulmonary Artery. (See pp. 772, 773, 774, 775.)

Fig. 25. Human Semilunar Valve distended with plaster of Paris, and one of the segments (g) removed, to show the precise shape of the lunulæ, or opposing surfaces ($b\,b'$), between which union takes place when the valve is in action. (See pp. 776, 777.)

Fig. 26. Shows the spiral relation of the Sinuses of Valsalva to the segments ($v\,w\,x$) of the semilunar valve in the Human Pulmonary Artery, and how the segments are spirally wedged into each other, and fixed by six non-symmetrical semilunar surfaces, to form three perpendicular crescentic walls ($r\,s\,o$). Seen from beneath. (See pp. 776, 777.)

Fig. 27. Shows the Semilunar Valves in the Heart of the Sheep ($v\,w\,x$) distended with plaster of Paris, or as they appear in action, together with the spiral nature of that action. (See pp. 776, 777.)

Fig. 28. Shows the same in the Aorta ($v\,w\,x$) of the Human Heart, and in addition, the bifid nature of the musculi papillares ($a\,b$, $c\,d$), and the distribution of the chordæ tendineæ to both segments of the mitral valve ($m\,n$). (See pp. 776, 777, 787, 788, 789.)

Fig. 29. Vertical Section carried through the Aorta and through the middle of one of the segments of the Semilunar Valve of the Whale. Shows the variation in the thickness of the vessel ($c\,e$), and the structure of the semilunar valve ($a\,r\,s'\,o$). (See pp. 770, 773, 774.)

Fig. 30. Shows Arterial and Auriculo-ventricular Orifices, with their fibrous rings ($x\,v\,n\,n'$). (See pp. 769, 770.)

Fig. 31. Human Mitral Valve, Chordæ tendineæ, and Musculi papillares inverted. Shows the bifid nature of the musculi papillares ($a\,b$), and the threefold distribution of the chordæ tendineæ (s). (See pp. 786, 787, 788, 788.)

Fig. 32. Shows Structure of the Mitral Valve in the Sheep. (See pp. 786, 787, 788, 789.)

Fig. 33. Anterior Segment of Human Mitral Valve. Shows the threefold distribution of the chordæ tendineæ, from above downwards, and from the mesial line towards the margin of the segments. (See pp. 786, 787, 788, 789.)

Fig. 34. Example of Mitral Valve in the Right Ventricle of the Human Heart. (See pp. 789, 790, 791.)

Fig. 35. Vertical Section, carried through the aorta ($a\,a'$) semilunar valve ($s\,s'\,s''$), left auricle ($d\,d'$, $y\,y'$), the segments of the mitral valve ($m\,m'$, $n\,n'$), and the left ventricle ($v\,v'$), of the Human Heart. Shows the greater thickness of the aorta, where the segments of the semilunar valve unite above ($b\,b'$, $c\,c'$), and its greater tenuity behind the centre of each segment (o); also how a portion of the aorta (p) is continued into the larger or anterior segment of the mitral valve ($m\,m'$). It also shows the relation of the left auricle ($y\,y'$, $d\,d'$) to the aorta ($a\,a'$), mitral valve ($m\,m'$, $n\,n'$), and left ventricle ($v\,v'$).

$r\,r'$, Openings of coronary arteries.
$n\,n'$, Segments of semilunar valve uniting above.
$x\,x'$, $e\,e'$, Auricle terminating by wedge-shaped process in mitral valve.
$z\,z'$, Left coronary artery.
$t\,t'\,t''$, Convex, or attached borders of segments of semilunar valve.
g, Septal wall of left ventricle.
$h\,h'$, Chordæ tendineæ.
$i\,i'$, Musculi papillares.
w', Apex of left ventricle. (See pp. 770, 771).

Fig. 36. Left Ventricle of Human Heart laid open to show the semilunar ($s\,s'$) and mitral (m) valves *in situ*. (See pp. 770, 771.)

Plate XXIX.

Fig. 37. Portion of Heart of Sturgeon. Shows auriculo-ventricular valve (a) with three chordæ tendineæ (b) proceeding from it. (See p. 781.)

Fig. 38. Ventricle and Bulbus Arteriosus of Skate laid open. Shows several rows of semilunar valves ($a\,b\,c$) increasing in size in a direction from below upwards. (See p. 779.)

Fig. 39. Bulbus Arteriosus and Ventricle of Sturgeon,—the former displaying five rows of semilunar valves (*a*), the latter an auriculo-ventricular valve (*b*), with numerous tendinous bands running into it. (See p. 779.)

Fig. 40. Bulbus Arteriosus and Portion of Ventricle of Lepidosteus. Shows the great thickness of the bulb (*a*), and of the valves (*b*) contained within it, and between which tendinous bands run. (See p. 780.)

Fig. 41. Portion of Bulbus Arteriosus of Basking Shark. Shows the great thickness of the bulb (*a*) and of the valves (*b*), and how the latter support each other. (See p. 780.)

Fig. 42. Heart of Crocodile. Shows spiral semi-muscular, semi-tendinous valve (*r*), situated between right auricle (*a*) and ventricle (*x*). (See p. 781.)

Fig. 43. Bulbus Arteriosus and Ventricle of Sun-fish. Shows three semilunar valves (*a b c*) at orifice of bulb, and an auriculo-ventricular valve (*f*), consisting of two segments. (See pp. 779, 781.)

Fig. 44. Heart of Serpent (*Python tigris*). Shows muscular semilunar valve at orifice of left superior cava (*s*); also spiral muscular slit (*r*), occurring between the ventricles. (See pp. 781, 782).

Fig. 45. Heart of Emu, showing spiral muscular valve (*g h*), occurring in right auriculo-ventricular orifice. (See p. 781.)

Fig. 46. Heart of Swan, with right and left ventricles laid open. Shows spiral muscular valve of right ventricle (*i*), and mitral valve (*v*) of left, and how one portion of the former (*j*) corresponds in position to the anterior musculus papillaris (*y*) of the latter. (See p. 781.)

Fig. 47. Heart of Frog-fish. Shows three sets of semilunar valves; one occurring where the large veins join the auricle (*c*); a second, where the auricle opens into the ventricle (*b*); the third being situated at the orifice of the bulbus arteriosus (*a*). (See p. 779.)

Fig. 48. Bulbus arteriosus and portion of Ventricle of Grey Shark. Shows semilunar valves, with tendinous chords running between them (*a*); also, auriculo-ventricular valve (*g*). (See pp. 779, 780.)

Fig. 49. Wax cast of Left Ventricle (*b*) and portion of Right Ventricle (*a*) of Deer. Shows spiral nature of the left ventricular cavity,—the spiral course or tracks of the musculi papillares (*x y*), and how between these, two spiral grooves (*j q*) occur, which direct the blood on to the segments of the mitral valve in spiral waves. (See pp. 784, 785.)

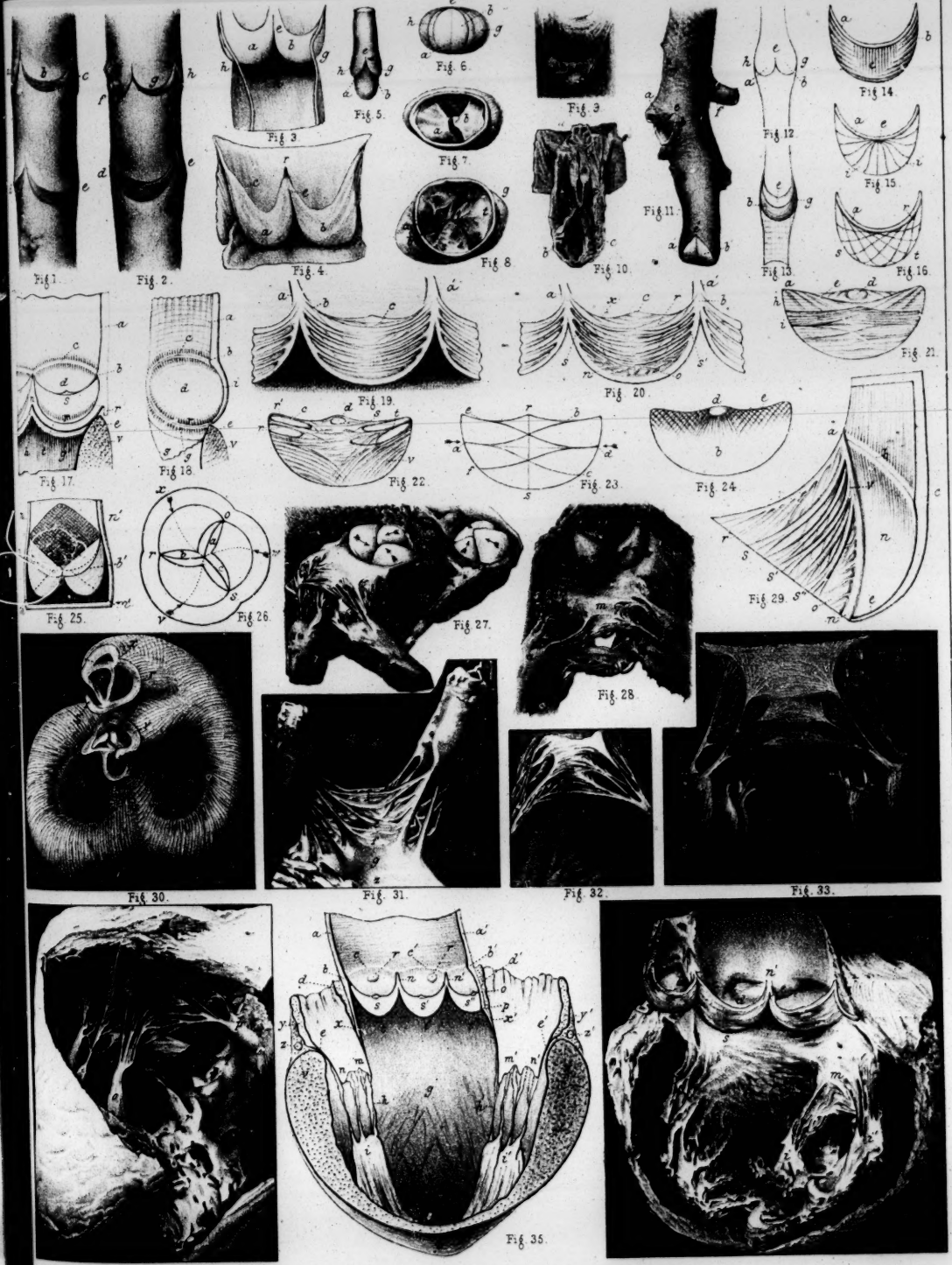
Fig. 50. Plaster of Paris cast of Right and Left Ventricles of Zebra. Shows infundibulum or conus arteriosus (*i*) of right ventricle, and analogous portion of left ventricle (*p o*); also three prominences on each (*d e k r v*), corresponding to the sinuses of Valsalva. It also shows the double cone formed by the left ventricular cavity, the one apex pointing towards the apex of the heart (*j*), the other towards the aorta (*h*). (See pp. 784, 785.)

Fig. 51. Same cast seen posteriorly. Shows the mitral (*m n*) and tricuspid (*g h*) valves in action, and how the blood, when these are closed, assumes a conical form (*o*) for pushing aside the segments of the semilunar valves, and causing them to fall back upon the sinuses of Valsalva (*v w*). It also shows how the right ventricular cavity (*c*) curves round the left one (*x*), and how the pulmonary artery (*b*) and aorta (*h*) pursue different directions. (See pp. 784, 785.)

Figs. 52, 53, and 54. Show the Mitral (*r s*) and Tricuspid (*m i n*) Valves of the Sheep in action. How the segments, acted upon by the spiral columns of blood, *roll up* from beneath towards the end of the diastole (fig. 52); how, at the beginning of the systole, they are wedged and twisted into each other, on a level with the auriculo-ventricular orifices (fig. 53); and how, if the pressure exerted be great, they project into the auricular cavities (fig. 54). (See pp. 796, 797, 798, 799, 800, 801.)

Figs. 55, 56, and 57. Show the same in the Human Heart, with this difference, that in the right ventricle, a true mitral valve (*m n*), as not unfrequently happens, has taken the place of the tricuspid. (See pp. 796, 797, 798, 799, 800, 801.)

Note.—The spiral *downward* movement of the mitral and tricuspid valves has only been partially represented (figs. 52 and 55) owing to the great difficulty experienced in representing spiral cavities.



Plates by W S Ayling and the Author Drawings by the Author

W H M.c Farlane, Lith.r Edin.r

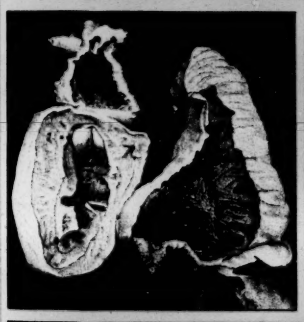
Fig. 37.

Fig. 38.

Fig. 39.

Fig. 40.

Fig. 41.



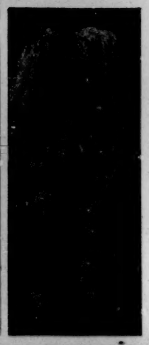
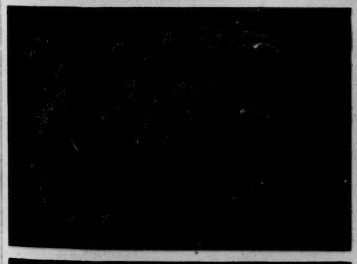
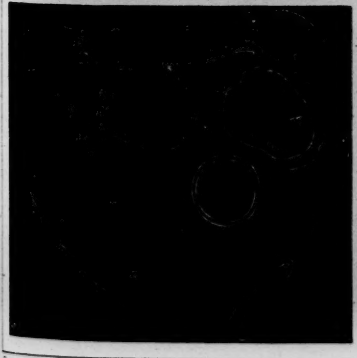
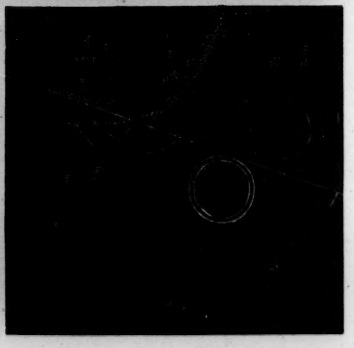


Fig. 50.

Fig. 51.







From Photo by W.S. Ayling and the Author. Drawings by H.S. Wilson M.D.

W.H. M.c Farlane, Lith.r Edin.r

ERRATA.

Page 21, line 4 from bottom, *for* value v *read* value of v.

„ 23, line 7 from top, *for* $(\cos t + \sin t)$ *read* $(\sin t - \cos t)$.

„ — line 4 from bottom, *for* e^{-t} *read* e^{-2t}.

„ — line 3 from bottom, the first *minus* sign in the line should be *plus*.

„ 26, line 2 from bottom, *for* 23° 23′ *read* 66° 37′.

„ 122, line 5 from top, *for* chemical *read* clinical.

„ 130, line 6 from top, *for* in his "Treatise upon Vital Motions," "where, *read* "in his 'Treatise upon Vital Motions' where.

„ 633, line 23 from top, *for* treatise *read* charge.

„ 644, line 2 from bottom, *for* 1778 *read* 1781.

„ 714, line 5 from bottom, *for* two primary affinities *read* two secondary affinities.

PROCEEDINGS

OF THE

STATUTORY GENERAL MEETINGS,

AND

LIST OF MEMBERS ELECTED AT THE ORDINARY MEETINGS,

SINCE JANUARY 6, 1862;

WITH

LIST OF DONATIONS TO THE LIBRARY,

FROM NOV. 25, 1861, TILL NOV. 23, 1864.

PROCEEDINGS, &c.

Monday, November 25, 1861.

At a Statutory General Meeting, Dr CHRISTISON, V.P., in the Chair, the Minutes of the General Meeting of 26th November 1860 were read and confirmed.

The following Office-Bearers were duly elected:—

His Grace the DUKE OF ARGYLL, K.T., President.
Sir DAVID BREWSTER, K.H.,
Dr CHRISTISON,
Professor KELLAND,
Hon. Lord NEAVES,
The Very Rev. Dean RAMSAY,
Principal FORBES,
} Vice-Presidents.
Dr JOHN HUTTON BALFOUR, General Secretary.
Dr LYON PLAYFAIR, C.B.,
Dr GEORGE JAMES ALLMAN,
} Secretaries to the Ordinary Meetings.
J. T. GIBSON-CRAIG, Esq., Treasurer.
Dr DOUGLAS MACLAGAN, Curator of Library and Museum.

COUNCILLORS.

Dr LOWE.	Dr SCHMITZ.
Professor W. J. M. RANKINE.	Dr SELLER.
JAMES DALMAHOY, Esq.	E. W. DALLAS, Esq.
Dr JOHN BROWN.	Rev. L. S. ORDE.
Professor FRASER.	Professor TAIT.
JAMES LESLIE, Esq., C.E.	JOHN MUIR, D.C.L.

The following Gentlemen were, on the motion of Dr BURT, appointed to audit the Treasurer's Accounts:—

Dr SELLER. A. BRYSON, Esq. W. T. THOMSON, Esq.

The Meeting then adjourned.

(Signed) R. CHRISTISON, *V.P.*

Monday, January 6, 1862.

At a meeting of the Society, held on Monday 6th January 1862, the Hon. Lord NEAVES in the Chair, the following Address to Her Majesty, expressing the sorrow of the Society on the death of H.R.H. the Prince Consort, was read and adopted :—

TO THE QUEEN.

May it please your Majesty,

We, the President and Fellows of the Royal Society of Edinburgh, beg leave most humbly to offer to your Majesty the sincere expression of our condolence for the great bereavement which your Majesty has sustained, and of our grief for the great loss which has befallen the nation by the death of His Royal Highness the Prince Consort, who, by his exemplary discharge of every duty, by his sympathy with the feelings, and his solicitude for the welfare, of his adopted country, and by his unwearied efforts to promote the progress of Learning, Science, Industry, and Art, had won for himself the just admiration, esteem, and gratitude of the whole community.

That in this heavy affliction, your Majesty may be supported by the consolations of religion, by the honoured memory of the departed, by the reverence and affection of your Majesty's children, and by the devotion of a loyal and united people, who recognise in your Majesty's government the great source and security, under Providence, of the high degree of prosperity and happiness which they enjoy, is the prayer of your Majesty's most dutiful subjects and servants.

Signed in name and by authority of this Society, by his Grace the Duke of ARGYLL, President.

(Signed) R. CHRISTISON, *V.P.*

Monday, January 20, 1862.

At a meeting of the Society, held on Monday 20th January 1862, JAMES T. GIBSON-CRAIG, Esq., Treasurer, in the Chair, the following reply to the Address to Her Majesty was read :—

WHITEHALL, 13*th January* 1862.

MY LORD DUKE,

I have the honour to acknowledge the receipt of the loyal and dutiful Address of the President and Fellows of the Royal Society of Edinburgh, on the occasion of the death

of His Royal Highness the Prince Consort, and to inform your Grace, that I shall take an early opportunity of laying the Address before Her Majesty.

I have the honour to be,

My Lord Duke,

Your Grace's obedient Servant,

(Signed) G. GREY.

His Grace the Duke of ARGYLL, &c. &c.

(Signed) R. CHRISTISON, *V.P.*

Monday, November 25, 1862.

At a Statutory General Meeting, Dr CHRISTISON, V.P., in the Chair, the Minutes of the General Meeting of 25th November 1861, and those of the 6th and 20th January 1862, were read and confirmed.

The following Office-Bearers were duly elected :—

His Grace the DUKE OF ARGYLL, K.T., President.
Sir DAVID BREWSTER, K.H.,
Dr CHRISTISON,
Professor KELLAND,
Hon. Lord NEAVES,
Principal FORBES,
Professor COSMO INNES,
} Vice-Presidents.
Dr JOHN HUTTON BALFOUR, General Secretary.
Dr LYON PLAYFAIR, C.B.,
Dr GEORGE JAMES ALLMAN,
} Secretaries to the Ordinary Meetings.
J. T. GIBSON-CRAIG, Esq., Treasurer.
Dr DOUGLAS MACLAGAN, Curator of Library and Museum.

COUNCILLORS.

Professor FRASER.	Professor TAIT.
JAMES LESLIE, Esq., C.E.	JOHN MUIR, D.C.L.
Dr SCHMITZ.	A. CAMPBELL SWINTON, Esq.
Dr SELLER.	Dr WILLIAM ROBERTSON.
E. W. DALLAS, Esq.	Dr E. RONALDS.
Rev. L. S. ORDE.	T. C. ARCHER, Esq.

The following Gentlemen were, on the motion of Dr BURT, seconded by Professor KELLAND, appointed to audit the Treasurer's Accounts:—

JAMES CUNNINGHAM, Esq. Dr WILLIAM SELLER. Sheriff CLEGHORN.

The Meeting then adjourned.

(Signed) R. CHRISTISON, *V.P.*

Monday, April 20, 1863.

At a meeting of the Society, held on Monday the 20th April 1863, Principal FORBES in the Chair, the following Address to His Royal Highness the PRINCE OF WALES was read and adopted:—

TO HIS ROYAL HIGHNESS THE PRINCE OF WALES.

May it please your Royal Highness,

We, the President and Fellows of the Royal Society of Edinburgh, desire humbly to approach your Royal Highness, with the expression of our dutiful and heartfelt congratulation on your Royal Highness's marriage.

Ever ready to rejoice at whatever affords a prospect of increased happiness to your Royal Highness, and a further security for the continued sway of a Royal House which has conferred on this realm so many benefits and blessings, we hail with especial interest and gratification the union of your Royal Highness with the Daughter of an ancient nation, distinguished at all times for noble and generous qualities, and which holds a high place among the countries of Europe in literature and science; and, above all, we regard it as an unspeakable boon, that the Royal Lady whom we now welcome to our shores is endowed with all the virtues and attractions which are best calculated to bless and adorn domestic life, to assist in cheering the widowed solitude of our beloved Sovereign, and to sustain in unsullied lustre the honour and dignity of the British Court.

We earnestly hope and pray, that this auspicious alliance may be productive of all the happiness with which we wish to see it attended.

(Signed) R. CHRISTISON, *V.P.*

Monday, November 23, 1863.

At a Statutory General Meeting, Professor CHRISTISON, V.P., in the Chair; the Minutes of the last General Statutory Meeting, and those of 20th April 1863, were read and confirmed.

The following Office-Bearers were then elected for 1863–64:—

His Grace the DUKE OF ARGYLL, K.T., President.
Sir DAVID BREWSTER, K.H.,
Dr CHRISTISON,
Professor KELLAND,
Hon. Lord NEAVES,
Principal FORBES,
Professor COSMO INNES,
} Vice-Presidents.
Dr JOHN HUTTON BALFOUR, General Secretary.
Dr LYON PLAYFAIR,
Dr GEORGE JAMES ALLMAN,
} Secretaries to Ordinary Meetings.
DAVID SMITH, Esq., Treasurer.
Dr DOUGLAS MACLAGAN, Curator of Library and Museum.

COUNCILLORS.

E. W. DALLAS, Esq.	T. C. ARCHER, Esq.
Rev. L. S. ORDE.	W. F. SKENE, Esq.
Professor TAIT.	A. KEITH JOHNSTON, Esq.
A. CAMPBELL SWINTON, Esq.	Rev. Dr STEVENSON.
Dr WILLIAM ROBERTSON.	Dr STEVENSON MACADAM.
Dr E. RONALDS.	Hon. Lord JERVISWOODE.

The Chairman read the following intimation from the Council:—

"The Council consider it expedient that one Vice-President should annually be removed from the list of Vice-Presidents, and that he should be ineligible for election for twelve months. On submitting future lists to the Society, the Council propose to omit the name of the senior Vice-President in the list for each year, and to submit another name in substitution."

On the motion of Dr BURT, seconded by Dr SELLER, Mr W. T. THOMSON, Mr JAMES CUNNINGHAM, and Dr WILLIAM ROBERTSON were appointed a Committee to examine the Treasurer's accounts.

The following letters from His Royal Highness the PRINCE OF WALES were read:—

MARLBOROUGH HOUSE, *July* 16, 1863.

Lieut.-General KNOLLYS has been desired by His Royal Highness the PRINCE OF WALES, to thank the President and Fellows of the Royal Society of Edinburgh for their Address on the occasion of His Royal Highness's marriage,—for their congratulations,—and for the gratifying sentiments they have expressed towards the Princess.

To His Grace the DUKE OF ARGYLL, President.

ABERGELDIE, BALLATER, ABERDEENSHIRE.

MY LORD DUKE,

I have the honour to acknowledge, by command of the PRINCE OF WALES, the receipt of the diploma constituting His Royal Highness an HONORARY Fellow of the Royal Society of Edinburgh; and am further commanded to convey to you His Royal Highness's extreme gratification at having received the same.

I have the honour to be,

My Lord Duke,

Your very obedient Servant,

W. KNOLLYS, Lieut.-Gen.

The meeting then adjourned.

To His Grace the DUKE OF ARGYLL.

LIST OF MEMBERS ELECTED.

January 6, 1862.

Rev. William G. Blaikie. Henry Cheyne, Esq., W.S.

January 20, 1862.

Alex. Mackenzie Edwards, Esq.

February 3, 1862.

Dr Walter Boyd M'Kinlay. Dr John P. Macartney.
Dr Edmund Ronalds.

February 17, 1862.

Thomas C. Archer, Esq. Rev. V. Grantham Faithfull.
Dr James Hector.

March 17, 1862.

Nicholas Alex. Dalzell, Esq.

April 7, 1862.

Hon Lord Barcaple. Rev. Robert Boog Watson.

December 1, 1862.

Robert Campbell, Esq. Dr H. F. C. Cleghorn.
Professor Blackie.

January 5, 1863.

Edward Meldrum, Esq. Charles Lawson, Esq.
James Hannay, Esq. Dr Alexander Peddie.

January 19, 1863.

The Right Hon. Lord Dunfermline. William Jameson, Esq.
William Brand, Esq. Dr Murray Thomson.

February 2, 1863.

Dr John Young. David Page, Esq.

February 16, 1863.

Dr J. G. Wilson. Dr J. Matthews Duncan.
George R. Maitland, Esq. W. Dittmar, Esq.

March 2, 1863.

Rev. Dr Nisbet.

March 16, 1863.

Hon. Lord Ormidale. Professor J. D. Everett.

April 6, 1863.

Hon. G. Waldegrave Leslie. Hon. Charles Baillie (Lord Jerviswoode).
James Sanderson, Esq.

April 20, 1863.

Charles Cowan, Esq. Dr John Alex. Smith.

December 7, 1863.

Dr Alex. Crum Brown. Dr Alex. Wood.

December 21, 1863.

Dr Andrew Wood. Robert William Thomson, Esq., C.E.

January 4, 1864.

James David Marwick, Esq.

January 18, 1864.

Rev. D. F. Sandford. Robert S. Wyld, Esq.

February 1, 1864.

Peter M'Lagan, Esq. William Lindsay, Esq.
Professor W. Y. Sellar. Robert Hutchison, Esq.
Rev. John Hannah, D.D., re-elected.

February 15, 1864.

William Wallace, Ph.D.

March 7, 1864.

Professor Robert Dyce, M.D.

May 2, 1864.

Arthur Abney Walker, Esq. Dr John Foulerton.

LIST OF THE PRESENT ORDINARY MEMBERS,

Corrected up to November 1, 1864

IN THE ORDER OF THEIR ELECTION.

His Grace the DUKE OF ARGYLL, K.T.,

PRESIDENT.

Date of Election.

1808 James Wardrop, Esq., London.
Sir David Brewster, K.H., LL.D., F.R.S. Lond., *Principal of the University of Edinburgh.*
1812 Sir George Clerk, Bart., F.R.S. Lond.
1815 Henry Home Drummond, Esq., *of Blair-Drummond.*
William Thomas Brande, F.R.S. Lond., *Professor of Chemistry in the Royal Institution.*
1818 Patrick Miller, M.D., *Exeter.*
1820 Charles Babbage, F.R.S. Lond.
Sir John F. W. Herschel, Bart., F.R.S. Lond.
Dr William Macdonald, *Professor of Natural History, St Andrews.*
1821 John Cay, Esq., *Advocate.*
Robert Kaye Greville, LL.D.
Robert Hamilton, M.D.
1822 James Smith, Esq., *of Jordanhill*, F.R.S. Lond.
William Bonar, Esq.
George A. Walker-Arnott, LL.D., *Professor of Botany, Glasgow.*
Sir James South, F.R.S. Lond.
Sir W. C. Trevelyan, Bart., *Wallington, Northumberland.*
1823 Captain Thomas David Stuart, *of the Hon. East India Company's Service.*
Warren Hastings Anderson, Esq.
Alexander Thomson, Esq., *of Banchory.*
Liscombe John Curtis, Esq., *Ingsdon House, Devonshire.*
Robert Christison, M.D., *Professor of Materia Medica.*

Date of Election.

1824 Robert E. Grant, M.D., *Professor of Comparative Anatomy, University College, London.*
Rev. Dr William Muir, *one of the Ministers of Edinburgh.*
1827 John Gardiner Kinnear, Esq.
Very Rev. Edward Bannerman Ramsay, A.M. Camb., LL.D.
1828 David Maclagan, M.D.
Sir William A. Maxwell, *of Calderwood*, Bart.
John Forster, Esq., *Architect, Liverpool.*
Thomas Graham, M.A., D.C.L., *Master of the Mint, London.*
David Milne-Home, Esq., *Advocate, of Milne-Graden and Wedderburn.*
Dr Manson, *Nottingham.*
1829 A. Colyar, Esq.
Sir William Gibson-Craig, Bart., *of Riccarton.*
Right Honourable Duncan M'Neill, *Lord Justice-General.*
Venerable Archdeacon Sinclair, *Kensington.*
James Walker, Esq., W.S.
1830 J. T. Gibson-Craig, Esq., W.S.
Sir Archibald Alison, Bart., *Sheriff of Lanarkshire.*
James Syme, Esq., *Professor of Clinical Surgery.*
Thomas Barnes, M.D., *Carlisle.*
1831 James D. Forbes, D.C.L., F.R.S. Lond., *Principal of the United College, St Andrews.*
1832 Montgomery Robertson, M.D.
1833 Rear-Admiral Sir Alexander Milne, R.N.
His Grace the Duke of Buccleuch, K.G., *Dalkeith Palace.*
David Craigie, M.D.
Sir John Stuart Forbes, Bart., *of Pitsligo.*
Alexander Hamilton, LL.B., W.S.
1834 Mungo Ponton, Esq., W.S., *Clifton, Bristol.*
Isaac Wilson, M.D., F.R.S. Lond.
Patrick Boyle Mure Macredie, Esq., *Advocate, of Perceton.*
William Sharpey, M.D., *Professor of Anatomy, University College, London.*
1835 John Hutton Balfour, A.M., M.D., F.R.S. Lond., *Professor of Botany.*
William Brown, Esq., F.R.C.S.
R. Mayne, Esq.
1836 David Rhind, Esq., *Architect.*
1837 John Archibald Campbell, Esq., W.S.
John Scott Russell, Esq., A.M., *London.*
Charles Maclaren, Esq.
Archibald Smith, Esq., M.A., Camb. *Lincoln's Inn, London.*
Richard Parnell, M.D.
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